Representation of the Dirac delta function

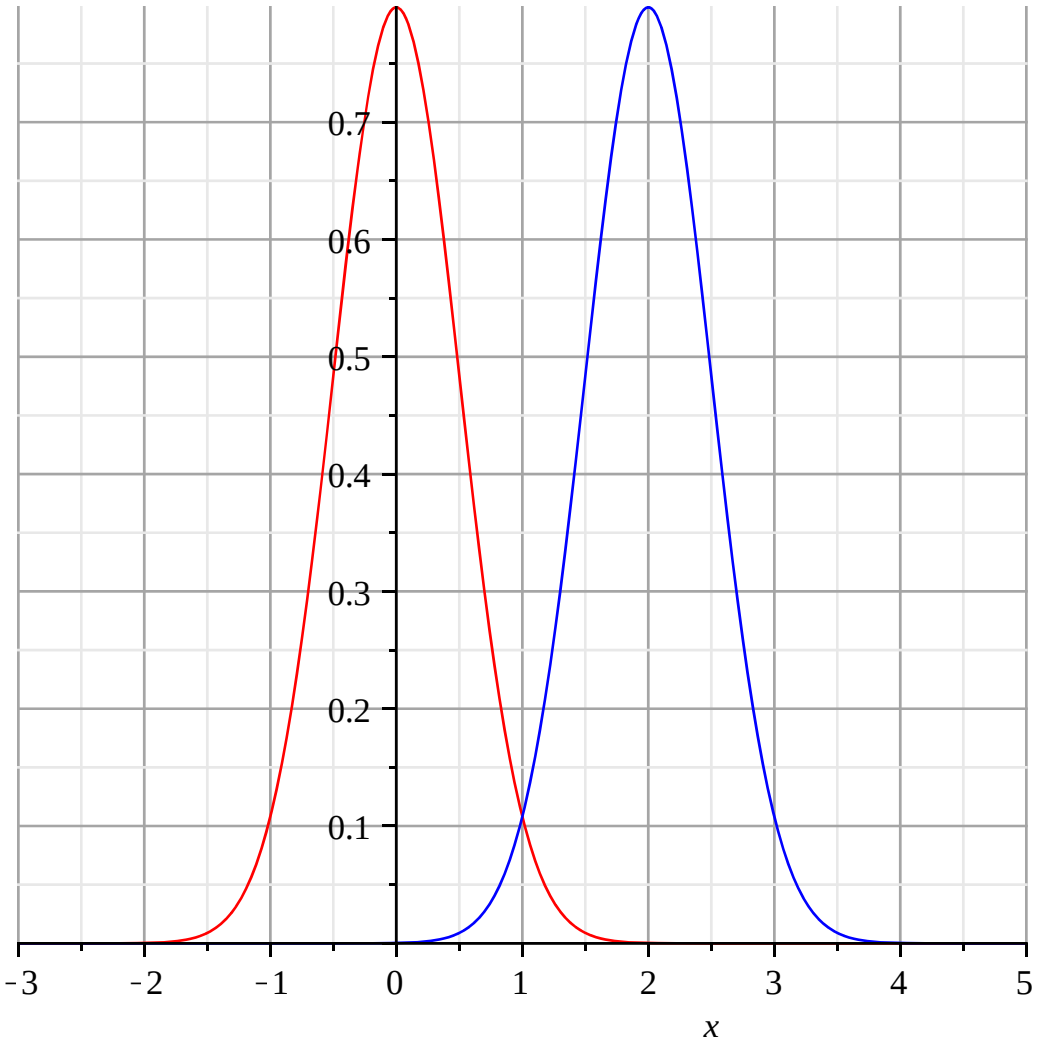
```
> restart;
>
```

A Gaussian distribution is a representation of the delta function as the half-width, σ → 0. *f(σ,a,x)* is a Gaussian shifted to x = a.

```
> f := (σ, a, x) → 1/sqrt(2·π) * exp(-(x-a)^2/(2·σ^2))/σ;
```

$$f := (\sigma, a, x) \rightarrow \frac{e^{-\frac{1}{2}\frac{(x-a)^2}{\sigma^2}}}{\sqrt{2\,\pi}\;\sigma} \qquad (1)$$

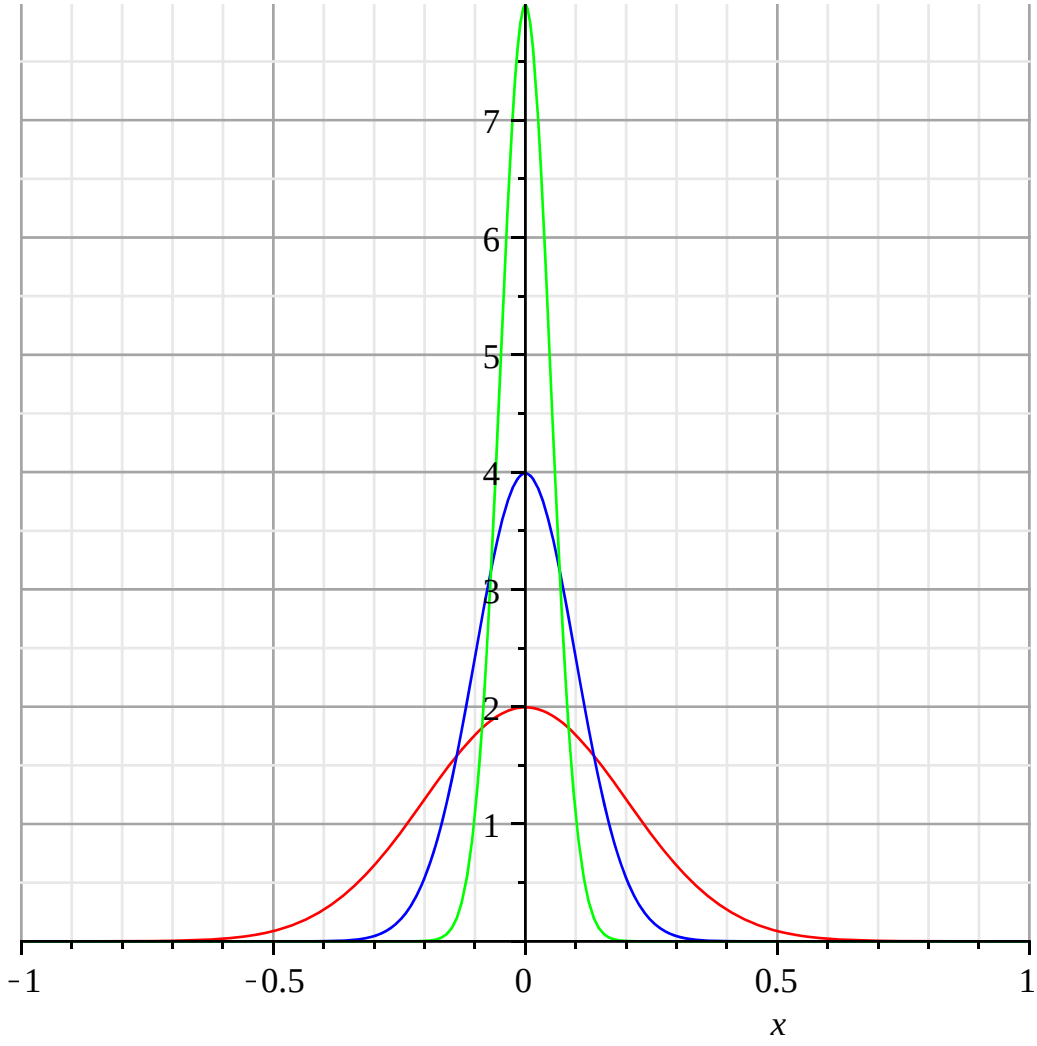
```
> plot([f(0.5, 0, x), f(0.5, 2, x)], x = -3..5, color = [red, blue]);
```




```
>
```

Below are plots of the Gaussion for various values of σ. As σ gets smaller, the Gaussian is more and more spiky.

```
> plot( [ f(0.2, 0, x) , f(0.1, 0, x), f(0.05, 0, x) ], x = -1 ..1, color = [red, blue, green]);
```




```
>
```

The integral of the Gaussian is the same, regardless of the value of σ. It is normalized to 1.

```
> int( f(0.1, 0, x), x = -100 ..100); int( f(0.01, 0, x), x = -100 ..100); int(f(0.001, 0, x), x = -100
  ..100);
```

1.
1.
1. **(2)**

```
>
```

What is the effect of integrating a function over a delta function?

```
> F := x^2 + 2 x + 5;
```

$$F := x^2 + 2\,x + 5 \tag{3}$$

```
> int(F*f(0.1, 0, x), x = -1 ..1); int(F*f(0.01, 0, x), x = -1 ..1); int(F*f(0.001, 0, x), x = -1
  ..1);
```

5.010000000
5.000100000
5.000001000 **(4)**

```
> int(F·f(0.1, 2, x), x = -3 ..3); int(F·f(0.01, 2, x), x = -3 ..3); int(F·f(0.001, 2, x), x = -3 ..3);
```

13.01000000
13.00010000
13.00000100 **(5)**

```
> Digits := 30; int(F·f(0.00000000000001, 2, x), x = 0 ..3);
```

$$Digits := 30$$

13.000000000000000000000000001 **(6)**

```
>
```