Point charge at a distance d above a grounded conducting plane

> *restart*;

> *with*(*VectorCalculus*) :

> $V := \frac{q}{4 \cdot \pi \cdot \varepsilon_o} \cdot \left(\left(x^2 + y^2 + (z-d)^2\right)^{-\frac{1}{2}} - \left(x^2 + y^2 + (z+d)^2\right)^{-\frac{1}{2}} \right);$

$$V := \frac{1}{4} \frac{q \left(\frac{1}{\sqrt{x^2 + y^2 + (z-d)^2}} - \frac{1}{\sqrt{x^2 + y^2 + (z+d)^2}} \right)}{\pi \varepsilon_o} \quad (1)$$

> $LV := Laplacian(V, [x, y, z]);$

$$LV := \frac{1}{4} \frac{1}{\pi \varepsilon_o} \left(q \left(\frac{3 x^2}{\left(x^2 + y^2 + (z-d)^2\right)^{5/2}} - \frac{1}{\left(x^2 + y^2 + (z-d)^2\right)^{3/2}} - \frac{3 x^2}{\left(x^2 + y^2 + (z+d)^2\right)^{5/2}} + \frac{1}{\left(x^2 + y^2 + (z+d)^2\right)^{3/2}} \right) \right)$$
$$+ \frac{1}{4} \frac{1}{\pi \varepsilon_o} \left(q \left(\frac{3 y^2}{\left(x^2 + y^2 + (z-d)^2\right)^{5/2}} - \frac{1}{\left(x^2 + y^2 + (z-d)^2\right)^{3/2}} - \frac{3 y^2}{\left(x^2 + y^2 + (z+d)^2\right)^{5/2}} + \frac{1}{\left(x^2 + y^2 + (z+d)^2\right)^{3/2}} \right) \right)$$
$$+ \frac{1}{4} \frac{1}{\pi \varepsilon_o} \left(q \left(\frac{3}{4} \frac{(2z - 2d)^2}{\left(x^2 + y^2 + (z-d)^2\right)^{5/2}} - \frac{1}{\left(x^2 + y^2 + (z-d)^2\right)^{3/2}} - \frac{3}{4} \frac{(2z + 2d)^2}{\left(x^2 + y^2 + (z+d)^2\right)^{5/2}} + \frac{1}{\left(x^2 + y^2 + (z+d)^2\right)^{3/2}} \right) \right) \quad (2)$$

> $simplify(LV);$

$$0 \quad (3)$$

> $gradV := -Gradient(V, [x, y, z]);$

$$gradV := -\frac{1}{4} \frac{q \left(-\frac{x}{\left(x^2 + y^2 + (z-d)^2\right)^{3/2}} + \frac{x}{\left(x^2 + y^2 + (z+d)^2\right)^{3/2}} \right)}{\pi \varepsilon_o} \bar{e}_x$$
$$- \frac{1}{4} \frac{q \left(-\frac{y}{\left(x^2 + y^2 + (z-d)^2\right)^{3/2}} + \frac{y}{\left(x^2 + y^2 + (z+d)^2\right)^{3/2}} \right)}{\pi \varepsilon_o} \bar{e}_y$$
$$- \frac{1}{4} \frac{q \left(-\frac{1}{2} \frac{2z - 2d}{\left(x^2 + y^2 + (z-d)^2\right)^{3/2}} + \frac{1}{2} \frac{2z + 2d}{\left(x^2 + y^2 + (z+d)^2\right)^{3/2}} \right)}{\pi \varepsilon_o} \bar{e}_z \quad (4)$$

> $z := 0;$

$$z := 0 \quad (5)$$

> *gradV*;

$$-\frac{1}{2}\frac{q\,d}{\pi\,\varepsilon_o\,(x^2+y^2+d^2)^{3/2}}\bar{e}_z \quad \textbf{(6)}$$

>

Let $s^2 = x^2 + y^2$, the surface charge density on the plane is $\sigma = \varepsilon_o E_z$

> $\sigma := -\varepsilon_o \cdot \frac{q \cdot d}{2 \cdot \pi \cdot \varepsilon_o \cdot (s^2 + d^2)^{\frac{3}{2}}}$;

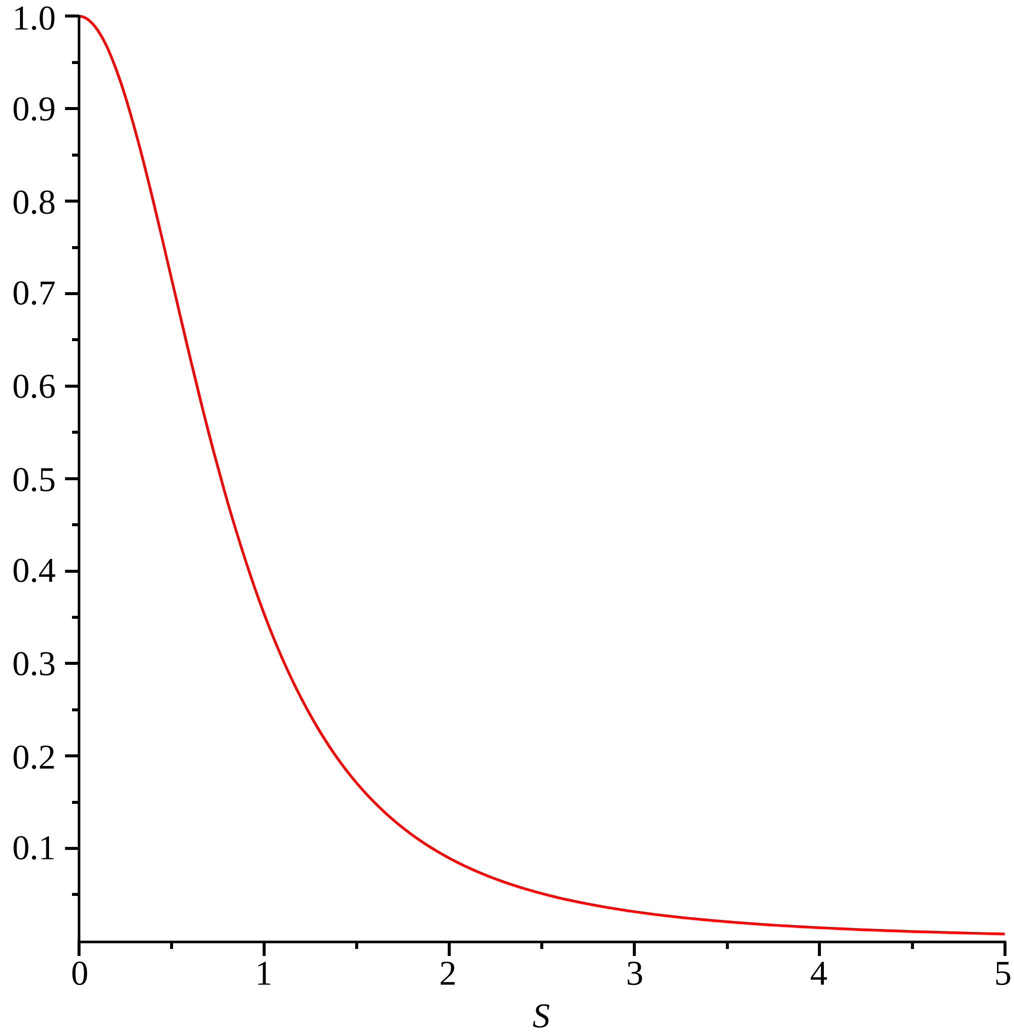
$$\sigma := -\frac{1}{2}\frac{q\,d}{\pi\,(s^2+d^2)^{3/2}} \quad \textbf{(7)}$$

Plot the normalized absolute value of the charge density, with S = s/d

> $\sigma Norm := \frac{1}{(1+S^2)^{\frac{3}{2}}}$;

$$\sigma Norm := \frac{1}{(1+S^2)^{3/2}} \quad \textbf{(8)}$$

> *plot*($\sigma Norm$, $S = 0..5$) ;



>

Calculate the total induced charge on the conducting plane

> $qInduced := simplify\left(\int_0^{\infty} \sigma \cdot 2 \cdot \pi \cdot s \, ds\right)$ assuming $d > 0$;

$$qInduced := -q \tag{9}$$

> $restart;$

> $with(plots):$

> $Vnorm := \left(x^2 + y^2 + (z-d)^2\right)^{-\frac{1}{2}} - \left(x^2 + y^2 + (z+d)^2\right)^{-\frac{1}{2}};$

$$Vnorm := \frac{1}{\sqrt{x^2 + y^2 + (z-d)^2}} - \frac{1}{\sqrt{x^2 + y^2 + (z+d)^2}} \tag{10}$$

> $x := 0;\ d := 2;$

$$x := 0$$
$$d := 2 \tag{11}$$

> $plot1 := contourplot(Vnorm, y = -3..3, z = 0..5, contours = [0, 0.01, 0.05, 0.1, 0.2, 0.3, 0.5, 1],$
$grid = [50, 50], coloring = [red, blue]):$

> $Eynorm := \frac{y}{\left(x^2+y^2+(z-d)^2\right)^{\frac{3}{2}}} - \frac{y}{\left(x^2+y^2+(z+d)^2\right)^{\frac{3}{2}}};$

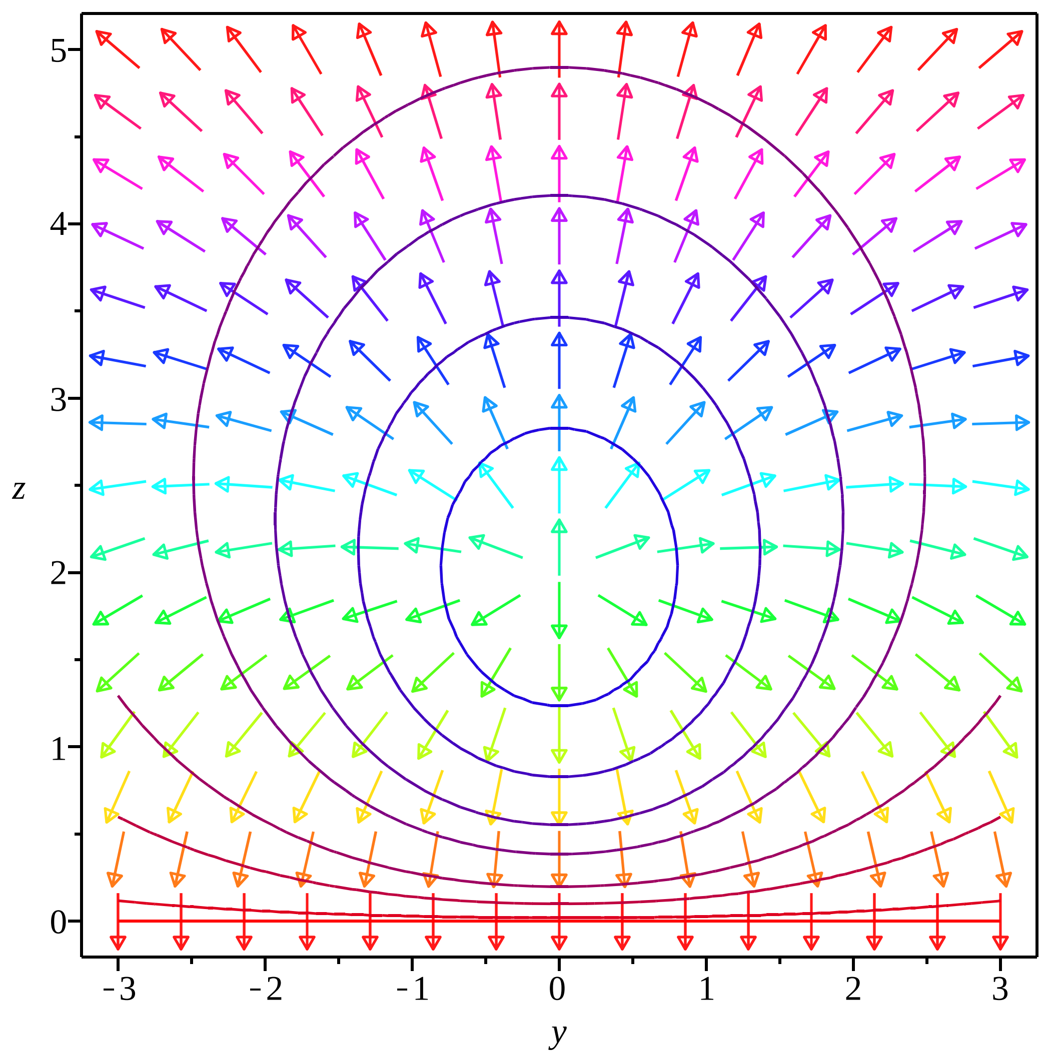
$$Eynorm := \frac{y}{\left(y^2+(z-2)^2\right)^{3/2}} - \frac{y}{\left(y^2+(z+2)^2\right)^{3/2}} \quad \textbf{(12)}$$

> $Eznorm := \frac{1}{2}\,\frac{2\,z-2\,d}{\left(x^2+y^2+(z-d)^2\right)^{\frac{3}{2}}} - \frac{1}{2}\,\frac{2\,z+2\,d}{\left(x^2+y^2+(z+d)^2\right)^{\frac{3}{2}}};$

$$Eznorm := \frac{1}{2}\,\frac{2\,z-4}{\left(y^2+(z-2)^2\right)^{3/2}} - \frac{1}{2}\,\frac{2\,z+4}{\left(y^2+(z+2)^2\right)^{3/2}} \quad \textbf{(13)}$$

> *plot2* := *fieldplot*([*Eynorm*, *Eznorm*], *y* = -3 ..3, *z* = 0 ..5, *arrows* = *SLIM*, *grid* = [15, 15], *fieldstrength* = *fixed*, *color* = *z*) :

> *display*({ *plot1*, *plot2* });



The electric field vectors are plotted as fixed strength to show only the direction not magnitude.