

## L-11 Rotational Inertia

- Rotational (angular) Momentum
- Conservation of angular momentum



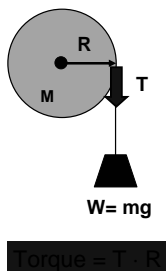
Why is a bicycle stable (it doesn't fall over) only when it is moving?

## Rotational inertia → symbol I

- Rotational inertia is a parameter that is used to quantify how much torque it takes to get a particular object rotating
- it depends not only on the mass of the object, but where the mass is relative to the hinge or axis of rotation
- the rotational inertia is bigger, if more mass is located *farther* from the axis.

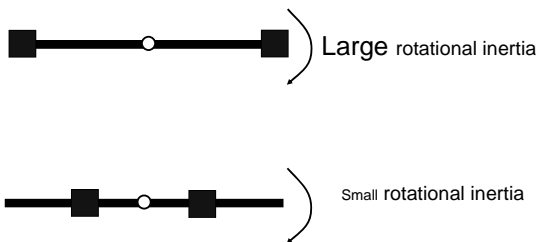
## Rotational inertia and torque

- To start an object spinning, a torque must be applied to it
- The amount of torque required depends on the rotational inertia (I) of the object
- The rotational inertia (I) depends on the mass of the object, its shape, *and on how the mass is distributed*
- Solid disk:  $I = \frac{1}{2} M R^2$
- The higher the rotation inertia, the more torque that is required to make an object spin



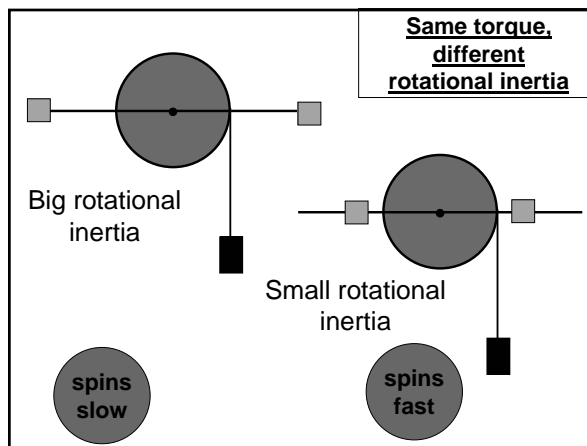
## rotational inertia examples

2 objects have identical mass and length

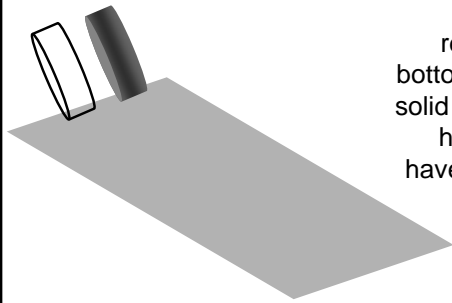


## How fast does it spin?

- For spinning or rotational motion, the rotational inertia of an object plays the same role as ordinary mass for simple motion
- For a given amount of torque applied to an object, its rotational inertia determines its rotational acceleration → the smaller the rotational inertia, the bigger the rotational acceleration



### Rolling down the incline



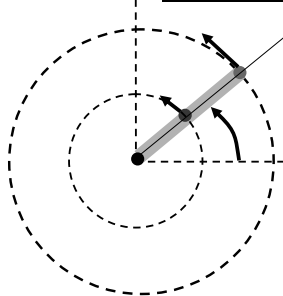
Which one reaches the bottom first, the solid disk or the hoop? They have the same mass and diameter.

The solid disk gets to the bottom faster because it has the smaller rotational inertia

### Speed of rotation

- For motion in a straight line we tell how fast you go by the velocity meters per second, miles per hour, etc.
- How do we indicate how fast something rotates?
- We use a parameter called **rotational velocity**, (symbol- Omega  $\Omega$ ) simply the number of revolutions per minute for example -- the number of times something spins say in a second or minute (rpm's- revs per min)
- for example the rotational speed of the earth spinning on it axis is 1 revolution per day or 1 revolution per 24 hours.

### Ordinary (linear) speed and rotational speed



- the rod is rotating around the circle in the counterclockwise direction
- ALL points on the rod have the **SAME** rotational speed
- The red point in the middle has only half the linear speed as the blue point on the end.

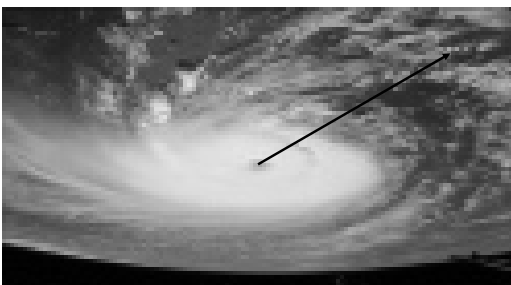
every point on the line moves through the same angle

### Ice Capades



Skaters farther from center must skate faster

### Hurricanes



Most dangerous winds are at edges of hurricane

### Conservation of linear momentum

- If an object is moving with velocity  $v$ , it has *linear momentum*:  $p = m v$
- If no outside forces disturb the object, its linear momentum is conserved
- If 2 objects interact (e.g., collide) the forces are equal and opposite and cancel each other so the linear momentum of the pair is conserved.

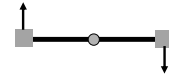
$$(p_A + p_B)_{\text{before}} = (p_A + p_B)_{\text{after}}$$

## Rotational (angular) momentum $J$

- A spinning object has rotational momentum  
→ symbol  $J$
- rotational momentum ( $J$ ) =  
rotational inertia ( $I$ ) x rotational velocity ( $\Omega$ )
- $J = I \times \Omega$

## Conservation of rotational momentum

- If no outside torques disturb a spinning object, its rotational momentum is conserved
- The rotating masses on the rod keep spinning until the friction in the bearing slows it down. Without friction, it would keep spinning.
- Note that the total linear momentum is zero so that does not come into play.



## Rotational momentum

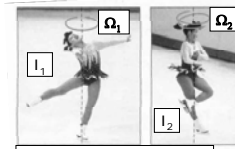
- $J = \text{rotational inertia } (I) \times \text{angular velocity } (\Omega)$
- since the rotational momentum can't change then if the rotational inertia changes, the rotational velocity must also change to keep the rotational momentum constant
- Or,  $I_1 \Omega_1 = I_2 \Omega_2$
- If the rotational inertia increases, then the rotational velocity must decrease
- if the rotational inertia decreases, then the rotational velocity must increase

## Rotational momentum demonstrations

- spinning ice skater
- divers
- Hobermann sphere
- bicycle wheel
- top
- gyroscope



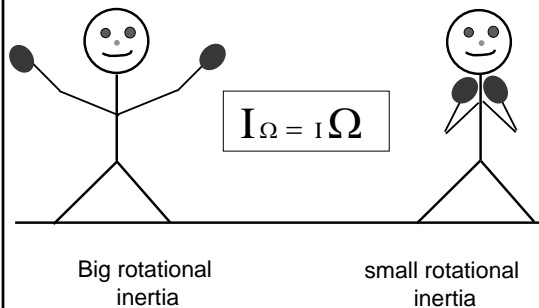
VIDEO



Conservation of  $J$ :  
 $I_2 < I_1 \rightarrow \Omega_2 > \Omega_1$

Objects that have rotational momentum (SPIN) tend not to lose it easily → Bicycles

## You can change your rotational inertia



## Spinning faster or slower

- When your arms are extended you have a big rotational inertia
- When you pull your arms in you make your rotational inertia smaller
- If you were spinning with your arms out, when you pull your arms in you will spin faster to keep your rotational momentum constant
- This works in figure skating and diving

## Example

- A figure skater has a rotational inertia  $I_1$  when her arms are stretched out, and  $I_2$  when her arms are pulled in close to her body. If her angular velocity is  $\Omega_1$  when she spins with her arms stretched out, what is her angular velocity when she pulls her arms in, so that  $I_2 = \frac{1}{2} I_1 = 0.5 I_1$
- Solution:** Angular momentum is conserved, so
 
$$I_1 \Omega_1 = I_2 \Omega_2$$
 since  $I_1/I_2 = 1 / 0.5 = 2$ ,  $\Omega_2 = 2 \Omega_1$ 
 → She doubles her angular speed.

## Spinning wheel defies gravity!

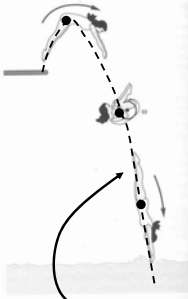
**Gyroscope**- an object that can spin and rotate about three axes



Once it starts spinning its axle wants to keep spinning in the same direction. It resists forces that try to change the direction of its spin axis.

**spinning wheel**

## Divers use rotational momentum conservation to spin



- the diver starts spinning when she jumps off the board
- when she pulls her arms and legs in she makes her rotational inertia smaller
- this makes her spin even faster!
- Her CG follows the same path as a projectile

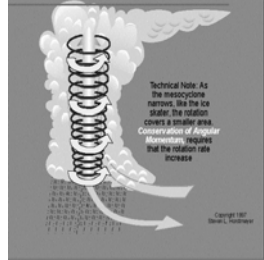
## Natural effects due to conservation of J

- The length of the day is determined by the time it takes the Earth to complete one full spin about its axis. Big earthquakes can alter the distribution of mass in the earth's crust. The distribution of mass determines the rotational inertia of the earth.
- The Moon is getting farther from the Earth because the Earth's daily rotation is slowing down due to friction of the ocean waters on the ocean bottom. The decrease in the angular momentum of the Earth is accompanied by an increase in the angular momentum of the Moon in its orbit around the Earth.

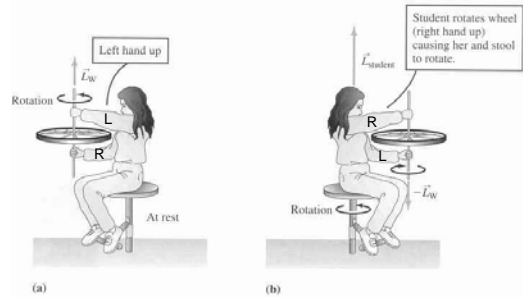
## Tornadoes



The intense updraft stretches the mesocyclone vertically and (like the ice skater with arms pulled in near the body) the rotation speed increases...



## Don't fall off the stool!



<http://www.youtube.com/watch?v=V3UsrfHa4MQ>