the solution arising from a short-lived but very rapid rate of thermal expansion or transition to the gaseous state. The argument then relies upon the formation of the ion arc and a filamentary jet of heavy ions moving at high speed. Indeed, excluding a thermal basis for the force, it is inescapable that the forces measured must involve high-speed jets of heavy ions, which, for some reason unconnected with the onset of ionization (the solution being already fully ionized), form by escalation to the arc condition. In this case, the axial electrodynamic force seems a likely cause, in spite of its anomalous nature. On the other hand, if a thermal basis for the force is not excluded and some reason can be found for retarding the effects of heating to account for the absence of force in the nonarc condition, then it becomes logical to regard the discharge as triggered by electron release to form an arc and concentrate the thermal action to produce attendant explosive forces.

Further research by the author will concentrate on efforts to produce axial forces under steady current conditions and to develop a fuller account of why the forces are less evident in the nonarc condition of Fig. 1. The theoretical aspect of this involves the fundamental basis of the Neumann potential and its derivation from the Fechner hypothesis [11], [12]. In its turn, this raises some questions about the effective mass of charge carriers other than electrons, opening the issue of how heavy ions, and notably the proton, can develop electrodynamic actions except via a lepton (electron-positron) field involvement. Unlike electron electrodynamics, theory for the gyromagnetic properties of the proton is relatively undeveloped, making study of heavy ion anomalous electrodynamic forces all the more interesting.

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Electrostatic Ion-Cyclotron Waves in a Plasma with Negative Ions

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Abstract—**The dispersion relation for electrostatic ion-cyclotron (EIC) waves in a plasma containing a fraction of negative ions is derived from the fluid picture. Two wave modes are generally possible. Some of their features are investigated.**

Electrostatic ion-cyclotron (EIC) waves have been studied for well over two decades both experimentally [1] and theoretically [2]. Their occurrence in the earth's ionosphere has also been under active investigation [3]. The situation that is usually envisaged is one in which the plasma consists of electrons and one species of positive ions. Cases with more than one positive ion species also have been studied in relation to the earth's ionosphere [4].

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In this note we consider the EIC wave modes possible when the plasma consists of the three following species: positive ions, negative ions, and electrons. This investigation is prompted by similar ones performed on ion-acoustic waves in a plasma containing negative ions [5]. All three species are treated as fluids, described by

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \boldsymbol{v}_j) = 0 \tag{1}$$

$$n_j m_j \frac{\partial \boldsymbol{v}_j}{\partial t} + n_j m_j \boldsymbol{v}_j \cdot \nabla \boldsymbol{v}_j + kT_j \nabla n_j - e_j n_j \boldsymbol{E} - e_j n_j \boldsymbol{v}_j \times \boldsymbol{B} = 0 \tag{2}$$

where the subscript j stands for $+$, $-$, and e to indicate positive ions, negative ions, and electrons, respectively. The charges e_j are equal to e for positive ions, and to $-e$ for negative ions and electrons. Charge quasi-neutrality is assumed throughout, i.e., $n_+ = n_-, + n_e$. For simplicity, both positive ion and negative ion temperatures are taken to be zero ($T_+ = T_- = 0$) while the electrons have a finite temperature T_e. Furthermore, since we will be dealing with low-frequency waves, the electron inertia is neglected in (2) for the electrons. The electrons are taken to be at all times in Boltzmann equilibrium, which is equivalent to setting $\boldsymbol{v}_e \times \boldsymbol{B} = 0$ in the electron momentum equation.

Equations (1) and (2) are linearized around a time-independent zero-order state in which all fluid velocities are zero, no electric field is present, and the plasma is spatially uniform, with densities $n_{+0} = n_0$ for the positive ions, $n_{-0} = \epsilon n_0$ for the negative ions, and $n_{e0} = (1 - \epsilon)n_0$ for the electrons. A static and uniform magnetic field $\boldsymbol{B}$ is present, directed along the positive z-axis of a Cartesian frame of reference. All first-order quantities are taken to vary as $\exp[i(K_x x + K_z z - \omega t)]$, with $K_z^2 \ll K_x^2$. A dispersion relation is easily obtained, which reads

$$[\omega^2 - (\omega_{c+}^2 + K_x^2 h_+^2)] \cdot [\omega^2 - (\omega_{c-}^2 + K_x^2 h_-^2)] = K^4 h_+^2 h_-^2 \tag{3}$$

where $\omega_{c+} = (eB/m_+)$, $\omega_{c-} = (eB/m_-)$, m_+ and m_- being the positive and negative ion masses, and

$$h_+^2 = \frac{C_+^2}{1 - \epsilon}$$

$$h_-^2 = \frac{\epsilon C_-^2}{1 - \epsilon}$$

where

$$C_+^2 = \frac{\kappa T_e}{m_+}$$

and

$$C_-^2 = \frac{\kappa T_e}{m_-}.$$

The roots of (3) are shown in Fig. 1, for the case $m_+/m_- = 4$, which nearly corresponds to, e.g., a Q-machine plasma with Cs^+ positive ions and Cl^- negative ions. The quantity ω/ω_{c+} is shown as a function of $d = (h_+ K_x/\omega_{c+})$, for several values of ϵ, the percentage of negative ions. Clearly, two modes are present, one with $\omega \geq \omega_{c+}$, the "low-frequency" mode, and another with $\omega \geq \omega_{c-}$, the "high-frequency" mode.

It is easily shown, from the linearization procedure employed to obtain (3), that the first-order densities and potential ϕ_1 are related

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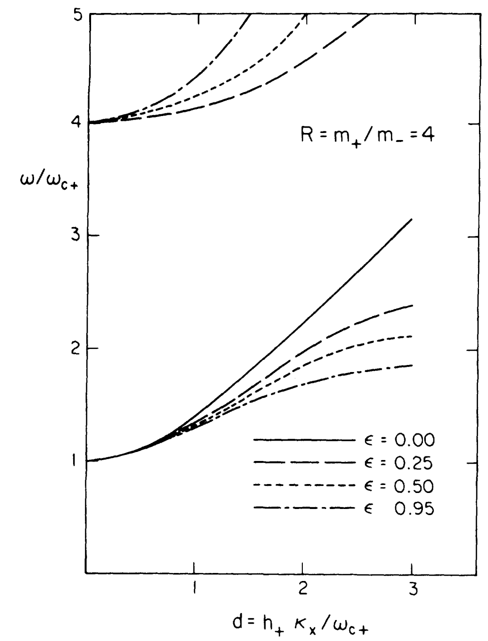

Fig. 1. The wave frequency versus perpendicular wavenumber, for a mass ratio $m_+/m_- = 4$ and several values of ϵ, the percentage of negative ions.

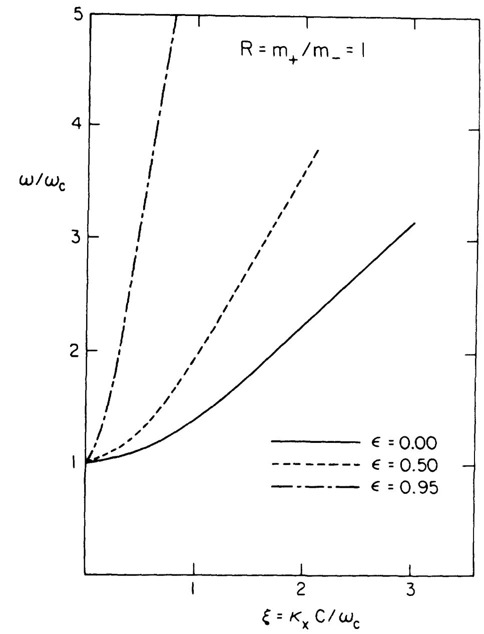

Fig. 2. The wave frequency versus perpendicular wavenumber, for a mass ratio $m_+/m_- = 1$ and several values of ϵ, the percentage of negative ions.

by

$$n_{+1} = \frac{K_x^2 C_+^2}{\omega^2 - \omega_{c+}^2} n_0 \frac{e\phi_1}{\kappa T_e} \tag{4}$$

$$n_{-1} = \left[\frac{K_x^2 C_+^2}{(\omega^2 - \omega_{c+}^2)(1 - \epsilon)} - 1\right](1 - \epsilon) n_0 \frac{e\phi_1}{\kappa T_e} \tag{5}$$

$$n_{e1} = (1 - \epsilon) n_0 \frac{e\phi_1}{\kappa T_e}. \tag{6}$$

From (4), (5), and (6), together with the dispersion relation (3), it is readily seen that, for the low-frequency mode, n_{+1}, n_{-1}, n_{e1}, and ϕ_1 all vary in phase. On the other hand, for the high-frequency mode, n_{+1}, n_{e1}, and ϕ_1 vary in phase, with n_{-1} being 180° out of phase. This is precisely the same behavior discussed in [5], when dealing with the "fast" ion-acoustic mode in a plasma with negative ions.

A situation which may be of particular interest is that of equal masses for the positive and negative ion species $m_+ = m_-$. This situation would be expected to occur when the two ion species are produced from the same parent gas, one by ionization and the other by electron attachment. In such a case there is one EIC wave mode, with the dispersion relation

$$\omega^2 = \omega_c^2 + \frac{1 + \epsilon}{1 - \epsilon} K_x^2 C^2 \tag{7}$$

where $\omega_c = \omega_{c+} = \omega_{c-}$ and $C_- = C_+ = C$. This special case is illustrated in Fig. 2, where ω/ω_c is shown as a function of $\xi = (K_x C/\omega_c)$, for several values of the percentage of negative ions ϵ. It can readily be seen that the wave phase velocity ω/K_x, in a direction normal to $\boldsymbol{B}$, increases drastically as ϵ varies from 0 toward 1. As for the phase relation of n_{+1}, n_{-1}, n_{e1}, and ϕ_1 of this mode, (4)–(6) and the dispersion relation equation (7) give n_{+1}, n_{e1}, and ϕ_1 oscillating in phase, while n_{-1} oscillates 180° out of phase from the other three quantities.

In conclusion, a fluid treatment has been used to identify the EIC wave modes possible in a plasma containing negative ions, and to illustrate some of the basic features of these modes. A kinetic theory treatment is best suited to study the conditions under which the modes can be excited.

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