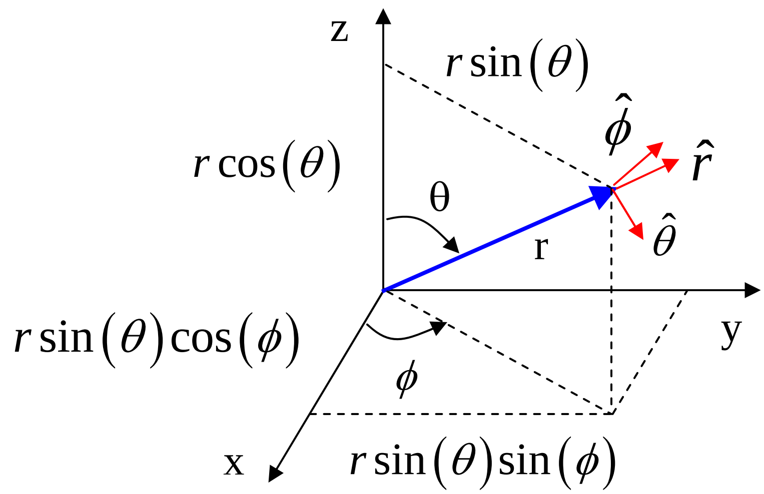
Spherical Coordinates



$$0 \le r \le \infty; \quad 0 \le \theta \le \pi; \quad 0 \le \phi \le 2\pi$$

$\hat{r}$, $\hat{\theta}$, *and* $\hat{\phi}$ are not fixed unit vectors

$$x = r\sin(\theta)\cos(\phi)$$

$$y = r\sin(\theta)\sin(\phi)$$

$$z = r\cos(\theta)$$

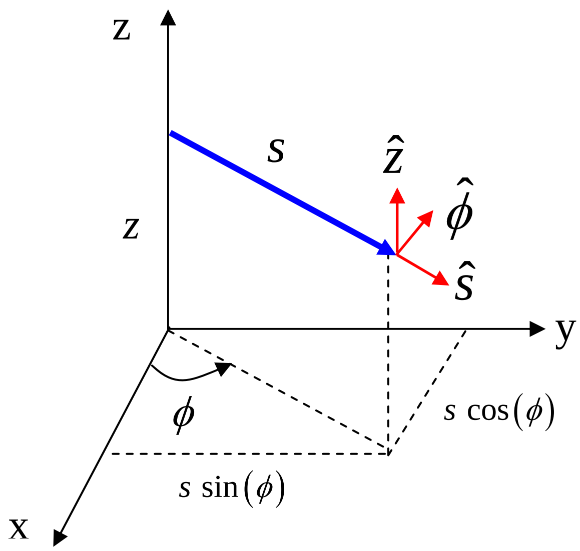
$$\hat{r} = \sin(\theta)\cos(\phi)\hat{x} + \sin(\theta)\sin(\phi)\hat{y} + \cos(\theta)\hat{z}$$

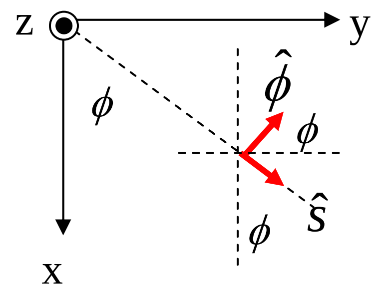
$$\hat{\theta} = \cos(\theta)\cos(\phi)\hat{x} + \cos(\theta)\sin(\phi)\hat{y} - \sin(\theta)\hat{z}$$

$$\hat{\phi} = -\sin(\phi)\hat{x} + \cos(\phi)\hat{y}$$

volume element: $d\tau = r^2 \sin(\theta)\, dr\, d\theta\, d\phi$

Cylindrical Coordinates





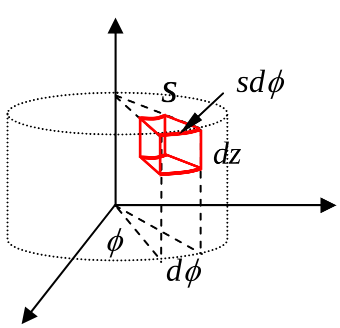
$\hat{s}$ and $\hat{\phi}$ are not fixed unit vectors, $\hat{z}$ is a fixed unit vector

$x = s\cos(\phi) \quad y = s\sin(\phi) \quad z = z$

$$\hat{s} = \cos(\phi)\,\hat{x} + \sin(\phi)\,\hat{y} \qquad \hat{x} = \hat{s}\cos(\phi) - \hat{\phi}\sin(\phi)$$

$$\hat{\phi} = -\sin(\phi)\,\hat{x} + \cos(\phi)\,\hat{y} \qquad \hat{y} = \hat{s}\sin(\phi) + \hat{\phi}\cos(\phi)$$

$$\hat{z} = \hat{z}$$



volume element: $d\tau = sds\ d\phi\ dz$

Gradient in cylindrical coordinates

$$f(x,y,z) = f(s,\phi,z); \quad s = s(x,y), \quad \phi = \phi(x,y)$$

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

the unit vectors are given above in terms of $\hat{s}$ and $\hat{\phi}$

use chain rule: $$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial s}\frac{\partial s}{\partial x} + \frac{\partial f}{\partial \phi}\frac{\partial \phi}{\partial x} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial x}$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial s}\frac{\partial s}{\partial y} + \frac{\partial f}{\partial \phi}\frac{\partial \phi}{\partial y} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial y}$$

but, $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = 0,$ because z is independent of x and y

$$s = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}(y/x)$$

$$\frac{\partial s}{\partial x} = \cos(\phi); \quad \frac{\partial s}{\partial y} = \sin(\phi); \quad \frac{\partial \phi}{\partial x} = -\frac{\sin(\phi)}{s}; \quad \frac{\partial \phi}{\partial y} = \frac{\cos(\phi)}{s}$$

$$\Rightarrow \qquad \nabla f = \hat{s}\frac{\partial f}{\partial s} + \frac{\hat{\phi}}{s}\frac{\partial f}{\partial \phi} + \hat{z}\frac{\partial f}{\partial z}$$