Point charge near a grounded conducting sphere

`> restart;`

`> ` $V := \dfrac{q}{4 \cdot \pi \cdot \varepsilon_o} \left(\dfrac{1}{\sqrt{x^2 + y^2 + (z-a)^2}} - \dfrac{R}{a \cdot \sqrt{x^2 + y^2 + \left(z - \dfrac{R^2}{a}\right)^2}} \right);$

$$V := \frac{1}{4}\, \frac{q \left(\dfrac{1}{\sqrt{x^2 + y^2 + z^2 - 2za + a^2}} - \dfrac{R}{a\sqrt{x^2 + y^2 + \left(z - \dfrac{R^2}{a}\right)^2}} \right)}{\pi \varepsilon_o} \tag{1}$$

`> with(VectorCalculus) :`

`> minusgradV := -Gradient(V, [x, y, z]);`

$$minusgradV := -\frac{1}{4}\, \frac{q \left(-\dfrac{x}{(x^2 + y^2 + z^2 - 2za + a^2)^{3/2}} + \dfrac{Rx}{a\left(x^2 + y^2 + \left(z - \dfrac{R^2}{a}\right)^2\right)^{3/2}} \right)}{\pi \varepsilon_o} \bar{e}_x \tag{2}$$

$$-\frac{1}{4}\, \frac{q \left(-\dfrac{y}{(x^2 + y^2 + z^2 - 2za + a^2)^{3/2}} + \dfrac{Ry}{a\left(x^2 + y^2 + \left(z - \dfrac{R^2}{a}\right)^2\right)^{3/2}} \right)}{\pi \varepsilon_o} \bar{e}_y$$

$$-\frac{1}{4}\, \frac{q \left(-\dfrac{1}{2}\dfrac{2z - 2a}{(x^2 + y^2 + z^2 - 2za + a^2)^{3/2}} + \dfrac{1}{2}\dfrac{R\left(2z - \dfrac{2R^2}{a}\right)}{a\left(x^2 + y^2 + \left(z - \dfrac{R^2}{a}\right)^2\right)^{3/2}} \right)}{\pi \varepsilon_o} \bar{e}_z$$

`> LV := Laplacian(V, [x, y, z]);`

$$LV := \frac{1}{4}\,\frac{1}{\pi \varepsilon_o}\left(q \left(\frac{3x^2}{(x^2 + y^2 + z^2 - 2za + a^2)^{5/2}} - \frac{1}{(x^2 + y^2 + z^2 - 2za + a^2)^{3/2}} \right. \right. \tag{3}$$

$$\left. \left. - \frac{3Rx^2}{a\left(x^2 + y^2 + \left(z - \dfrac{R^2}{a}\right)^2\right)^{5/2}} + \frac{R}{a\left(x^2 + y^2 + \left(z - \dfrac{R^2}{a}\right)^2\right)^{3/2}} \right) \right)$$

$$+ \frac{1}{4}\,\frac{1}{\pi \varepsilon_o}\left(q \left(\frac{3y^2}{(x^2 + y^2 + z^2 - 2za + a^2)^{5/2}} - \frac{1}{(x^2 + y^2 + z^2 - 2za + a^2)^{3/2}} \right. \right.$$

$$- \frac{3\,R\,y^2}{a\left(x^2+y^2+\left(z-\frac{R^2}{a}\right)^2\right)^{5/2}} + \frac{R}{a\left(x^2+y^2+\left(z-\frac{R^2}{a}\right)^2\right)^{3/2}}\Bigg)\Bigg)$$

$$+ \frac{1}{4}\,\frac{1}{\pi\,\varepsilon_o}\Bigg(q\Bigg(\frac{3}{4}\,\frac{(2\,z-2\,a)^2}{(x^2+y^2+z^2-2\,z\,a+a^2)^{5/2}} - \frac{1}{(x^2+y^2+z^2-2\,z\,a+a^2)^{3/2}}$$

$$- \frac{3}{4}\,\frac{R\left(2\,z-\frac{2\,R^2}{a}\right)^2}{a\left(x^2+y^2+\left(z-\frac{R^2}{a}\right)^2\right)^{5/2}} + \frac{R}{a\left(x^2+y^2+\left(z-\frac{R^2}{a}\right)^2\right)^{3/2}}\Bigg)\Bigg)$$

> `simplify(%);`

$$0 \tag{4}$$

> `y := 0;`

$$y := 0 \tag{5}$$

> `minusgradV;`

$$-\frac{1}{4}\,\frac{q\left(-\dfrac{x}{(x^2+z^2-2\,z\,a+a^2)^{3/2}} + \dfrac{R\,x}{a\left(x^2+\left(z-\frac{R^2}{a}\right)^2\right)^{3/2}}\right)}{\pi\,\varepsilon_o}\,\bar{e}_x \tag{6}$$

$$-\frac{1}{4}\,\frac{q\left(-\dfrac{1}{2}\,\dfrac{2\,z-2\,a}{(x^2+z^2-2\,z\,a+a^2)^{3/2}} + \dfrac{1}{2}\,\dfrac{R\left(2\,z-\frac{2\,R^2}{a}\right)}{a\left(x^2+\left(z-\frac{R^2}{a}\right)^2\right)^{3/2}}\right)}{\pi\,\varepsilon_o}\,\bar{e}_z$$

> `Exnorm := -( -\frac{x}{(x^2+z^2-2za+a^2)^{3/2}} + \frac{Rx}{a(x^2+(z-R^2/a)^2)^{3/2}} );`

$$Exnorm := -\left(- \frac{x}{\left(x^2+z^2-2\,z\,a+a^2\right)^{\frac{3}{2}}} + \frac{R\,x}{a\left(x^2+\left(z-\frac{R^2}{a}\right)^2\right)^{\frac{3}{2}}} \right);$$

$$Exnorm := \frac{x}{(x^2+z^2-2\,z\,a+a^2)^{3/2}} - \frac{R\,x}{a\left(x^2+\left(z-\frac{R^2}{a}\right)^2\right)^{3/2}} \tag{7}$$

> `Eznorm := -( -\frac{1}{2}\frac{2z-2a}{(x^2+z^2-2za+a^2)^{3/2}} + \frac{1}{2}\frac{R(2z-2R^2/a)}{a(x^2+(z-R^2/a)^2)^{3/2}} );`

$$Eznorm := -\left(-\frac{1}{2}\,\frac{2\,z-2\,a}{\left(x^2+z^2-2\,z\,a+a^2\right)^{\frac{3}{2}}} + \frac{1}{2}\,\frac{R\left(2\,z-\frac{2\,R^2}{a}\right)}{a\left(x^2+\left(z-\frac{R^2}{a}\right)^2\right)^{\frac{3}{2}}} \right);$$

$$Eznorm := \frac{1}{2} \frac{2z - 2a}{\left(x^2 + z^2 - 2za + a^2\right)^{3/2}} - \frac{1}{2} \frac{R\left(2z - \frac{2R^2}{a}\right)}{a\left(x^2 + \left(z - \frac{R^2}{a}\right)^2\right)^{3/2}}$$ (8)

> $Vnorm := \dfrac{1}{\sqrt{x^2 + y^2 + z^2 - 2za + a^2}} - \dfrac{R}{a\sqrt{x^2 + y^2 + \left(z - \frac{R^2}{a}\right)^2}} ;$

$$Vnorm := \frac{1}{\sqrt{x^2 + z^2 - 2za + a^2}} - \frac{R}{a\sqrt{x^2 + \left(z - \frac{R^2}{a}\right)^2}}$$ (9)
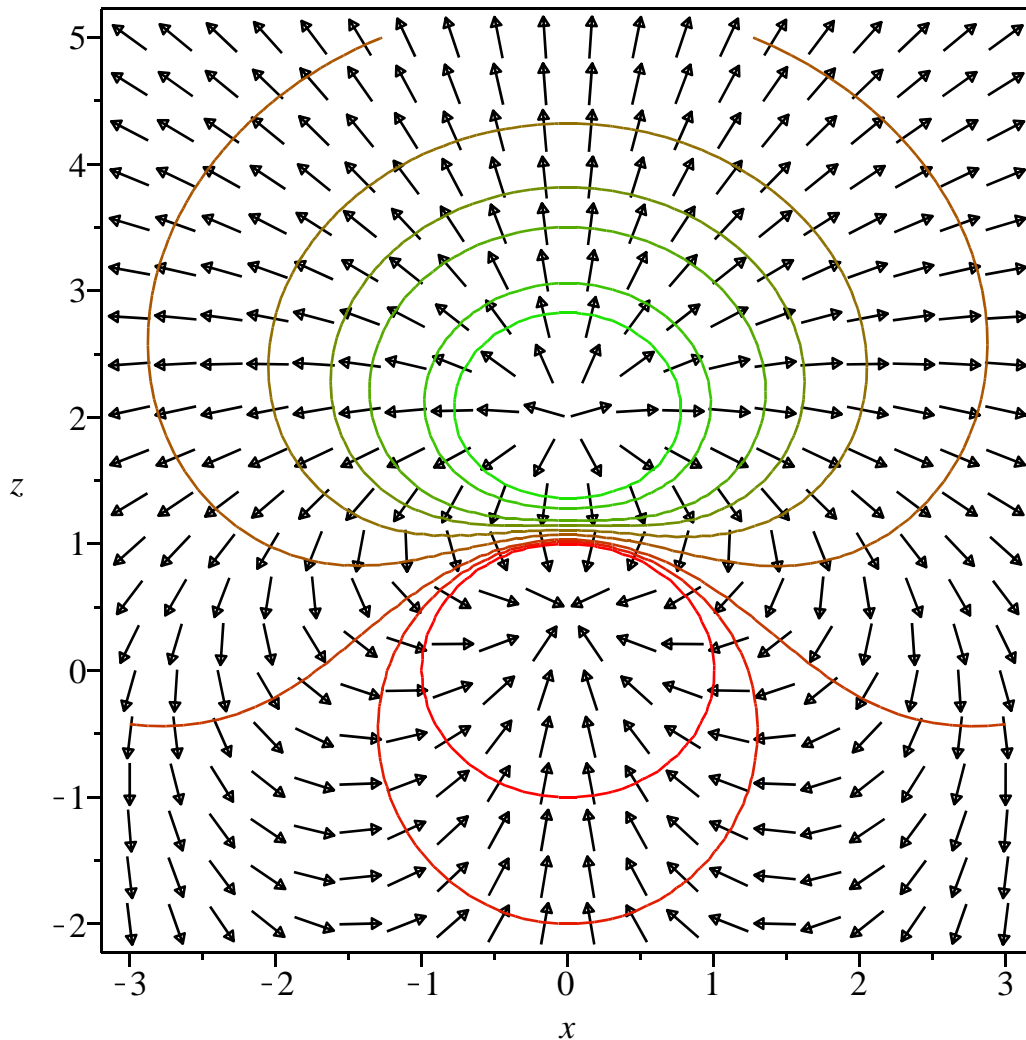
> $R := 1; a := 2;$

$$R := 1$$
$$a := 2$$

(10)

> $with(plots):$

> $plot1 := contourplot(Vnorm, x = -3..3, z = -2..5, grid = [50, 50], contours = [0, 0.05, 0.1, 0.2, 0.3, 0.4, \ 0.5, 0.75, 1.0], coloring = [red, green]):$

> $plot2 := fieldplot([Exnorm, Eznorm], x = -3..3, z = -2..5, arrows = SLIM, grid = [20, 20], fieldstrength = fixed):$

> $display(\{plot1, plot2\});$

>