29:129 Notes for Nov. 5, 2012

Example (a).—A spherical conducting shell of radius R is surrounded by a dielectric material out to a radius a. The conductor has a uniform surface charge density σ. Find **D** and **E**.

for r < R , E = 0.

Due to the symmetry in the problem we can use Gauss's law to get D from the free charge.

$$\int_S \vec{D}\cdot d\vec{a} = Q_f \rightarrow D(4\pi r^2) = \sigma 4\pi R^2 \rightarrow \vec{D} = \sigma\left(\frac{R}{r}\right)^2 \hat{r}$$

for r > a, P = 0, so $\vec{E} = \frac{\sigma}{\varepsilon_o}\left(\frac{R}{r}\right)^2 \hat{r}$. Inside the dielectric we cannot find E because we do not know P.

Example (b).—A think spherical dielectric shell of inner radius a and outer radius b has a polarization $\vec{P} = \frac{k}{r}\hat{r}$ and no free charge. Find E everywhere.

<u>Method 1</u>: from the bound charges:

$$\sigma_b(a) = \frac{k}{a}\hat{r}\cdot(-\hat{r}) = -\frac{k}{a}; \quad \sigma_b(b) = \frac{k}{b}\hat{r}\cdot(\hat{r}) = \frac{k}{b} \quad \rho_b = -\nabla\cdot\vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 P_r\right) = -\frac{k}{r^2}$$

For r < a, E = 0, for r > b,

$$E = \frac{Q_{b,total}}{4\pi\varepsilon_o r^2}, \quad Q_{b.total} = Q_b(a) + Q_b(b) + \int_a^b \rho_b 4\pi r^2 dr = 0 \Rightarrow E = 0.$$

For a < r < b (inside the dielectric),

$$\int_S \vec{E}\cdot d\vec{a} = Q_S/\varepsilon_o \rightarrow \varepsilon_o E\left(4\pi r^2\right) = \sigma_b(a) 4\pi a^2 + \int_a^r \rho_b 4\pi r^2 dr \Rightarrow \vec{E} = -\frac{k}{\varepsilon_o r}\hat{r}.$$

<u>Method 2</u>: using D. Since $Q_f = 0$, D = 0 everywhere, so that $\vec{E} = -\left(\vec{P}/\varepsilon_o\right)$.

For r < a, P = 0, so E = 0; for a < r < b, $\vec{P} = \left(k/r\right)\hat{r} \Rightarrow \vec{E} = -\left(k/\varepsilon_o r\right)\hat{r}$.