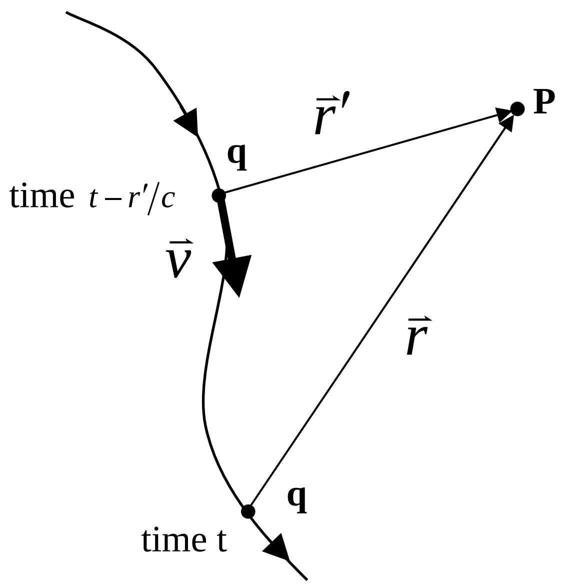
Electric and magnetic fields of a point charge moving in an arbitrary manner*



A point charge q moves in an arbitrary manner. We want to write down the electric and magnetic fields at point P at time t. Now, electromagnetic disturbances propagate at the speed of light c, and therefore the fields at time t are determined by the position of the charge at the earlier time t – r'/c, which is called the 'retarded' time. r'/c is the time delay for information to travel from the position of the particle at t – r'/c to point P.

$$\vec{E}_P(t) = \frac{q}{4\pi\varepsilon_o}\left[\frac{\hat{r}'}{r'^2} + \frac{r'}{c}\frac{d}{dt}\left(\frac{\hat{r}'}{r'^2}\right) + \frac{1}{c^2}\frac{d^2}{dt^2}\left(\hat{r}'\right)\right]$$

$$\hat{r}' = \frac{\vec{r}'}{r'}$$

$$\vec{B}_P(t) = \frac{1}{c}\left(\hat{r}' \times \vec{E}\right)$$

The first term is just the field of a point charge –Coulomb's law. The second term is the correction to the Coulomb field when the charge is moving. A moving point charge produces both an electric and magnetic field, and the magnetic effects are contained in the second term. The third term in the equation for the electric field is the one that gives rise to electromagnetic radiation (e.g. light waves). This term varied as 1/r'. The magnetic field due to this term also varied as 1/r', so their product varies as $1/r'^2$. The first and second terms go to zero faster as r' goes to infinity, so only the last term remains finite --- these are the radiation fields.

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*Reference: Feynman Lectures, Vol 2, Ch 28.