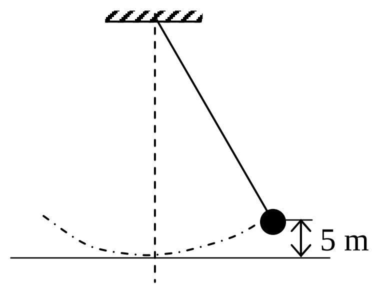
29:006

Exercises on simple harmonic motion
(Try the exercises before looking at the solutions.)

1. A 10 kg mass is attached to a rope that is 100 m long to form a huge pendulum. The mass is pulled aside so that it is 5 meters above its resting point.

(a) How much potential energy (PE) does it have when it is 5 m above its resting point?



(b) When the pendulum is released how much kinetic energy (KE) will it have when it passes through its lowest point? *Hint*: energy is conserved.

(c) How fast will it be moving when it passes through its lowest point?

(d) If it takes 2 seconds for the pendulum to reach its lowest point after it is released, when will it return to its initial position? What is this time called?

2. The period, T of a harmonic oscillator is 4 seconds. What is its frequency f?

3. The frequency, f of a harmonic oscillator is 0.1 Hz. What is its period of oscillation?

4. What force is needed to keep a spring stretched by 10 cm if the spring constant is 20 N/m?

5. When cart of mass m is connected to a hoop spring of spring constant k on the air track, the cart undergoes simple harmonic motion with a period of 5 seconds. The experiment is repeated with a different cart of mass M and it is found that the period is 10 seconds. What is the relationship between m and M?

6. A huge pendulum is made by hanging a 100 kg mass at the end of a rope that is 40 m long.
(a) What is the period of this pendulum?
(b) How many *complete* cycles will this pendulum execute in one minute?

Formulas:

Kinetic Energy KE = ½ mv^2, gravitational potential energy GPE = mgh

frequency f = 1/T, where T is the period of oscillation

Period of a mass-spring oscillator: $T = 2\pi\sqrt{\frac{m}{k}}$, period of a pendulum length L: $T = 2\pi\sqrt{\frac{L}{g}}$

Solutions:

1. (a) PE = m g h = 10 kg · 10 m/s^2 · 5 m = 500 J

(b) all of the PE is converted to KE at the bottom, so KE = PE = 500 J

(c) KE = ½ m v^2 → $v = \sqrt{\frac{2 \bullet KE}{m}} = \sqrt{\frac{2 \bullet 500\, J}{10\, kg}} = \sqrt{\frac{1000}{10}} = \sqrt{100} = 10\, m/s$

(d) It takes 2 seconds to get to the bottom, another 2 seconds to rise to its highest point on the left side, 2 seconds to get back down to the bottom and another 2 seconds to get back to its starting point, so the total is 2 + 2 + 2 + 2 = 8 seconds. This time is called the period, T of oscillation.

2. $f = \frac{1}{T} = \frac{1}{4\, s} = \frac{1}{4} Hz = 0.25\, Hz$

3. $T = \frac{1}{f} = \frac{1}{0.1\, Hz} = \frac{1}{\frac{1}{10} Hz} = 10s$

4. Magnitude of force, F = k(N/m) ·x (amount of stretch in m) = 20 N/m · 0.10 m = 2 N.

5. $T = 2\pi\sqrt{\frac{m}{k}}$ → to double T, the mass must increase by a factor of 4 , since $\sqrt{4} = 2$.

Therefore M = 4 ·m.

6. (a) $T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{40m}{10m/s^2}} = 2\pi\sqrt{4} = 4\pi = 12.6\ s$

(b) f = 1/T = 0.079 Hz or 0.079 cycles per second, thus in one minute (60 s) this pendulum will execute 0.079 cycles/s × 60 s = 4.76 cycles, or 4 complete cycles