Legendre Polynomials

```
> restart;
```

Maple has a package of orthogonal functions called Orthopoly.
This package must first be called up:

```
> with(orthopoly);
```

$$[G, H, L, P, T, U] \qquad (1)$$

The P's are the Legendre Ploynomials, for example to see a few of them:

```
> P(0, cos(θ)); P(1, cos(θ)); P(2, cos(θ)); P(3, cos(θ)); P(4, cos(θ)); P(5, cos(θ)); P(6,
    cos(θ));
```

$$1$$

$$\cos(\theta)$$

$$-\frac{1}{2} + \frac{3}{2}\cos(\theta)^2$$

$$\frac{5}{2}\cos(\theta)^3 - \frac{3}{2}\cos(\theta)$$

$$\frac{3}{8} + \frac{35}{8}\cos(\theta)^4 - \frac{15}{4}\cos(\theta)^2$$

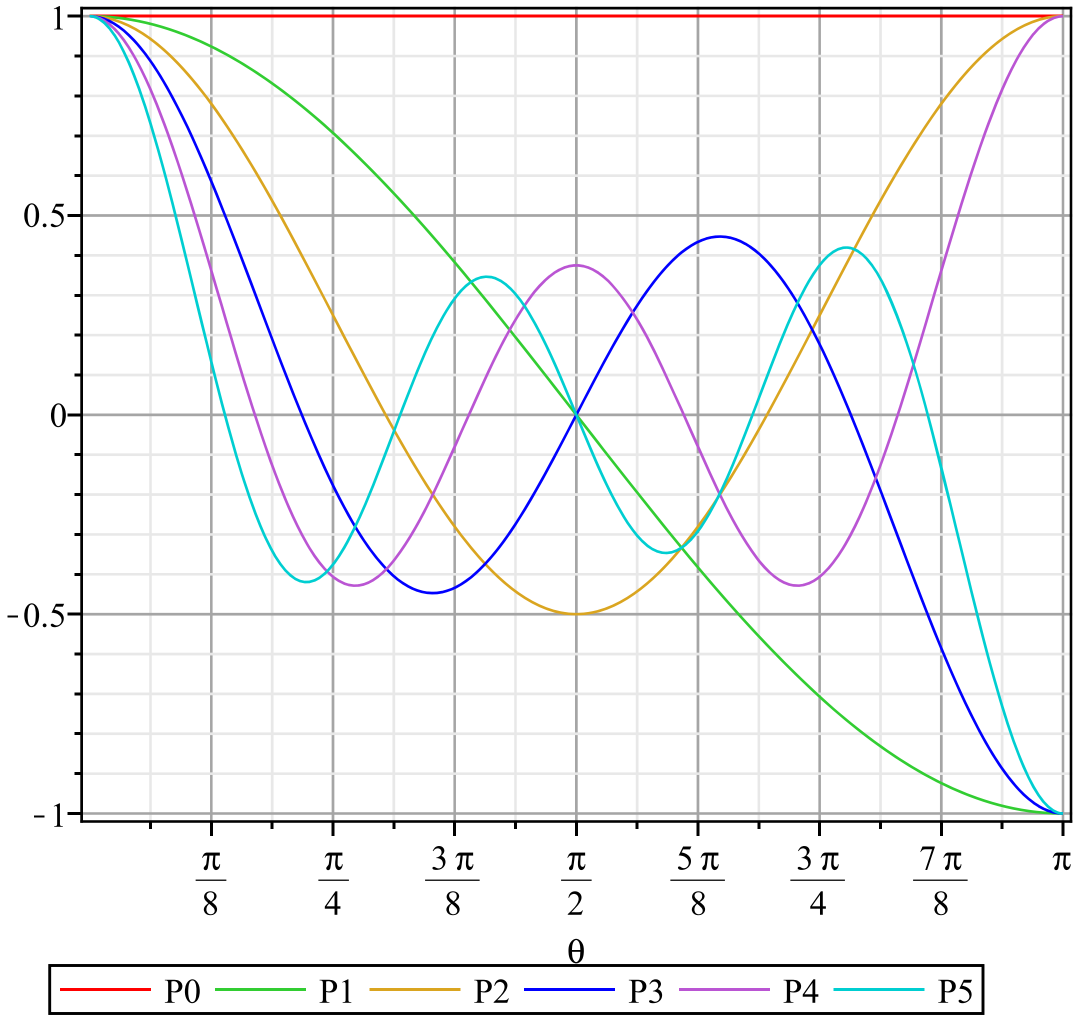
$$\frac{63}{8}\cos(\theta)^5 - \frac{35}{4}\cos(\theta)^3 + \frac{15}{8}\cos(\theta)$$

$$-\frac{5}{16} + \frac{231}{16}\cos(\theta)^6 - \frac{315}{16}\cos(\theta)^4 + \frac{105}{16}\cos(\theta)^2 \qquad (2)$$

```
>
```

plot the first few Legendre polynomials

```
> plot([P(0, cos(θ)), P(1, cos(θ)), P(2, cos(θ)), P(3, cos(θ)), P(4, cos(θ)), P(5,
    cos(θ))], θ = 0 ..π);
```



The Legendre polynomials are orthogonal: The inner product of two different P's = 0, while the inner product of any P with itself gives 2/(2ℓ+1).

Some examples are shown:

> $\int_{-1}^{1} P(0, x) \cdot P(1, x) \, dx$; $\int_{-1}^{1} P(1, x) \cdot P(2, x) \, dx$; $\int_{-1}^{1} P(2, x) \cdot P(3, x) \, dx$; $\int_{-1}^{1} P(3, x) \cdot P(5, x) \, dx$;

$$0$$
$$0$$
$$0$$
$$0 \qquad (3)$$

> $\int_{-1}^{1} P(0, x) \cdot P(0, x) \, dx$;

$$2 \qquad (4)$$

> $\int_{-1}^{1} P(1, x) \cdot P(1, x) \, dx$;

$$\frac{2}{3} \qquad (5)$$

> $\int_{-1}^{1} P(2, x) \cdot P(2, x) \, dx;$

$$\frac{2}{5} \tag{6}$$

> $\int_{-1}^{1} P(3, x) \cdot P(3, x) \, dx;$

$$\frac{2}{7} \tag{7}$$

> $\int_{-1}^{1} P(4, x) \cdot P(4, x) \, dx;$

$$\frac{2}{9} \tag{8}$$

>