Dust Ion-Acoustic Shocks in a Q Machine Device

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Dust ion-acoustic shocks in a Q machine device are considered. For their description the so-called hydrodynamic ionization source model is used. The model is appropriate for the description of the laboratory experiments in a Q machine device and contains the most important basic mechanisms responsible for the formation of the dust ion-acoustic shocks. A comparative analysis of various dissipative processes occurring on ion-acoustic time scales during the excitation and propagation of nonlinear dust ion-acoustic perturbations in a complex (dusty) plasma is performed in terms of a purely kinetic approach and a hydrodynamic approach. It is found that the most important dissipative processes are the charging of dust grains, the absorption of ions by grains, the transfer of the ion momentum to the grains, and Landau damping.

1 Introduction

A complex (dusty) plasma is the plasma containing electrons, ions, neutrals, and dust microscopic particles which are composed of either solid or liquid material [1–6]. At present, the problem of the excitation and propagation of nonlinear ion-acoustic perturbations occupies an important place in the physics of complex plasma. Interest in this kind of research is associated primarily with the fact that the processes of dust grain charging are far from equilibrium precisely on ion-acoustic time scales, so that the anomalous dissipation, which, by its very nature, originates from the charging process, often plays a decisive role [7]. It is this anomalous dissipation mechanism that is responsible for the existence of a new kind of shock waves [8] that are "collisionless" in the sense that they are almost completely insensitive to electron-ion collisions. However, in contrast to classical collisionless shock waves, the dissipation due to dust charging involves interaction of electrons and ions with dust grains through microscopic electron and ion currents to the grain surfaces. The anomalous dissipation plays a very important role in the propagation of other dust ion-acoustic nonlinear structures, *e.g.*, in the case of the so-called "weakly dissipative" dust ion-acoustic solitons [9], whose shape is described by soliton solutions in a certain range of values of the Mach number. Because of the anomalous dissipation, these solitons are slowed down and damped.

Dust ion–acoustic shock waves were observed in a Q machine device at the University of Iowa (USA) [10] and in a double plasma device at the Institute of Space and Astronautical Science (Japan) [11] almost simultaneously. Experiments on shock waves in complex plasmas are conducted in a number of major research laboratories throughout the world. In particular, in strongly coupled complex plasmas Mach cone shocks were reported [12, 13]. An experimental observation of a shock wave of sufficient strength to melt a two-dimensional plasma crystal, through which it propagates was presented [14]. Dust acoustic shocks were observed in a microgravity experiment onboard the International Space Station [15]. There are also plans to carry out experiments on dust ion–acoustic shock waves during the mission of the International Space Station.

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Dust ion–acoustic shocks can have important applications in the description of natural phenomena such as those occurring in the interaction of the solar wind with dusty cometary comas [16] and also may find significant technological applications in, *e.g.*, the so-called hypersonic aerodynamics [17].

The advantage of the experiments carried out in a Q machine device is the negligible role of neutral atoms in the dissipation processes (in particular, ion-atom collisions are unimportant), which is caused by relatively low neutral atom densities. The negligible role of the dissipation processes with the participation of neutral atoms enables to study the relative roles played by the dissipative processes such as the charging of dust grains, the absorption of ions by grains, the transfer of the ion momentum to the grains, and Landau damping.

In the case when one can neglect the dissipation due to the interaction with neutrals, the nonlinear dust ion-acoustic structures can be described theoretically by solving a set of hydrodynamic equations that is specially derived for a complex plasma from the kinetic equations for electrons, ions, and dust grains [18]. These equations are the basis for the so-called hydrodynamic ionization source model which allows us to obtain [19, 20] dust ion-acoustic shock structures as a result of evolution of an initial perturbation and to explain the experimental value of the width of the shock wave front.

Of course, in the hydrodynamic derivation, such an important, purely kinetic effect as Landau damping is not taken into consideration. However, in some situations, Landau damping can play an important role. Thus, the fact that, in laboratory experiments in a Q machine device, dust ion-acoustic waves were not observed at sufficiently low dust densities was attributed precisely to this damping [10].

It should be noted that there are different approaches to describing Landau damping in complex plasmas. First of all, we must mention the papers in which the corresponding damping rates were calculated without allowance for the grain charging (see, *e.g.*, [21]). As early as 1993 [7], it became clear that the charging of dust grains has a significant impact on the damping described at the kinetic level (which will be referred below to as kinetic damping), part of which is Landau damping. Consequently, dust grain charging should be taken into account in calculations. Nevertheless, theoretical studies are still often conducted based on the results of [21], in particular, because the expression for the kinetic damping rate of ion-acoustic waves that takes into account dust grain charging and could be used to analyze the results of complex plasma experiments has not yet been given in a compact form. As for the results that are presented in [7, 22] and could be used to calculate the corresponding damping rates, they either have a complicated integro-operator form or refer to the limiting cases irrelevant to the present-day experiments. Moreover, in [7, 22], the final formula for the dielectric function of a complex plasma, which is important for deriving the expression for the kinetic damping rate, involves contradictions. All this goes to show that it is necessary to refine the expression for the kinetic damping rate of dust ion-acoustic waves and to reduce it to a compact form, convenient for analyzing the experimental results.

Since dissipation is one of the processes that play a key role in the formation and propagation of shock waves, the hydrodynamic approach to describing these nonlinear structures is valid only if dissipative processes that are taken into account in the hydrodynamic equations turn out to be more important than Landau damping. Hence, it is important to classify dissipative processes and determine the ranges of plasma parameters in which some particular processes dominate. Our objective here is to give a theoretical description of dust ion-acoustic shock structures observed in a Q machine device, and using these theory findings as well as the laboratory results to analyze dissipative processes occurring during the propagation of ion-acoustic shock structures in a complex plasma. In Section 2, we present the results of numerical simulations of shock waves in the experiments [10] by means of a hydrodynamic ionization source model. In Section 3, we analyze anomalous dissipative processes in terms of the hydrodynamic equations, which are usually used to describe nonlinear dust ion-acoustic structures, investigate the effect of kinetic damping, and compare this effect with the relative effects of anomalous dissipative processes in the experiments with dust ion-acoustic shocks in a Q machine device. We use the derivation of an expression for the kinetic damping rate of the ion-acoustic waves on the basis of a purely kinetic approach to describing a complex plasma, which is carried out in Appendix A. In Section 4, we summarize the main results and conclusions of our study.

2 Interpretation of experiments

The experiments [10] performed in a Q machine device (that was modified to allow the introduction of dust grains into the plasma) were performed with Cs^+ ions. The plasma parameters of the experiments were $T_e \approx T_i \approx 0.2$

eV, $n_{i0} \sim 10^6 - 10^7$ cm^{-3}, $a \sim 0.1 - 1$ μm. The quantity $\epsilon Z_{d0} \equiv n_{d0} Z_{d0}/n_{i0}$ (estimated from measurements of the ion and electron saturation currents to the Langmuir probe [23, 24]) was varied from 0 to 0.95. Here, $T_{e(i)}$ is the electron (ion) temperature, n_i is the ion density, n_d is the dust density, $q_d = -Z_d e$ is the dust grain charge, $-e$ is the electron charge, a is the grain radius, and the subscript 0 stands for the unperturbed plasma parameters.

Here we present an interpretation of the experiments [10] with the aid of the so-called hydrodynamic ionization source model [19, 20]. For this purpose, we modify this model in the following way. In [19, 20], the ionization source term was chosen to correspond to conventional electron impact ionization of neutrals (as is traditionally done in describing complex plasmas) and, accordingly, was proportional to the electron density. However, in the laboratory experiments performed in a Q machine device, cesium ions in the plasma were produced through ionization of cesium atoms at the hot plate surface. Consequently, the ionization source term in the evolutionary equation for the ion density should be independent of the electron density n_e [25]. Additionally, in contrast to the model of [19, 20], we take into account the effect of the gas-kinetic ion pressure on the evolution of the complex plasma.

The evolutionary equations for the ions in planar geometry, thus, take the form [25]

$$\partial_t n_i + \partial_x (n_i v_i) = -\nu_{\mathrm{ch}} n_i + S_i, \tag{1}$$

$$\partial_t (n_i v_i) + \partial_x \left(n_i v_i^2\right) = -\frac{e n_i}{m_i}\,\partial_x \varphi - \frac{T_i}{m_i}\,\partial_x n_i - \tilde{\nu} n_{0i} v_i. \tag{2}$$

Here, v_i is the ion velocity, S_i is the ionization source intensity (its value is chosen so that it exactly cancels the term describing the absorption of ions by dust grains in an unperturbed complex plasma), m_i is the ion mass, φ is the electrostatic potential, the rate ν_{ch} at which the ions are absorbed by the dust grains is equal to

$$\nu_{\mathrm{ch}} = \nu_q \frac{Z_{d0} d}{1 + Z_{d0} d} \frac{(\tau + z_0)}{z_0 (1 + \tau + z_0)}, \tag{3}$$

$d = n_{d0}/n_{e0}$, $\tau = T_i/T_e$, $\nu_q = \omega_{pi}^2 a (1 + z_0 + \tau)/\sqrt{2\pi} v_{Ti}$ is the grain charging rate, $\omega_{pi} = \sqrt{4\pi n_{i0} e^2/m_i}$ is the ion plasma frequency, $z = Z_d e^2/a T_e$, $v_{Ti} = \sqrt{T_i/m_i}$ is the ion thermal velocity, the rate $\tilde{\nu}$ at which the ions lose their momentum as a result of their absorption on the grain surfaces and their Coulomb collisions with the grains has the form

$$\tilde{\nu} = \nu_q \frac{Z_{d0} d}{(1 + Z_{d0} d)\, z_0 (1 + \tau + z_0)} \left(z_0 + \frac{4\tau}{3} + \frac{2 z_0^2}{3\tau} \Lambda \right), \tag{4}$$

$\Lambda = \ln\left(\lambda_{Di}/\max\{a, b\}\right)$ is the Coulomb logarithm, $\lambda_{Di} = \omega_{pi}/v_{Ti}$ is the ion Debye length, and $b = Z_{d0} e^2/T_i$. Expressions (3) and (4) are valid in the range $v_i/c_s < 1$.

In addition, this modification of the hydrodynamic ionization source model includes the equation describing Boltzmann distribution for electrons, Poisson's equation for the electrostatic potential, and the equations of the orbit-limited probe model [26, 27] describing the variation of the dust particle charge, which are the same as in Refs. [19, 20].

We test the agreement of the conclusions of the theory based on the hydrodynamic ionization source model with the main experimental results [10] on shocks in a Q machine device:

(i) Dust ion-acoustic shocks are generated at sufficiently high dust densities (under the experimental conditions of [10], at dust densities such that $\epsilon Z_{d0} \equiv n_{d0} Z_{d0}/n_{i0} \geq 0.75$). The conclusion about the formation of a shock wave is drawn from the fact that the perturbation front steepens as time elapses. At sufficiently low dust densities, the perturbation front does not steepen but instead widens.

(ii) When the shock wave structure has formed, the shock front width $\Delta\xi$ is described by the theoretical estimate, which is based on the model developed in [8] (see also [20])

$$\Delta\xi \sim M c_s/\nu_q, \tag{5}$$

where $M c_s$ is the shock wave speed, M is the Mach number, $c_s = \sqrt{T_e/m_i}$ is the ion-acoustic speed.

(iii) The velocity of the dust ion-acoustic shocks increases considerably with increasing ϵZ_{d0}.

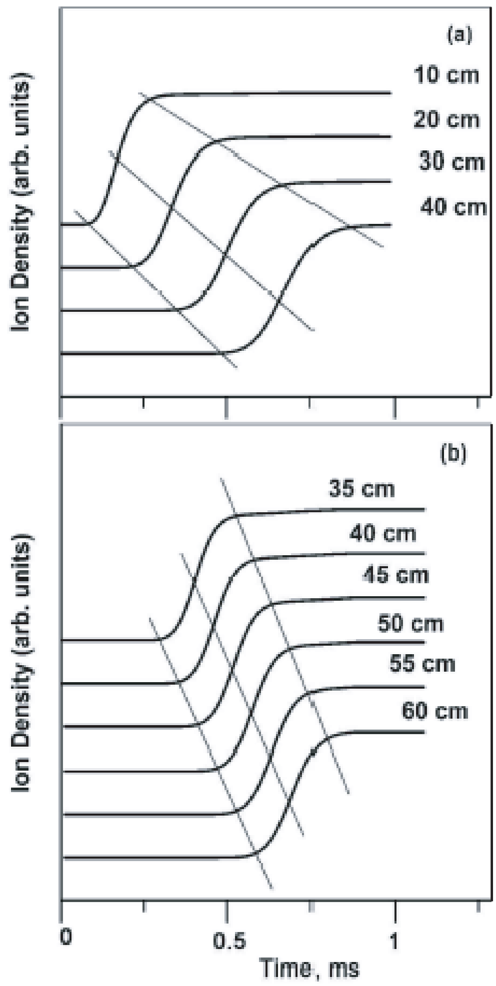

Fig. 1 Time evolutions of the ion density (heavy curves) at different distances from the grid for $\epsilon Z_{d0} =$ (a) 0 and (b) 0.75, obtained from the calculations on the basis of the hydrodynamic ionization source model. The remaining parameters of the complex plasma and of the perturbation are as follows: $T_e = T_i = 0.2$ eV, $n_{i0} = 1.024 \cdot 10^7$ cm^{-3}, $a = 0.1$ μm, $\Delta x = 25$ cm, and $\Delta n_i/n_{i0} = 2$. The light lines show the widening of the wave front (at $\epsilon Z_{d0} = 0$) and its steepening (at $\epsilon Z_{d0} = 0.75$), which agrees with the experimental data.

(iv) In the case where dust is present, the amplitude of the shock decreases as we move away from the grid (which is used to produce the plasma density pulses [10]). Such a decrease in the shock amplitude is associated, in particular, with an attenuation of the ion flux as the ions pass through the region of the dust and can characterize the dissipation in the system.

Our calculations were based on the computational method developed in [20] in order to investigate the evolution of the initial perturbation in a complex plasma with variable-charge dust grains. We used the following values of the plasma parameters: the electron and ion temperatures were equal to one another, $T_e = T_i = 0.2$ eV; the background ion density $n_{i0} = 1.024 \cdot 10^7$ cm^{-3} was the same for all series of simulations; the grain radius was $a = 0.1$ μm; the width of the rectangular initial perturbation was $\Delta x = 25$ cm; and the excess initial perturbed ion density above the background ion density in the remaining unperturbed plasma was $\Delta n_i/n_{i0} = 2$ (see Fig. 2 in [10]). The calculations were carried out for different values of the parameter ϵZ_{d0}.

In Fig. 1 (which is constructed analogously to Fig. 2 in [10]), we illustrate the time evolution of the ion density at different distances from the grid. The time evolutions (heavy curves) were calculated for $\epsilon Z_{d0} = 0$ (a) and $\epsilon Z_{d0} = 0.75$ (b). The light curves show the widening of the wave front (at $\epsilon Z_{d0} = 0$) and its steepening (at $\epsilon Z_{d0} = 0.75$). This agrees with the experimental data.

The extent to which the shock front widens was calculated to be $\Delta\xi/Mc_s \sim 0.3$ ms (see Fig. 1(b)), which corresponds to that observed experimentally (see Fig. 2(b) in [10]) and also to estimate (5), obtained using the theoretical model of [8].

The initial perturbation evolves in such a way that its front velocity V_p becomes nearly constant about 1 ms after it starts propagating through the background plasma. Fig. 2 shows the dependence of the perturbation front

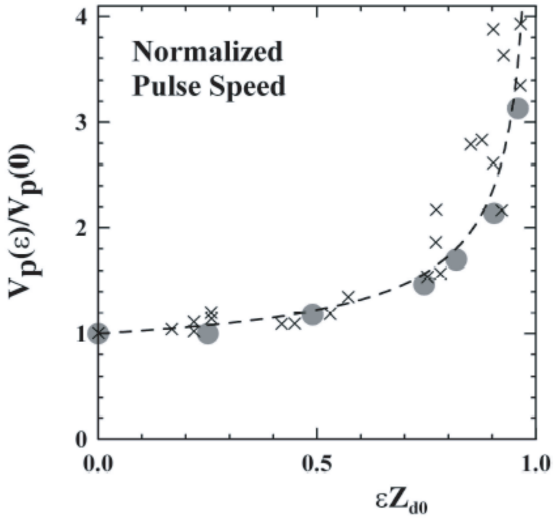

Fig. 2 Dependence of the perturbation front velocity (normalized to its value in the absence of dust) on ϵZ_{d0}. The crosses refer to the experimental points and the calculated results are represented by the closed circles. The linear dust ion–acoustic wave speed is shown as the dashed curve.

velocity (normalized to its value in the absence of dust, $\epsilon = 0$) on the parameter ϵZ_{d0}. For comparison, we also plot the experimental points (crosses). The calculated results are represented by closed circles. The agreement between theory and experiment is quite good. We emphasize that the linear dust ion-acoustic wave speed given in [10] and shown as the dashed curve in Fig. 2 also provides a good description of the experimental results. The reason is a weak dependence of the velocity of a nonlinear dust ion-acoustic wave on the dust density (or, more precisely, on ϵZ_{d0}). The velocity of the nonlinear dust ion-acoustic wave therewith depends strongly on the amplitude of the wave. Such a conclusion is confirmed, in particular, by the results of the investigation of the dust ion-acoustic solitons [9], where it was shown that the form of the evolving compressive soliton (damped and slowing down due to the interaction with dust) is given by the "conservative" soliton solution for the corresponding Mach number at any moment. This can be explained only if the velocity of the evolving dust ion-acoustic soliton depends mainly on its amplitude.

The effect of a decrease in the shock amplitude as we move away from the grid (see Fig. 2(b) in [10]) is not most conspicuous in the numerical plot (Fig. 1(b)) calculated for rather small dust size $a = 0.1$ μm and ion density $n_{i0} = 1.024 \cdot 10^7$ cm^{-3}. The attenuation of the ion flux in the vicinity of the shock front (as the shock passes through the region of the dust) calculated numerically for the same data as Fig. 1(b) is only about seven per cent. Numerical analysis shows that the decrease in the shock amplitude and the attenuation of the ion flux manifest themselves stronger with increase in the dust size and the ion density.

Hence, the theoretical hydrodynamic ionization source model makes it possible to describe all the main experimental results on dust ion-acoustic shock waves. A further development of the model and refinement of the results involve an account of the effects of dust density nonuniformity in experimental devices.

3 Dissipative Processes

Dissipation is one of the processes that play a key role in the formation and propagation of dust ion-acoustic shocks. An analysis of the dispersion properties of ion-acoustic waves on the basis of the set of equations of the hydrodynamic ionization source model yields the following expression for the linear damping rate

$$\gamma_{\mathbf{k}} \approx -\Gamma \equiv -\frac{\nu_{\rm ch} + \tilde{\nu}}{2}. \tag{6}$$

It is clear that, in terms of the hydrodynamic ionization source model, the dissipation in a complex plasma is governed by the processes of absorption of ions by dust grains and also by Coulomb collisions between ions and dust grains. All these processes are closely related to the mechanisms by which the grains are charged. In fact, on the one hand, we have $\Gamma \propto \nu_q$; and, on the other, we see that the absorbed ions participate directly in dust grain charging.

The Landau damping can also be of significant importance. It is well known [28] that in the absence of dust in a Q machine plasma with $T_e = T_i$, ion-acoustic waves are very heavily damped by the Landau damping. The above hydrodynamic approach to describing these structures is valid only if dissipative processes that are taken into account in the hydrodynamic equations turn out to be more important than the Landau damping. In this Section we classify dissipative processes and determine the ranges of plasma parameters in which some particular processes dominate.

To evaluate the role of different dissipative processes in the propagation of dust ion-acoustic shocks we derive (see Appendix A) an expression for the kinetic damping rate of the ion–acoustic waves on the basis of a purely kinetic approach to describing a complex plasma. In the case $\omega_{\mathbf{k}}^s \gg \nu_q$ it takes the form

$$\gamma_{\mathbf{k}}^L = \gamma_{\mathbf{k}}^{L,R} + \gamma_{\mathbf{k}}^{L,q}, \tag{7}$$

where

$$\gamma_{\mathbf{k}}^{L,R} \approx -\sqrt{\frac{\pi}{8}\frac{m_e}{m_i}\frac{n_{i0}}{n_{e0}}}\frac{\omega_{\mathbf{k}}^s}{(1+|\mathbf{k}|^2\lambda_{De}^2)^{3/2}}$$
$$\left(1+\frac{n_{i0}}{n_{e0}}\sqrt{\frac{T_e^3}{T_i^3}}\sqrt{\frac{m_i}{m_e}}\exp\left[-\frac{T_e n_{i0}}{2T_i n_{e0}(1+|\mathbf{k}|^2\lambda_{De}^2)}\right]\right), \tag{8}$$

$$\gamma_{\mathbf{k}}^{L,q} = -\nu_q\sqrt{\frac{\pi}{2}}\frac{Z_{d0}d}{z_0}\frac{(t+z_0)}{(1+t+z_0)(1+|\mathbf{k}|^2\lambda_{De}^2)}, \tag{9}$$

m_e is the electron mass, $\lambda_{De} = \sqrt{T_e/4\pi n_{e0}e^2}$ is the electron Debye length, $\mathbf{k}$ is a wave number, $d = n_{d0}/n_{e0}$, $t = T_i/T_e$, and

$$\omega_{\mathbf{k}}^s \approx \frac{|\mathbf{k}|c_s\sqrt{n_{i0}/n_{e0}}}{\sqrt{1+|\mathbf{k}|^2\lambda_{De}^2}} \tag{10}$$

is the linear dispersion relation for dust ion-acoustic waves.

The first term $\gamma_{\mathbf{k}}^{L,R}$ on the right-hand side of relationship (7) describes ordinary Landau damping on electrons and ions, and the second term $\gamma_{\mathbf{k}}^{L,q}$ describes damping due to the interaction of electrons and ions with dust grains. The rates of these two damping processes are both referred to as the kinetic damping rate. The introduction of the common term is justified because, in a complex plasma, these processes are inseparable. This is most strikingly exemplified in [7], in which the damping of dust ion-acoustic waves was considered in the case $\omega_{\mathbf{k}}^s \ll \nu_q$, opposite to the case treated here. It follows from this example that, even when the resonant denominators describing the damping in the dielectric response functions of the electrons and ions correspond to conventional Landau poles, a new kind of collisionless damping arises that differs from ordinary Landau damping and is associated with the dust grain charging processes.

We emphasize, that there are other approaches to describing Landau damping of dust ion-acoustic waves (see [21]). In [21], only the first term $\gamma_{\mathbf{k}}^{L,R}$ in kinetic damping rate (7) was taken into account. However, in some situations typical of present-day experiments with complex plasmas, the second term $\gamma_{\mathbf{k}}^{L,q}$ predominates over the first term. In fact, for the data of the the experiments on dust ion-acoustic shocks in a Q machine device, cesium vapor plasma with $T_e = T_i = 0.2$ eV, $a = 0.1$ μm, and the characteristic wave vector $|\mathbf{k}| \sim 2\pi/\Delta\xi \sim \nu_q/Mc_s$ corresponding to the characteristic width (5) of the front of the shock wave associated with anomalous dissipation, the second term (with ν_q) in relationship (7) is larger than the first term under the condition $\epsilon Z_{d0} > 0.6$. To obtain this condition we take into account the dependence of the Mach number M on ϵZ_{d0} (see Fig. 2). The dust ion-acoustic shock (front steepening) is observed in a Q machine device when the condition $\epsilon Z_{d0} > 0.6$ is satisfied. We thus arrive at the conclusion that dust grain charging processes can substantially modify the rate of kinetic damping of dust ion-acoustic perturbations; moreover, in many situations, it is these charging processes that dominate the kinetic damping mechanism.

The hydrodynamic equations would not generally apply to ion-acoustic waves, since the kinetic effects are not included in them. However, even in the absence of dust the ion-acoustic waves can be described by the fluid equations, which are identical to those for a neutral fluid and it is known that those equations have solutions that

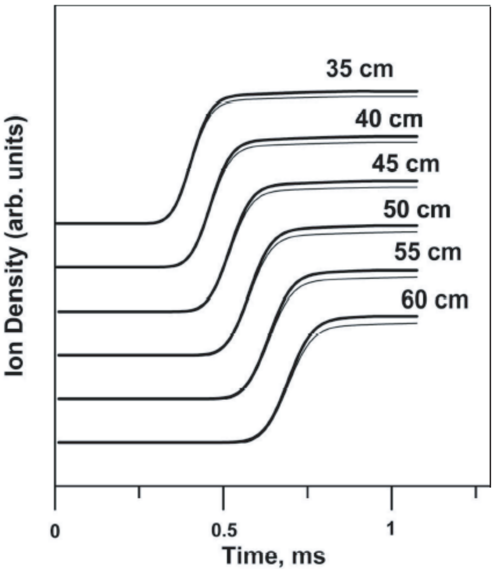

Fig. 3 Time evolutions of the ion density at different distances from the grid for $\epsilon Z_{d0} = 0.75$. The parameters of the complex plasma and of the perturbation are $T_e = T_i = 0.2$ eV, $n_{i0} = 1.024 \cdot 10^7$ cm^{-3}, $a = 0.1$ μm, $\Delta x = 25$ cm, and $\Delta n_i/n_{i0} = 2$. The heavy curves show the time evolutions of the ion density calculated by using the ionization source model. The light curves illustrate the deviations from these time evolutions due to kinetic damping (including Landau damping).

show pulse steepening [29]. The increase in the dust density leads to the diminution of the role of the Landau damping. This is consistent with the data of the experiments [30] which have established that the presence of negatively charged dust greatly reduces the strength of the Landau damping of dust ion-acoustic waves, even in a plasma with equal ion and electron temperatures. Therefore hydrodynamic equations which contain the steepening effect can be applied in the presence of dust too.

A simple criterion for determining whether the hydrodynamic ionization source model is applicable to nonlinear dust ion-acoustic structures is the condition $\Gamma > \gamma^L_{\mathbf{k}}$. The validity of this condition means that the dissipative processes that are taken into account in the hydrodynamic equations are more important than the kinetic damping. For cesium vapor plasma with $T_e = T_i = 0.2$ eV, $a = 0.1$ μm, $n_{i0} = 1.024 \cdot 10^7$ cm^{-3}, and the characteristic wave vector $|\mathbf{k}| \sim 2\pi/\Delta\xi \sim \nu_q/Mc_s$, the condition $\Gamma > \gamma^L_{\mathbf{k}}$ is valid if $\epsilon Z_{d0} > 0.07$. Thus for a wide range of the plasma and dust grain parameters in a Q machine device the hydrodynamic ionization source model is applicable for the description of the ion-acoustic nonlinear structures.

For $T_e \sim T_i$, the Landau damping is the most significant dissipation process for small ϵZ_{d0} (in our case for $\epsilon Z_{d0} \lesssim 0.07$). The significant role of the Landau damping in this case means that it can make a contribution to a spreading out of the pulse as it propagates down the plasma column.

The increase in the parameter ϵZ_{d0} leads to the diminution of the role of the Landau damping, while the processes of the charging of dust grains, of the absorption of ions by grains, and of the transfer of the ion momentum to the grains become more important. The small effect of the kinetic damping on the evolution of the dust ion-acoustic shocks in the experiments in a Q machine device is illustrated in Fig. 3, which was obtained for the same parameter values as in Fig. 1(b), but with allowance for the change in the rates characterizing dissipative processes in the hydrodynamic ionization source model.

Since $\gamma^{L,q}_{\mathbf{k}}$ in the kinetic description and Γ in the hydrodynamic description are related to the anomalous dissipation caused by the interaction of plasma particles with dust grains, *i.e.*, they have the same origin, it is of interested to compare these rates. Fig. 4 presents the relief of the ratio $\gamma^{L,q}_{\mathbf{k}}/\Gamma$ on the plane $(n_{i0}e^2/n_{d0}aT_i,\ n_{i0}/n_{e0})$ for the parameters of complex plasmas in the experiments in a Q machine device. It corresponds to the experimental conditions $T_e = T_i = 0.2$ eV, $n_{i0} = 1.024 \cdot 10^7$ cm^{-3}, Cs$^+$ ions, $a = 0.1$ μm. The characteristic value of $|\mathbf{k}|$ in the wave spectrum corresponding to a particular shock-wave profile is estimated by taking the Fourier transform of the profile. Thus, for the shock-wave profile that is shown in Fig. 1(b) and is calculated for a distance of 60 cm from the grid, we obtain $|\mathbf{k}| \approx 0.12$ cm^{-1}. The closed circle in Fig. 3 fits the above experimental data and $\epsilon Z_{d0} = 0.75$. It can be seen that the experimental parameters satisfy the inequality $\gamma^{L,q}_{\mathbf{k}}/\Gamma < 1$.

The inequality $\gamma^{L,q}_{\mathbf{k}}/\Gamma < 1$ is the necessary condition under which the ionization source model is applicable to nonlinear dust ion-acoustic shocks. As it has been shown, it is fulfilled in a Q machine plasma. The analysis analogous to that carried out above shows that in typical experiments in complex plasmas the ratio $\gamma^{L,q}_{\mathbf{k}}/\Gamma$ is

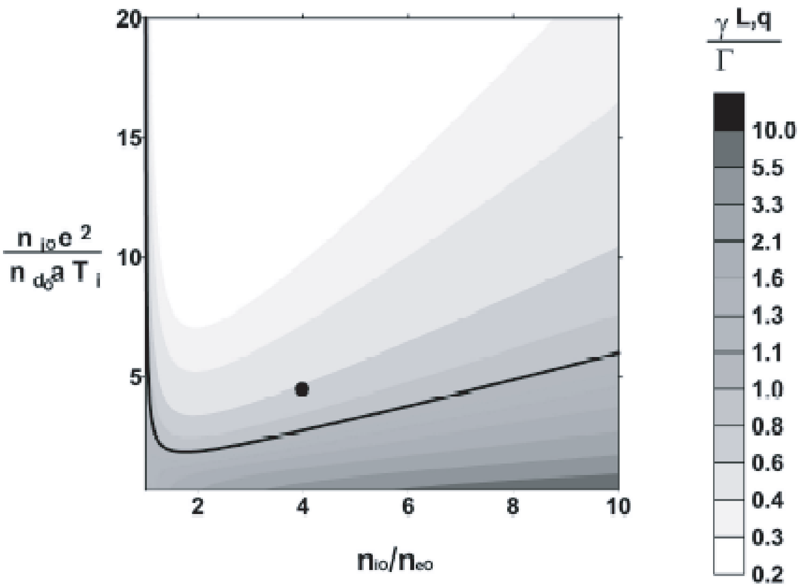

Fig. 4 Relief of the ratio $\gamma_{\mathbf{k}}^{L,q}/\Gamma$ on the plane $(n_{i0}e^2/n_{d0}aT_i,\ n_{i0}/n_{e0})$ for the plasma parameters $T_e = T_i = 0.2$ eV, $n_{i0} = 1.024 \cdot 10^7$ cm^{-3}, Cs^+ ions, $a = 0.1$ μm. The closed circle corresponds to $\epsilon Z_{d0} = 0.75$. The heavy curve corresponds to $\gamma_{\mathbf{k}}^{L,q} = \Gamma$.

often larger than unity. Such a situation takes place, *e.g.*, in experiments with glow discharges [31] for $n_i/n_e = 1/(1-\epsilon Z_{d0}) > 1$ and in experiments with RF discharge plasmas onboard the International Space Station [15] for $n_i/n_e = 1/(1-\epsilon Z_{d0}) > 3$. This means that, over fairly wide ranges of the dust grain parameters, dust ion-acoustic structures in typical experiments carried out with complex plasmas on devices based on glow and RF discharges should be described in terms of kinetic theory.

The dissipation related to the processes of momentum loss by ions as a result of their absorption on the grain surfaces and their Coulomb collisions with the grains forbids the existence of stationary shocks. There is no external source of ion momentum which is able to compensate for this momentum loss. This statement is valid independently whether we use hydrodynamic or kinetic approach to nonlinear dust ion-acoustic waves. In the dust ion-acoustic nonstationary shocks a balance between nonlinearity and dissipation is possible in the vicinity of their front, that results in the formation of the shock front during the time $\tau \sim \nu_q^{-1}$ much shorter than the characteristic time of shock propagation. But the amplitude of such shocks decreases.

In conclusion of this Section, we note that Nakamura et al. [11] attempted to describe their experimental results on the basis of the Korteweg-de Vries-Burgers (KdVB) equation with the dissipative viscosity coefficient proportional to the ion-grain collision rate (see also [32,33]). The correctness of the use of the viscosity concept for the description of dust ion-acoustic shocks was analyzed in [34]. It was shown that the viscosity concept can not be introduced to analyze the observations on dust ion-acoustic shocks: the viscosity should be taken to be zero. The description of the experiments [11] within the hydrodynamic ionization source model is beyond the scope of this paper. Nevertheless, we note that the ionization source model is capable of describing the main features of this experiment. In particular, it provides an explanation of the experimental fact that oscillations in the shock-wave profile due to the separation of charges (electrons and ions) are suppressed by the dust.

4 Summary

We have presented the results of investigation on generation of dust ion-acoustic shocks in a Q machine device. The advantage of the experiments carried out in a Q machine device is the negligible role of neutral atoms in the dissipation processes that enables to observe dust ion-acoustic shocks related to anomalous dissipation, which, by its very nature, originates from the dust grain charging process.

A fairly good agreement between theory and experiment is provided by the hydrodynamic ionization source model. In this model, the dissipation in a complex plasma is governed by the processes of absorption of ions by dust grains and also by Coulomb collisions between ions and grains. All these processes are closely related to the mechanisms by which the grains are charged.

The kinetic damping rate which includes Landau damping rate has been derived based on a purely kinetic approach to describing complex plasmas. The expression for the kinetic damping rate is found to contain the conventional terms that account for Landau damping in a plasma consisting of electrons, ions, and dust grain with a constant (unperturbed) charge and the terms that arise from the variable charge of the dust grains. It is shown that, in certain situations, the latter group of terms may play a dominant role. In particular, the condition under which the terms incorporating the variable grain charge into the kinetic damping rate predominate over the remaining terms is satisfied in a Q machine device for rather large dust densities, when dust ion-acoustic shocks are observed.

Under the conditions of a plasma in a Q machine device, the Landau damping is the dominant dissipation process for small dust densities. The increase in the dust density leads to the diminution of the role of the Landau damping, while the dust grain charging processes in the absorption of ions by dust grains and in Coulomb collisions between ions and grains become more important.

The dissipation related to the processes of momentum loss by ions as a result of their absorption on the grain surfaces and their Coulomb collisions with the grains forbids the existence of stationary shocks. In dust ion-acoustic shocks a balance between dispersion and dissipation is possible in the vicinity of their front, that results in the formation of the shock front during the time much shorter than the characteristic time of shock propagation. But the amplitude of such shocks decreases.

A criterion for determining whether or not the hydrodynamic approach is applicable to nonlinear dust ion-acoustic structures observed in experiments has been obtained. The ionization source model is applicable to the dust ion-acoustic shocks in a Q machine device for a wide range of the plasma and dust grain parameters. This is another advantage of application of a Q machine device for observation of the dust ion-acoustic shocks. In typical experiments with complex plasmas in devices based on glow and RF discharges, the effect of kinetic damping on the generation and propagation of dust ion-acoustic structures is significant over fairly wide ranges of the plasma and dust grain parameters, so that it becomes necessary to apply the kinetic approach.

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Appendix

A Kinetic damping of dust ion-acoustic waves

Here we study the kinetic damping of dust ion-acoustic waves, part of which is Landau damping. The dust ion-acoustic mode in a complex plasma is analogous to the ion-acoustic mode in a conventional two-component plasma consisting of electrons and one ion species. The difference in the dispersion relations for the modes is explained as being due to the effects peculiar to complex plasmas (processes at the surfaces of dust grains, fluctuations of the grain charges, recombination of electrons and ions on the grain surfaces, *etc.*). An essential feature of dust ion-acoustic waves is that they can exist at $T_e \sim T_i$ [35]. We present the derivation of a dispersion relation and expressions for the kinetic damping rate of dust ion-acoustic waves on the basis of a standard, purely kinetic approach, with the use of the method developed by Tsytovich and de Angelis [22].

A.1 Dielectric Function

We begin with the derivation of an expression for the dielectric function of a complex plasma. Since we are interested here in ion-acoustic time scales, each dust grain can be considered to be immobile. We assume that the grains are influenced by an electrostatic field (the extent of this influence is determined by the variable grain charge q) and also by the plasma particles (electrons and ions). The cross section for the interaction of plasma particles with dust grains is given by the formula (see, *e.g.*, [4])

$$\begin{cases} \sigma_\alpha = \pi a^2 \left(1 - 2e_\alpha q/am_\alpha|\mathbf{v}_\alpha|^2\right), \\ \text{for} \qquad 2e_\alpha q/am_\alpha|\mathbf{v}_\alpha|^2 < 1, \\ \sigma_\alpha = 0, \qquad \text{for} \qquad 2e_\alpha q/am_\alpha|\mathbf{v}_\alpha|^2 \geq 1. \end{cases} \tag{A1}$$

Here, e_α and v_α are the charge and velocity of plasma particles of species α; and the subscript $\alpha = e, i$ stands for the electrons and ions, respectively.

The distribution function $f_\alpha(t, \mathbf{r}, \mathbf{p})$ of plasma particles of species α is chosen to be normalized to their density n_α as follows:

$$n_\alpha = \int f_\alpha(t, \mathbf{r}, \mathbf{p}) \frac{d^3\mathbf{p}}{(2\pi)^3}. \tag{A2}$$

Since the dust grains are assumed to be immobile, their distribution function depends only on their charge q, $f_d(q)$. The normalization of the grain distribution function is chosen to satisfy the relationship

$$n_d = \int f_d(q) dq. \tag{A3}$$

The distribution function of dust grains can be represented as the sum of the unperturbed function $\Phi_d = \langle f_d \rangle$ and the perturbation δf_d, which is induced, *e.g.*, by the electric field of a dust ion-acoustic wave. Here, the angular brackets stand for averaging over the statistical ensemble. Since the distribution of plasma particles is also perturbed in their interaction with dust grains, we can write

$$f_\alpha = \Phi_\alpha + \delta f_\alpha, \qquad \Phi_\alpha =< f_\alpha > . \tag{A4}$$

The currents to a dust grain can also be represented as the sum of the unperturbed and perturbed components. The interaction of plasma electrons and ions with dust grains is described by the equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + e_\alpha \mathbf{E} \frac{\partial}{\partial \mathbf{p}}\right) f_\alpha = S_\alpha - \int \sigma_\alpha(q, \mathbf{v})|\mathbf{v}| f_d(q) f_\alpha dq, \tag{A5}$$

where the terms S_α account for all external sources of the particles of species α.

The kinetic equation for dust grains has the form

$$\frac{\partial f_d}{\partial t} + \frac{\partial}{\partial q}\left(I_{\text{ext}} + \sum_\alpha I_\alpha\right) f_d = 0, \tag{A6}$$

where $I_\alpha(q, \mathbf{r}, t)$ are the electron and ion currents to a grain at the point $\mathbf{r}$ and at the time t. The remaining currents, which are generated, *e.g.*, by the photoelectric effect and/or secondary electron emission, are incorporated into the term I_{ext}.

The currents to a dust grain of charge q at the point $\mathbf{r}$ can be determined using interaction cross sections (A1):

$$I_\alpha(q, \mathbf{r}, t) = \int e_\alpha \sigma_\alpha(q, \mathbf{v})|\mathbf{v}| f_\alpha(t, \mathbf{r}, \mathbf{p}) \frac{d^3\mathbf{p}}{(2\pi)^3}. \tag{A7}$$

The equation for the unperturbed distribution function of the plasma particles can be written as

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}}\right) \Phi_\alpha = S_\alpha - \nu_{d,\alpha} f_\alpha + J_\alpha, \tag{A8}$$

where

$$\nu_{d,\alpha} = \int \sigma_\alpha(q, \mathbf{v})|\mathbf{v}| \Phi_d(q) dq, \tag{A9}$$

and the collision integral J_α has the form

$$J_\alpha = -e_\alpha \left\langle \delta\mathbf{E}\frac{\partial \delta f_\alpha}{\partial \mathbf{p}} \right\rangle - \int |\mathbf{v}|\sigma_\alpha(q,\mathbf{v})\langle \delta f_\alpha \delta f_d(q)\rangle dq. \tag{A10}$$

For the perturbed distribution function, we obtain

$$\begin{aligned} \left(\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}}\right)\delta f_\alpha &= -e_\alpha \delta\mathbf{E}\frac{\partial \Phi_\alpha}{\partial \mathbf{p}} - e_\alpha\left(\delta\mathbf{E}\frac{\partial \delta f_\alpha}{\partial \mathbf{p}} - \left\langle \delta\mathbf{E}\frac{\partial \delta f_\alpha}{\partial \mathbf{p}}\right\rangle\right) \\ - \int |\mathbf{v}|\sigma_\alpha(q,\mathbf{v})&(\Phi_d(q)\delta f_\alpha + \delta f_d(q)\Phi_\alpha + \delta f_d(q) f_\alpha - \langle \delta f_\alpha \delta f_d(q)\rangle)dq. \end{aligned} \tag{A11}$$

We ignore the nonlinear terms in the equation for the perturbed distribution function and assume that the terms containing the perturbed quantities change only slightly with time. As a result, we arrive at the following equation for the Fourier components:

$$\delta f_{\alpha,\mathbf{k},\omega} = \frac{1}{i(\omega - \mathbf{k}\cdot\mathbf{v} + i\nu_{d,\alpha})} \times \left(e_\alpha \delta\mathbf{E}_{\mathbf{k},\omega}\frac{\partial \Phi_{\alpha,\mathbf{p}}}{\partial \mathbf{p}} + \delta\nu_{d,\alpha,\mathbf{k},\omega}\Phi_{\alpha,\mathbf{p}}\right), \tag{A12}$$

where $\delta\nu_{d,\alpha,\mathbf{k},\omega} = \int |\mathbf{v}|\sigma_\alpha(q,\mathbf{v})\delta f_d(q)dq$.

The unperturbed part of the distribution function of the dust grains is given by the equation

$$\frac{\partial \Phi_d}{\partial t} = -\frac{\partial}{\partial q}\left(\left(I_{\mathrm{ext}} + \sum_\alpha \langle I_\alpha(q)\rangle\right)\Phi_d + \sum_\alpha \langle \delta I_\alpha(q)\delta f_d(q)\rangle\right), \tag{A13}$$

whose right-hand side describes the kinetics of dust charging.

Assuming that all the currents to a dust grain, except for the currents of the plasma electrons and ions, are unperturbed, we obtain from kinetic equation (A6) the following equation for the perturbed part of the distribution function of the dust grains:

$$\begin{aligned} \frac{\partial \delta f_d}{\partial t} = -\frac{\partial}{\partial q}\Bigg(&\left(I_{\mathrm{ext}} + \sum_\alpha \langle I_\alpha(q)\rangle\right)\delta f_d(q) + \sum_\alpha \delta I_\alpha(q)\Phi_d(q) \\ &+ \sum_\alpha \delta I_\alpha(q)\delta f_d(q) - \sum_\alpha \langle \delta I_\alpha(q)\delta f_d(q)\rangle\Bigg). \end{aligned} \tag{A14}$$

Again, we neglect the nonlinear terms and assume that the terms containing the perturbed quantities are nearly constant in time to arrive at the following expression for the Fourier component of the perturbed distribution function of the dust grains

$$i\omega\delta f_{d,\mathbf{k},\omega} - \frac{\partial}{\partial q}\left\{\left(I_{\mathrm{ext}} + \sum_\alpha \langle I_\alpha(q)\rangle\right)\delta f_{d,\mathbf{k},\omega}\right\} = \frac{\partial}{\partial q}\left\{\left(\sum_\alpha \delta I_{\alpha,\mathbf{k},\omega}\right)\Phi_d\right\}, \tag{A15}$$

where

$$\delta I_{\alpha,\mathbf{k},\omega}(q) = \int e_\alpha |\mathbf{v}|\sigma_\alpha(q,\mathbf{v})\delta f_{\alpha,\mathbf{k},\omega}\frac{d^3\mathbf{p}}{(2\pi)^3}. \tag{A16}$$

Equations (A12) and (A15) constitute a set of equations for describing the perturbations of the plasma and of the dust.

The general solution δf_d to Eq. (A15) is the sum of the homogeneous solution $\delta f_d^{(0)}$, which satisfies Eq. (A15) with zero on the right-hand side and describes free fluctuations of the dust grain charges, and the inhomogeneous solution $\delta f_d^{(\mathrm{ind})}$, which describes the perturbations of the dust distribution function that are induced by the electric field of a dust ion-acoustic wave and by the perturbed currents to the grain surfaces. To determine the plasma dielectric function, it is sufficient to take into account only the perturbed distribution function $\delta f_d^{(\mathrm{ind})}$. Hence, in what follows, we will assume that $\delta f_d = \delta f_d^{(\mathrm{ind})}$.

In order to solve (A15), we consider an equation determining the equilibrium charge q_0 of a grain, *i.e.*, the charge at which the averaged current to the grain surface vanishes:

$$I_{\rm ext} + \sum_\alpha \langle I_\alpha(q_0)\rangle = I_{\rm ext} + \sum_\alpha \int e_\alpha |\mathbf{v}| \sigma_\alpha(q_0, \mathbf{v}) \Phi_\alpha \frac{d^3\mathbf{p}}{(2\pi)^3} = 0. \tag{A17}$$

If we regard the quantity $\Delta q = q - q_0$ as a new variable, then, for small Δq values, we can use the expansion

$$I_{\rm ext} + \sum_\alpha \langle I_\alpha(q_0)\rangle \approx \Delta q \frac{\partial}{\partial q} \sum_\alpha \langle I_\alpha(q)\rangle|_{q=q_0}. \tag{A18}$$

We introduce the slowly varying charging frequency ν_q through the standard relationship [4]

$$\nu_q = -\frac{\partial}{\partial q} \sum_\alpha \langle I_\alpha(q)\rangle|_{q=q_0}. \tag{A19}$$

In terms of charging frequency (A19), the dynamic equation for the charge of a dust grain, $\dot{q} = I_{\rm ext} + \sum_\alpha I_\alpha = 0$, has the form

$$\Delta \dot{q} = -\nu_q \Delta q, \tag{A20}$$

and Eq. (A15) becomes

$$\left(\frac{i\omega}{\nu_q} + \frac{\partial}{\partial \bar{q}} \bar{q}\right) \delta f_{d,\mathbf{k},\omega} = \frac{1}{\nu_q} R_{\mathbf{k},\omega}(q), \tag{A21}$$

where $R_{\mathbf{k},\omega}(q)$ denotes the right-hand side of Eq. (A15) and $\bar{q} = \Delta q / \Delta q_{t=0}$. The quantity $\Delta q_{t=0}$ is the solution to Eq. (A20) at the initial time $t = 0$ and has the meaning of the initial charge of a dust grain.

The solution to inhomogeneous equation (A21) can be written in terms of a Green's function:

$$\delta f_{d,\mathbf{k},\omega} = \hat{G} R_{\mathbf{k},\omega}(q) = \int G(q, q', \omega) R_{\mathbf{k},\omega}(q') dq'. \tag{A22}$$

Since $R_{\mathbf{k},\omega}(q) = (\partial/\partial q)\left\{\left(\sum_\alpha \delta I_{\alpha,\mathbf{k},\omega}(q)\right) \Phi_d\right\}$ is the total derivative with respect to q, the Green's function has the form [22]:

$$G(q, q', \omega) = \frac{1}{i(\omega + i\nu_q)}. \tag{A23}$$

The perturbed currents to a dust grain are described by an integral equation, which can be solved approximately under the condition that the charge of the dust grains is close to the equilibrium charge. Using expression (A16) and Eq. (A12), we get

$$\sum_\alpha \delta I_{\alpha,\mathbf{k},\omega}(q) = S_{\mathbf{k},\omega}(q) \delta E_{\mathbf{k},\omega} + \int \tilde{S}_{\mathbf{k},\omega}(q, q') \delta f_d(q') dq, \tag{A24}$$

where $\mathbf{E}_{\mathbf{k},\omega} = (\mathbf{k}/k) E_{\mathbf{k},\omega}$ and

$$S_{\mathbf{k},\omega}(q) = \sum_\alpha \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\sigma_\alpha(q, \mathbf{v}) e_\alpha^2 |\mathbf{v}|}{i(\omega - \mathbf{k}\cdot\mathbf{v} + i\nu_{d,\alpha}(\mathbf{v}))} \left(\frac{\mathbf{k}}{|\mathbf{k}|} \cdot \frac{\partial \Phi_\alpha}{\partial \mathbf{p}}\right), \tag{A25}$$

$$\tilde{S}_{\mathbf{k},\omega}(q, q') = \sum_\alpha \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\sigma_\alpha(q, \mathbf{v}) e_\alpha^2 |\mathbf{v}| \sigma_\alpha(q', \mathbf{v})}{i(\omega - \mathbf{k}\cdot\mathbf{v} + i\nu_{d,\alpha}(\mathbf{v}))} \Phi_\alpha. \tag{A26}$$

Substituting solution (A22) into formula (A24) and integrating over q' under the assumption that the charge of the dust grains is nearly equilibrium, we obtain an algebraic equation for $\sum_\alpha \delta I_{\alpha,\mathbf{k},\omega}(q)$. The solution to this equation has the form

$$\sum_\alpha \delta I_{\alpha,\mathbf{k},\omega}(q) = \beta_{\mathbf{k},\omega}\delta E_{\mathbf{k},\omega}, \tag{A27}$$

where

$$\beta_{\mathbf{k},\omega} = \frac{S_{\mathbf{k},\omega}(q_{\rm eq})}{1-\widetilde{S}'_{\mathbf{k},\omega}(q_0)\chi_{\mathbf{k},\omega}}, \tag{A28}$$

and where we have introduced the notation

$$\tilde{S}'_{\mathbf{k},\omega}(q_0) = \sum_\alpha \int \frac{\sigma_\alpha(q_0,\mathbf{v})e_\alpha|\mathbf{v}|^2}{i(\omega-\mathbf{k}\cdot\mathbf{v}+i\nu_{d,\alpha}(\mathbf{v}))}\left.\frac{\partial\sigma_\alpha(q,\mathbf{v})}{\partial q}\right|_{q=q_0}\Phi_\alpha\frac{d^3\mathbf{p}}{(2\pi)^3}, \tag{A29}$$

$$\chi_{\mathbf{k},\omega} = \frac{in_d}{\omega+i\nu_q}. \tag{A30}$$

Inserting solution (A27) into (A22) and expressing $\delta f_{d,\mathbf{k},\omega}$ in terms of $\delta E_{\mathbf{k},\omega}$, we reduce Eq. (A12) for the perturbed distribution function of the plasma particles to

$$\delta f_{\alpha,\mathbf{k},\omega} = \frac{\delta E_{\mathbf{k},\omega}}{i(\omega-\mathbf{k}\cdot\mathbf{v}+i\nu_{d,\alpha})}\left\{e_\alpha\left(\frac{\mathbf{k}}{|\mathbf{k}|}\cdot\frac{\partial\Phi_\alpha}{\partial\mathbf{p}}\right)-\Phi_\alpha\beta_{\mathbf{k},\omega}|\mathbf{v}|\left.\frac{\partial\sigma_\alpha(q,\mathbf{v})}{\partial q}\right|_{q=q_0}\frac{n_d}{i(\omega+i\nu_q)}\right\}. \tag{A31}$$

Using Poisson's equation

$$\nabla\cdot\mathbf{E} = 4\pi\sum_\alpha\int e_\alpha f_\alpha\frac{d^3\mathbf{p}}{(2\pi)^3}+4\pi\int q' f_d dq', \tag{A32}$$

we arrive at the equation

$$i|\mathbf{k}|\delta E_{\mathbf{k},\omega} = 4\pi\sum_\alpha\int e_\alpha\delta f_{\alpha,\mathbf{k},\omega}\frac{d^3\mathbf{p}}{(2\pi)^3}+4\pi\int q'\delta f_{d,\mathbf{k},\omega}dq', \tag{A33}$$

which can be rewritten as $i\varepsilon_{\mathbf{k},\omega}|\mathbf{k}|\delta E_{\mathbf{k},\omega}=0$, where the dielectric function $\varepsilon_{\mathbf{k},\omega}$ of the complex plasma is given by the formula

$$\begin{aligned}\varepsilon_{\mathbf{k},\omega} &= 1+\sum_\alpha\int\frac{4\pi e_\alpha^2}{|\mathbf{k}|^2(\omega-\mathbf{k}\cdot\mathbf{v}+i\nu_{d,\alpha})}\left(\mathbf{k}\cdot\frac{\partial\Phi_\alpha}{\partial\mathbf{p}}\right)\frac{d^3\mathbf{p}}{(2\pi)^3}\\ &+\frac{4\pi}{|\mathbf{k}|}\frac{n_d}{(\omega+i\nu_q)}\beta_{\mathbf{k},\omega}\left(-1+i\sum_\alpha\int\frac{e_\alpha|\mathbf{v}|\Phi_\alpha}{(\omega-\mathbf{k}\cdot\mathbf{v}+i\nu_{d,\alpha})}\left.\frac{\partial\sigma_\alpha(q,\mathbf{v})}{\partial q}\right|_{q=q_0}\frac{d^3\mathbf{p}}{(2\pi)^3}\right).\end{aligned} \tag{A34}$$

In the notation of Tsytovich and Havnes [7], expression (A34) coincides with the expression presented by them for the dielectric function of a complex plasma. Note that, in a paper by Tsytovich and de Angelis [22], the more general expression for $\varepsilon_{\mathbf{k},\omega}$ contains a misprint: the sign between the terms in the square brackets in the second row of formula (55) is incorrect. In particular, for dust grains with a zero velocity, formula (55) of [22] does not pass over to the formula that was obtained earlier in [7] for the dielectric function of a complex plasma.

A.2 Kinetic Damping Rate

The expression for the damping rate of the dust ion-acoustic waves can be derived using formula (A34). In this way, we restrict ourselves to considering Maxwellian equilibrium distribution functions Φ_α. Our objective here is to investigate the dispersion relation $\varepsilon_{\mathbf{k},\omega}=0$ in the frequency range $|\mathbf{k}|v_{Ti}\ll\omega\ll|\mathbf{k}|v_{Te}$ of ion-acoustic waves, where $v_{Te}=\sqrt{T_e/m_e}$ is the electron thermal velocity. This will be done by using the following small parameters:

$$\xi_e = \frac{\omega}{|\mathbf{k}| v_{Te}} \ll 1, \qquad \xi_i^{-1} = \frac{|\mathbf{k}| v_{Ti}}{\omega} \ll 1, \qquad \xi_{dw} = \frac{\pi a^2 n_d}{|\mathbf{k}|} \ll 1. \tag{A35}$$

The last small parameter has a simple physical meaning: it implies that the perturbation wavelength does not exceed the mean free path of dust grains with respect to their interaction with each other. The assumption that this parameter is small is quite justified because, on ion-acoustic time scales, the dust grains can be treated as being immobile. We also assume that the charging rate of the dust grains is much less than the wave frequency, $\nu_q \ll \omega$. This case is of primary interest for the description of dust ion-acoustic perturbations because the main contribution to their spectrum comes, as a rule, from modes with frequencies $\omega \gg \nu_q$. As will be shown below, the results of analyzing this case by the kinetic approach that incorporates dust grain charging differ *qualitatively* from the results obtained by Rosenberg [21] in the approach in which dust grain charging was not taken into account.

The electron, ion, and dust densities are assumed to satisfy the plasma quasineutrality condition

$$n_{e0} + n_{d0} Z_{d0} = n_{i0}. \tag{A36}$$

In the approximation adopted here, the dielectric function of a complex plasma has the form

$$\begin{aligned}
\varepsilon_{\mathbf{k},\omega} = {} & 1 + \frac{1}{|\mathbf{k}|^2 \lambda_{De}^2} \left(1 + \xi_{dw} \left(\sqrt{\pi z_0} e^{-z_0} + \pi (z_0 - 1/2)(1 + \mathrm{erf}(\sqrt{z_0}))\right)\right) - \frac{{\omega_{pi}}^2}{\omega^2} \\
& + i \left(\sqrt{\frac{\pi}{2}} \frac{{\omega_{pe}}^2 \omega}{|\mathbf{k}|^3 {v_{Te}}^3} \left(1 + \frac{n_{i0}}{n_{e0}} \sqrt{\frac{T_e^3}{T_i^3}} \sqrt{\frac{m_i}{m_e}} \exp\left[-\frac{\omega^2}{2 |\mathbf{k}|^2 v_{Ti}^2} \right] + \frac{4 \xi_{dw}}{\pi} \left(z_0 \Gamma(0, z_0) - e^{-z_0} \right) \right) \right. \\
& \left. + \frac{4 \xi_{dw} e^{-z_0}}{\sqrt{2\pi} \lambda_{De}^2 |\mathbf{k}|^2 \xi_e} \right),
\end{aligned} \tag{A37}$$

where $\omega_{pe} = \sqrt{4\pi n_{e0} e^2 / m_e}$ is the electron plasma frequency, and $\Gamma(\alpha, z_0) = \int_{z_0}^{\infty} x^{\alpha-1} \exp(-x) dx$.

Note that expression (A37) contains terms proportional to ξ_{dw}. It is these terms that account for the presence of dust grains with variable charges. The largest of them is the last term with the small quantity ξ_e in the denominator and it is the only one among the terms proportional to ξ_{dw} that will be considered hereafter. This term can be interpreted as a correction that takes into account the presence of dust grains with *variable charge* in the conventional expression for the plasma dielectric function, which was used, *e.g.*, in [21], where the dust can only be accounted for in terms of the dependence of the ion plasma frequency and electron Debye length on the ion density and electron density, respectively. In [21], no account was taken of the effect of the variable dust charge on the plasma dielectric function and, accordingly, the terms proportional to ξ_{dw} were not taken into consideration.

The dispersion relation $\varepsilon_{\mathbf{k},\omega} = 0$ has a solution in the form $\omega_{\mathbf{k}} = \omega_{\mathbf{k}}^s + i\gamma_{\mathbf{k}}^L$, which yields the well-known dispersion relation for ion-acoustic waves, (10), and the new expression for the kinetic damping rate, (7). Relationships (7) and (10) constitute a solution to an initial-value problem. They determine the complex solutions $\omega_{\mathbf{k}}$ to the dispersion relation at real $\mathbf{k}$ values.

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