The preceding calculation for **B** and the associated magnetic force could hardly be simpler. Nevertheless we shall approach the problem in yet another way to emphasize the relativistic nature of the magnetic field.

To investigate the magnetic field at P due to the moving linear distribution of charge, we must place a *moving* test charge at P. To simplify matters, we shall let the test charge q_2 move *parallel to the x axis* with the velocity **u** [Fig. 8–12(a)].

In this reference frame S, the force on q_2 contains both an electric part and a magnetic part:

$$\mathbf{F} = \mathbf{F}_{\mathcal{E}} + \mathbf{F}_B \tag{8–14}$$

The electric force is, by Eq. (8–11), given by

$$\mathbf{F}_{\mathcal{E}} = \boldsymbol{\mathcal{E}} q_2 = 2k\frac{\lambda}{b} q_2 \mathbf{e}_y \tag{8–15}$$

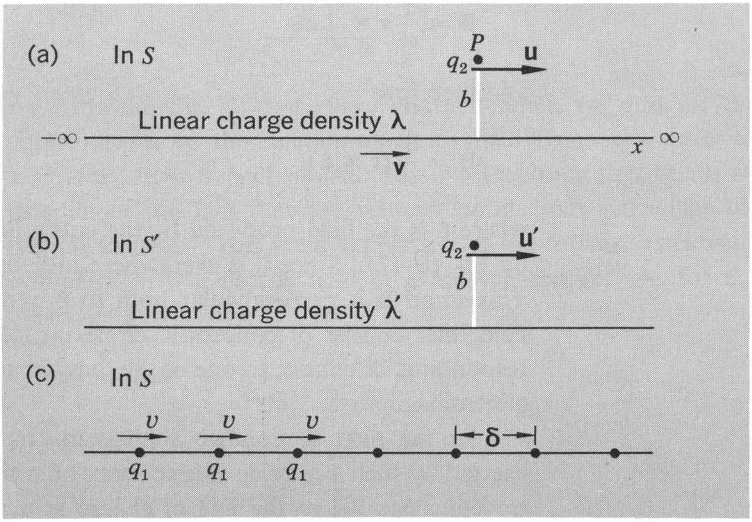

Fig. 8–12 (a) A test charge moves parallel to a line of moving charge. (b) The situation is transformed to a frame in which the line charge is stationary. (c) The linear charge density q_1/δ is not an invariant.

The magnetic force is

$$\mathbf{F}_B = q_2 \mathbf{u} \times \mathbf{B} \tag{8–16}$$

and if we can find $\mathbf{F}_B$, then we shall know something about $\mathbf{B}$.

Let us look at the system from a reference frame S' moving with velocity $\mathbf{v}$ relative to S. In S', the linear array is stationary and q_2 is moving with a velocity $\mathbf{u}'$ [Fig. 8–12(b)]. The distance from q_2 to the line is transverse to the direction of relative motion between S' and S and therefore is unchanged. But what happens to the linear charge density along the line? We can approximate the linear array by spacing an infinite collection of moving discrete charges q_1 a distance δ apart in frame S, and choosing q_1 and δ such that $q_1/\delta = \lambda$ [Fig. 8–12(c)].

If now we let q_1 and δ both approach zero in such a way that the ratio q_1/δ always remains constant and equal to λ, then we approach our idealized array—a line distribution of charge with constant linear density λ, moving with velocity $\mathbf{v}$.

In S' the charges q_1 are at rest. The distances between them are therefore greater, by the factor γ, than they are in S. Hence the charge density in S' is $\lambda' = q_1/\gamma\delta = \lambda/\gamma$ and is less than it is in S.

Now in S' the source charges are stationary. Thus the entire force on q_2 is purely electric and is given [cf. Eq. (8–11)] by

$$F' = 2k\frac{\lambda'}{b}q_2 = \frac{1}{\gamma}\left(2k\frac{\lambda}{b}\right)q_2$$

The force F' is in the y direction. Equations (8–7) tell us how to transform F' to find the total force F on q_2 as observed in S:

$$F_y' = \frac{F_y}{\gamma(1 - u_x v/c^2)}$$

Applied to the present case, this gives

$$F_y = \gamma\left(1 - \frac{uv}{c^2}\right)F_y'$$

That is,

$$\mathbf{F} = 2k\frac{\lambda}{b}q_2\left(1 - \frac{uv}{c^2}\right)\mathbf{e}_y \tag{8–17}$$

Thus from Eqs. (8–14), (8–15) and (8–17), the magnetic force observed in S is

$$\mathbf{F}_B = \mathbf{F} - \mathbf{F}_{\mathcal{E}}$$

Therefore,

$$\mathbf{F}_B = -\frac{uv}{c^2}\left(2k\frac{\lambda}{b}\right)q_2\mathbf{e}_y = -q_2\frac{uv}{c^2}\mathcal{E}$$

in complete agreement with Eq. (8–13). Using Eq. (8–16), we can then infer that the y component of **B** is zero, and that its z component is given by

$$B_z = \frac{v}{c^2}\left(2k\frac{\lambda}{b}\right) = \frac{1}{c^2}v\mathcal{E} \qquad (8\text{–}18)$$

in agreement with Eq. (8–10). To verify that this is the whole of **B**—that the x component of **B** is zero—we should have to let our test charge q_2 move in some other direction, say the y direction. But we shall leave that as an exercise for the reader. Once again we see that a force which is of purely electric origin in one frame (S') has both electric and magnetic constituents from the standpoint of another frame (S).

THE MAGNETIC FORCE ON A MOVING CHARGE DUE TO A CURRENT-BEARING WIRE

For ordinary velocities, the magnetic force between two electric charges is very, very small compared to the electric force—smaller by the factor uv/c^2, for example, for charges moving side by side at speeds u and v. It can be easily observed only if we can manage to get rid of the electric force. Fortunately we can do that, for nature provides both positive and negative charges.

Consider, for example, a long, electrically neutral copper wire in which a current is flowing. The positive copper ions remain stationary and the free negative electrons move, say with a velocity **v**.[1] What is the force on a moving test charge outside the wire? For simplicity let us first consider a test charge q_2 (e.g., an electron) moving at the same velocity **v** as we have assumed for the electrons in the wire [Fig. 8–13(a)]. The densities of positive and negative charges in the wire are equal and opposite, say $\pm\lambda_0$. They produce electric fields, at q_2, which are equal in magnitude ($\mathcal{E} = 2k\lambda_0/b$) but opposite in direction. Therefore the electric force on q_2 is zero. If q_2 were stationary, that would

[1]This is, of course, a gross oversimplification. The conduction electrons in a wire have all sorts of speeds and directions, but **v** represents a steady mean *drift* velocity associated with the net current flow.