Problems on the 1st Law of thermodynamics and heat engines

Principles

1) **The 1st Law of thermodynamics:** The change in the internal energy of a system is equal to the heat that it absorbs \((Q_{in})\) minus the work that it does \((W_{out})\)

\[
\text{Change in internal energy} = Q_{in} - W_{out}
\]

2) **Heat engines:**

    (a) Heat engines operate in a cycle in which the system is always returned to its original condition; therefore the change in internal energy for the cycle is ZERO. The engine absorbs an amount of heat \(Q_{in}\), performs an amount of work \(W_{out}\), and discards an amount of heat \(Q_{out}\). The energy balance for this engine is then: \(Q_{in} = W_{out} + Q_{out}\)

    (b) **engine efficiency:** \(\text{efficiency} = \frac{W_{out}}{Q_{in}}\), this can be expressed either as a fraction or a percentage.
Heat, work, and internal energy are typically expressed either in Joules (J), calories (cal) or British Thermal Units (BTU).

(Answers are on the next page, but try to do the problems first!)

1. 1000 BTUs of heat are absorbed by a gas while the gas expands and performs 700 BTUs of work. What is the change in the internal energy of the gas in this process?

2. An expanding gas that is in contact with a heat source performs 3000 BTU of work while its internal energy decreases by 1000 BTU. How much heat did the gas absorb from the source?

3. 4000 J of work is done on a gas while it is being compressed. If the internal energy of the gas increased by 2500 J, how much heat flowed into or out of the gas in this process?

4. An engine operating in a cycle absorbs 5000 cal of heat from a heat source and performs 2000 cal of work. How much heat was discarded in this cycle and what is the efficiency of this engine?

5. An engine operating in a cycle absorbs 15,000 BTU of heat and discards 10,000 BTU to a cold reservoir. How much work is done by this engine and what is its efficiency?
SOLUTIONS

Some useful hints: In applying the first law you must keep track of the signs of the quantities involved.

- if the change in internal energy (IE) is positive, then the IE increased; if the change in IE is negative, then IE decreased
- if heat enters (is absorbed by) a system then $Q_{in}$ is positive; if heat leaves the system, then $Q_{in}$ is negative
- if the system does work, then $W_{out}$ is positive; if work is done on the system, then $W_{out}$ is negative

1. Change in IE = $Q_{in} - W_{out} = 1000 \text{ BTU} - 700 \text{ BTU} = 300 \text{ BTU}$
   $\Rightarrow$ IE increases by 300 BTU

2. Change in IE = $Q_{in} - W_{out} \Rightarrow -1000 \text{ BTU} = Q_{in} - 3000 \text{ BTU}$
   $\Rightarrow Q = -1000 \text{ BTU} + 3000 \text{ BTU} = +2000 \text{ BTU}$
   $\Rightarrow$ the gas absorbed 2000 BTU
3. Here, \( W_{\text{out}} = -4000 \text{ J} \) because work is done ON the system

\[
\text{Change in IE} = Q_{\text{in}} - W_{\text{out}} \rightarrow +2500 \text{ J} = Q_{\text{in}} - (-4000 \text{ J}) \\
= Q_{\text{in}} + 4000 \text{ J}
\]

\[ \rightarrow Q_{\text{in}} = 2500 \text{ J} - 4000 \text{ J} = -1500 \text{ J} \]

\[ \rightarrow 1500 \text{ J} \text{ of heat flowed OUT OF the gas} \]

4. \( Q_{\text{in}} = W_{\text{out}} + Q_{\text{out}} \)

\[
5000 \text{ cal} = 2000 \text{ cal} + Q_{\text{out}} \rightarrow Q_{\text{out}} = 3000 \text{ cal}
\]

\[\text{efficiency} = \frac{W_{\text{out}}}{Q_{\text{in}}} = \frac{2000J}{5000J} = 0.4, \text{ or } 40\% \]

5. \( Q_{\text{in}} = W_{\text{out}} + Q_{\text{out}} \)

\[
15,000 \text{ BTU} = W_{\text{out}} + 10,000 \text{ BTU} \rightarrow W_{\text{out}} = 5000 \text{ BTU}
\]

\[\text{efficiency} = \frac{W_{\text{out}}}{Q_{\text{in}}} = \frac{5000\text{BTU}}{15,000} = 0.33, \text{ or } 33\% \]