

Conducting sphere in a uniform electric field (Griffiths Ex. 3.6)

It is better to plot the electric field in the y-x plane. To convert the field in (r, θ) coordinates to (y,z), use

$$r = \sqrt{y^2 + z^2}, \quad \cos(\theta) = \frac{z}{r}, \quad \sin(\theta) = \frac{y}{r},$$

$$u_r = \cos(\theta)u_y + \sin(\theta)u_z, \quad u_\theta = \cos(\theta)u_y - \sin(\theta)u_z$$

where the u's indicate unit vectors

The y and z components of the electric field are given below. The electric field is normalized by E_0 and y and z are normalized by R, so the sphere has unit radius

> restart;

$$> Ey := \frac{3 \cdot y \cdot z}{r^5}; \quad Ez := \frac{1}{r^2} \cdot (y^2 + z^2) + \frac{1}{r^5} \cdot (2 \cdot z^2 - y^2);$$

$$Ey := \frac{3 y z}{r^5}$$

$$Ez := \frac{y^2 + z^2}{r^2} + \frac{2 z^2 - y^2}{r^5} \quad (1)$$

$$> r := \sqrt{y^2 + z^2};$$

$$r := \sqrt{y^2 + z^2} \quad (2)$$

> Ey, Ez;

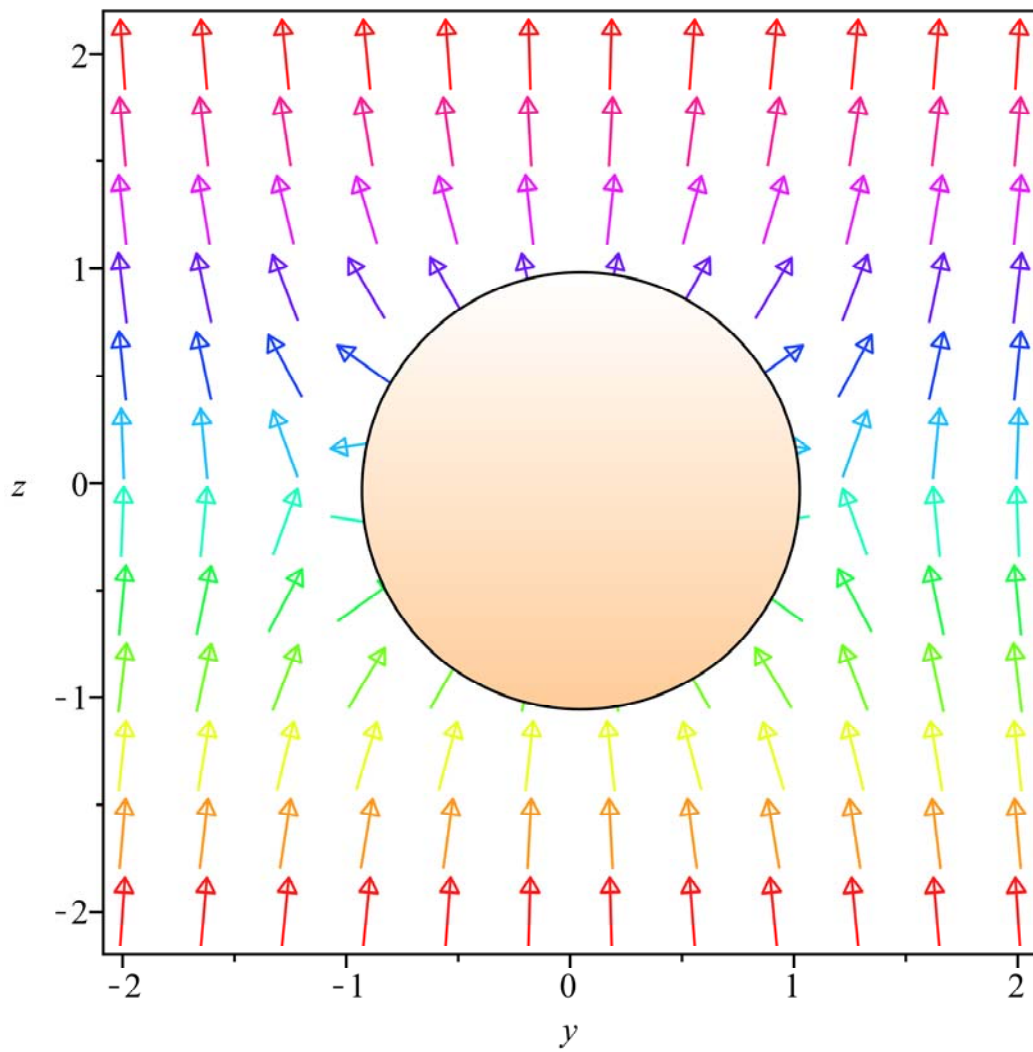
$$\frac{3 y z}{(y^2 + z^2)^{5/2}} + 1 + \frac{2 z^2 - y^2}{(y^2 + z^2)^{5/2}} \quad (3)$$

> with(plots) :

The fieldplot will be given with vectors of constant length so that the directions can be more easily seen. The first plot is on a scale -2 to +2, the sphere has a unit radius. Note that the electric field is VERTICAL

in these plots. Notice that at the surface, the electric field is perpendicular to the sphere.

> fieldplot([Ey, Ez], y=-2..2, z=-2..2, fieldstrength=fixed, arrows=SLIM, grid=[12, 12], color=z);



This plot is on a much larger scale (-10 to +10) so that the uniform nature of E is clear at large distances

```
> fieldplot([Ey, Ez], y=-10..10, z=-10..10, fieldstrength=fixed, arrows=SLIM, color=z, grid
=[12, 12]);
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