If we neglect air resistance, all objects, regardless of their mass, fall to earth with the same acceleration: $g \approx 10 \text{ m/s}^2$.

This means that if they start at the same height, they will both hit the ground at the same time.

**Free fall – velocity and distance**

<table>
<thead>
<tr>
<th>time (s)</th>
<th>speed (m/s)</th>
<th>distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.45</td>
<td>4.5</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>125</td>
</tr>
</tbody>
</table>

Effect of air resistance: terminal velocity

Air resistance increases with speed.

A person who has his/her hands and legs outstretched attains a terminal velocity of about 125 mph.

**Motion with constant acceleration**

- A ball falling under the influence of gravity is an example of what we call motion with constant acceleration.
- Acceleration is the rate at which the velocity changes with time (increases or decreases).
- If we know where the ball starts and how fast it is moving at the beginning we can figure out where the ball will be and how fast it is going at any later time.

**Simplest case: constant velocity**

If the acceleration = 0 then the velocity is constant.

In this case the distance an object will travel in a certain amount of time is given by $\text{distance} = \text{velocity} \times \text{time}$.

For example, if you drive at 60 mph for one hour you go $60 \text{ mph} \times 1 \text{ hr} = 60 \text{ mi}$.

**Example – running the 100 m dash**

Justin Gatlin won the 100 m dash in just under 10 s (9.85 s). Did he run with constant velocity, or was his motion accelerated?

- He started from rests and accelerated, so his velocity was not constant.
- Although his average speed was about $100 \text{ m}/10 \text{ s} = 10 \text{ m/s}$, he probably did not maintain this speed all through the race.
running the 100 m dash

the winner has the highest average speed = 100 m / time

BWS M series Coupe

• BMW claims that it’s M series coupe reaches 60 mph in 5.5 sec – what is it’s average acceleration?
• 60 mph $\Rightarrow$ 88 feet/sec
  
  $\Rightarrow$ acceleration = $\frac{88 \text{ ft/s}}{5.5 \text{ s}} = 16 \text{ ft/s}^2$
  
  • this means that on average the car’s speed increases by 16 ft/s every second

The velocity of a falling ball

• Suppose that at the moment you start watching the ball it has an initial velocity equal to $v_0$
• Then its present velocity ($v$) is related to the initial velocity and acceleration ($a$) by

  present velocity $= v = v_0 + a \times t$

Ball dropped from rest

• If the ball is dropped from rest then that means that its initial velocity is zero, $v_0 = 0$
• Then its present velocity $= a \times t$, where $a$ is the acceleration of gravity $g \approx 10 \text{ m/s}^2$ or $32 \text{ ft/s}^2$, for example:
• What is the velocity of a ball 5 seconds after it is dropped from rest from the top of the Sears Tower?
  
  $\Rightarrow$ $v = 32 \text{ ft/s}^2 \times 5 \text{ s} = 160 \text{ ft/s}$

The position of a falling ball

• Suppose we would like to know where a ball would be at a certain time after it was dropped
• Or, for example, how long would it take a ball to fall to the ground from the top of the Sears Tower (1450 ft).
• Since the acceleration is constant ($g$) we can figure this out!
**Falling distance**

- Suppose the ball falls from rest so its initial velocity is zero.
- After a time \( t \) the ball will have fallen a distance:
  \[
  \text{distance} = \frac{1}{2} \times \text{acceleration} \times t^2
  \]
- or \( d = \frac{1}{2} \times g \times t^2 \)

**Falling from the Sears Tower**

- After 5 seconds, the ball falling from the Sears Tower will have fallen a distance:
  \[
  \text{distance} = \frac{1}{2} \times 32 \text{ ft/s}^2 \times (5 \text{ s})^2 = 16 \times 25 = 400 \text{ feet.}
  \]
- We can turn the formula around to figure out how long it would take the ball to fall all the way to the ground (1450 ft):
  \[
  t = \sqrt{\frac{2 \times \text{distance}}{g}}
  \]

**Look at below!**

- or \( \text{time} = \sqrt{\frac{2 \times \text{distance}}{g}} \)
- \( \text{time} = \sqrt{\frac{2 \times 1450 \text{ ft}}{32 \text{ ft/s}^2}} = \sqrt{\frac{2900}{32}} = \sqrt{90.6} = 9.5 \text{s} \)
- when it hit the ground it would be moving at \( v = g \times t = 32 \text{ ft/s}^2 \times 9.5 \text{ sec} = 305 \text{ ft/s} \) or about 208 mph (watch out!)

**How high will it go?**

- Let's consider the problem of throwing a ball straight up with a speed \( v \). How high will it go?
- As it goes up, it slows down because gravity is pulling on it.
- At the very top its speed is zero.
- It takes the same amount of time to come down as go it did to go up.

**An amazing thing!**

- When the ball comes back down to ground level it has exactly the same speed as when it was thrown up, but its velocity is reversed.
- This is an example of the law of conservation of energy.
- We give the ball some kinetic energy when we toss it up, but it gets it all back on the way down.

**Problem**

- A volleyball player can leap up at 5 m/s. How long is she in the air?
- SOLUTION \( \Rightarrow \) total time \( t_{\text{total}} = t_{\text{up}} + t_{\text{down}} \)
- time to get to top \( t_{\text{up}} = \frac{v_0}{g} \)
- \( t_{\text{up}} = \frac{5 \text{ m/s}}{10 \text{ m/s}^2} = \frac{1}{2} \text{ sec} \)
- \( t_{\text{total}} = \frac{1}{2} + \frac{1}{2} = 1 \text{ sec} \)
So how high will it go?

- If the ball is tossed up with a speed \( v \), it will reach a maximum height \( h \) given by
  \[
  h = \frac{v^2}{2g} \quad \Rightarrow \quad v = \sqrt{\frac{2gh}{2g}}
  \]

- Notice that if \( h = 1 \text{m} \),
  \[
  v = \sqrt{2 \times 10 \times 1} = \sqrt{20} = 4.5 \text{ m/s}
  \]

- this is the same velocity that a ball will have after falling 1 meter.

Problem

- To spike the ball, a volleyball player leaps 125 cm straight up.
  - What was her speed when she left the court?
  - formula \( v = \sqrt{2gh} \)
  - 125 cm = 1.25 m
  - \[
  v = \sqrt{2 \times 10 \text{ m/s}^2 \times 1.25 \text{ m}}
  = \sqrt{25 \text{ m}^2/\text{s}^2}
  = 5 \text{ m/s}
  \]

Example

- Randy Johnson once threw a fastball at 102 mph (45.6 m/s).
  - If he could throw a ball straight up, how high would it go?
  - \[
  h = \frac{v^2}{2g} = (45.6)^2 \div 2 \times 10
  = 2079 \div 20 = 104 \text{ meters (341 ft)}
  \]
  - more than 100 yards, or the length of a football field!

Escape from planet earth

(Not everything that goes up must come down!)

- To escape from the gravitational pull of the earth an object must be given a velocity at least as great as the so called escape velocity
  - For earth the escape velocity is 7 mi/sec or 11,000 m/s, 11 kilometers/sec or about 25,000 mph.
  - An object given this velocity (or greater) on the earth’s surface can escape from earth!