Formulas

- Kinetic Energy \( KE = \frac{1}{2} mv^2 \)
- gravitational potential energy \( GPE = mgh \)
- frequency \( f = \frac{1}{T} \), where \( T \) is the period of oscillation
- Period of a mass-spring oscillator: \( T = 2\pi \sqrt{\frac{m}{k}} \)
- period of a pendulum: \( T = 2\pi \sqrt{\frac{L}{g}} \)

Exercises

1. A 10 kg mass is attached to a rope that is 100 m long to form a huge pendulum. The mass is pulled aside so that it is 5 meters above its resting point.

   (a) How much potential energy (PE) does it have when it is 5 m above its resting point?

   (b) When the pendulum is released how much kinetic energy (KE) will it have when it passes through its lowest point? \textit{Hint:} energy is conserved.

   (c) How fast will it be moving when it passes through its lowest point?

   (d) If it takes 2 seconds for the pendulum to reach its lowest point after it is released, when will it return to its initial position? What is this time called?

2. The period, \( T \) of a harmonic oscillator is 4 seconds. What is its frequency \( f \)?

3. The frequency, \( f \) of a harmonic oscillator is 0.1 Hz. What is its period of oscillation?

4. What force is needed to keep a spring stretched by 10 cm if the spring constant is 20 N/m?
5. When cart of mass \( m \) is connected to a hoop spring of spring constant \( k \) on the air track, the cart undergoes simple harmonic motion with a period of 5 seconds. The experiment is repeated with a different cart of mass \( M \) and it is found that the period is 10 seconds. What is the relationship between \( m \) and \( M \)?

6. A huge pendulum is made by hanging a 100 kg mass at the end of a rope that is 40 m long.
   (a) What is the period of this pendulum?
   (b) How many complete cycles will this pendulum execute in one minute?

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**Solutions**

1. (a) \( PE = m \cdot g \cdot h = 10 \text{ kg} \cdot 10 \text{ m/s}^2 \cdot 5 \text{ m} = 500 \text{ J} \)

   (b) all of the PE is converted to KE at the bottom, so \( KE = PE = 500 \text{ J} \)

   (c) \( KE = \frac{1}{2} \cdot m \cdot v^2 \rightarrow v = \sqrt{\frac{2 \cdot KE}{m}} = \sqrt{\frac{2 \cdot 500 \text{ J}}{10 \text{ kg}}} = \sqrt{\frac{1000}{10}} = \sqrt{100} = 10 \text{ m/s} \)

   (d) It takes 2 seconds to get to the bottom, another 2 seconds to rise to its highest point on the left side, 2 seconds to get back down to the bottom and another 2 seconds to get back to its starting point, so the total is \( 2 + 2 + 2 + 2 = 8 \text{ seconds} \). This time is called the period, \( T \) of oscillation.

2. \( f = \frac{1}{T} = \frac{1}{4 \text{ s}} = \frac{1}{4} \text{ Hz} = 0.25 \text{ Hz} \)

3. \( T = \frac{1}{f} = \frac{1}{0.1 \text{ Hz}} = \frac{1}{10 \text{ Hz}} = 10 \text{ s} \)

4. Magnitude of force, \( F = k(N/m) \cdot x \) (amount of stretch in m) = \( 20 \text{ N/m} \cdot 0.10 \text{ m} = 2 \text{ N} \).

5. \( T = 2\pi \sqrt{\frac{m}{k}} \rightarrow \) to double \( T \), the mass must increase by a factor of 4, since \( \sqrt{4} = 2 \).

   Therefore \( M = 4 \cdot m \).

6. (a) \( T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{40 \text{ m}}{10 \text{ m/s}^2}} = 2\pi \sqrt{4} = 4\pi = 12.6 \text{ s} \)

   (b) \( f = 1/T = 0.079 \text{ Hz} \) or 0.079 cycles per second, thus in one minute (60 s) this pendulum will execute 0.079 cycles/s \( \times 60 \text{ s} = 4.76 \) cycles, or 4 complete cycles