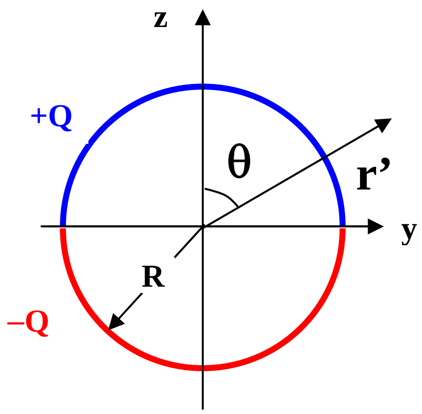
29:129 Electricity and Magnetism I Fall 2012



Calculate the electric dipole moment of a spherical charge distribution that is formed from two hemispherical shells of surface charge densities $+\sigma_o$ on the top hemisphere, and $-\sigma_o$ on the bottom hemisphere.

For a surface charge distribution, the dipole moment is

$$\vec{p} = \int \vec{r}'\sigma(\theta)da \,, \qquad [1]$$

where, in this problem: $\sigma(\theta) = +\sigma_o$ for $0 < \theta < \pi/2$, and $-\sigma_o$ for $\pi/2 < \theta < \pi$. If you consider the charges in this distribution, for every positive charge in the upper hemisphere, there is a negative charge immediately below it in the bottom hemisphere and together these make a single dipole. Thus the actual distribution is a collection of dipoles with dipole moments having different magnitudes but all pointing in the positive z direction, so that $\vec{p} = p_z\hat{z}$. Then the intergral [1] reduces to

$$p = p_z = \int z'\sigma(\theta)da = \int_0^{\frac{\pi}{2}} z'\sigma_o da + \int_{\frac{\pi}{2}}^{\pi} z'(-\sigma_o)da \,, \qquad [2]$$

where $da = 2\pi R^2 \sin(\theta)d\theta$, and $z' = R\cos(\theta)$. The integrals are easy ones and the result is

$$p_z = 2\pi R^3\sigma_o \,. \qquad [3]$$

Since $\sigma_o = Q/2\pi R^2$, [3] reduces to simply, $p_z = QR$.

(Problem 3.28 is similar to this problem.)