Abstract—A fluid analysis of the excitation of dust ion-acoustic (DIA) waves in a collisional dusty plasma is presented. The DIA waves are excited by a relative drift of the electrons and ions produced by a steady-state electric field applied to the plasma. The DIA instability is more easily excited if the relative concentration of negatively charged dust is increased.

Index Terms—Current-driven instabilities, dust ion-acoustic waves, dusty plasmas

I. INTRODUCTION

Generally, when negative ions are introduced into an electron/positive ion plasma, the properties of the plasma wave modes are modified and new wave modes appear. For example, the dispersion relation for ion-acoustic waves yields two modes of propagation, a “fast” mode and a “slow” mode when negative ions are present [1]. A similar effect occurs in dusty plasmas, i.e., a plasma consisting of electrons, positive ions, and negatively charged heavy dust grains. Quite different wave modes are possible depending on whether the dust grains are considered to be static or mobile. Rao et al. [2] first analyzed the case in which the dynamics of the dust grains is important and demonstrated the existence of a new very low frequency acoustic mode which they termed “dust acoustic (DA) waves.” Shukla and Silin [3] investigated the dust ion-acoustic (DIA) mode, which is the usual ion-acoustic mode modified by the presence of immobile negatively charged dust grains. The presence of negatively charged dust increases the phase velocity of the DIA waves, and as a consequence the Landau damping is expected to be significantly reduced [3], [4]. This was confirmed by Rosenberg [5] who used Vlasov theory to analyze the DIA instability driven by an electron drift relative to the ions.

For the case of a magnetized dusty plasma, D’Angelo [4] found that, in addition to the ion-acoustic modes, two electrostatic ion-cyclotron modes are possible. An electrostatic dust ion-cyclotron (EDIC) mode is present, which is the usual ion EIC mode modified by the fact that some of the negative charge in the plasma resides on immobile dust grains. Also, a very low frequency electrostatic dust cyclotron (EDC) wave appears which is associated with the dynamics of the magnetized dust grains.

The current interest in dusty plasmas is due to the realization of their importance in various astrophysical and geophysical environments (e.g., interstellar space, comet tails, planetary ring systems, and the polar mesosphere) as well as in industrial plasma processing devices used in semiconductor manufacturing.

With the introduction of devices to disperse dust grains into plasmas [6], [7], it has recently become possible to perform laboratory experiments on waves in dusty plasmas. Some of the initial investigations have included studies of the effects of dust on the excitation of EIC waves (EDIC) [8], and the propagation of DIA waves [9]. In devices in which the dust grains are confined in the plasma (e.g., by levitating them electrostatically [10]), it has also been possible to observe the low-frequency dust acoustic mode [11]–[13].

Laboratory devices used for semiconductor processing inevitably contain dusty plasmas, since the process itself often leads to the formation of micron-sized particles [14]. Some of the features common to many of the laboratory dusty plasma devices include: 1) weakly ionized plasmas with relatively high neutral gas concentrations and 2) quasistatic electric fields which give rise to ion and electron drifts [15]. Rosenberg [16] recently investigated an ion dust-streaming instability for conditions that might prevail in processing plasmas. A kinetic theory was used with the effects of collisions between charged particles and the neutrals also included. A fluid theory of the excitation of the DA instability in a collisional dusty plasma has also been performed [17]. For neutral gas pressures ≈ 100 mtorr, it was found that dc electric fields ≈ 10 V/m were sufficient for the growth of DA waves of ≈ 0.6 cm wavelength. In addition to laboratory applications, neutral-charged particle collisions may also be important in dusty plasmas in the near-Earth space environment [18].

In this paper the dispersion relation for current-driven, DIA waves in a weakly ionized plasma is obtained using fluid theory. Current-driven, ion-acoustic (IA) waves in collisional plasmas were previously analyzed by Self [19] and Kaw [20]. The system consists of four components: electrons, positive ions, negative dust grains (for comparison one calculation involving positively charged grains will be presented), and neutral gas atoms. Since we will be interested in waves in the ion-acoustic frequency range, the dust grains will be treated as an immobile \((md \rightarrow \infty)\) charge neutralizing background species whose main effect is to remove electrons from the plasma. A steady-state (zero order) electric field is present which causes the electrons and positive ions to drift in opposite directions with a drift speed determined by the combined
influence of the electric field and the drag force due to collisions with the neutral atoms.

The dispersion relation will be obtained in Section II and the numerical results will be presented in Section III. Section IV contains a discussion of the results and conclusions.

II. DISPERSION RELATION

The system consists of electrons, positive ions, dust grains, and neutral gas atoms of density \( n_e, n_i, n_d, \) and \( N, \) respectively. The dust grains are taken to be immobile \((m_d \rightarrow \infty)\) and except where noted otherwise negatively charged, \( q_d = -eZ. \) The electrons and ions have temperatures \( T_e \) and \( T_i; \) Electrons and ions undergo collisions with the (stationary) neutrals with collision frequencies \( \nu_i = N\sigma_{in}v_{th}\) and \( \nu_e = N\sigma_{en}v_{th}\) where \( \sigma_{in} \) is the ion (electron) neutral collision cross section and \( v_{th} \) is the electron (ion) thermal velocity.

An electric field \( E \) is present in the plasma. The positive ion fluid obeys the continuity and momentum equations:

\[
\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_iv_i) = 0
\]

\[
n_i m_i \frac{\partial v_i}{\partial t} + n_i m_i v_i \frac{\partial v_i}{\partial x} + kT_i \frac{\partial n_i}{\partial x} - e v_i E = -\nu_i n_i m_i v_i
\]

where \( v_i \) is the ion fluid velocity. The ions are described also by the continuity and momentum equations

\[
\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_ev_e) = 0
\]

\[
kT_e \frac{\partial n_e}{\partial x} + e v_e E = -\nu_e n_e m_e v_e
\]

where \( v_e \) is the electron fluid velocity. For low-frequency ion waves, the electron inertia can be neglected in (4) except in the collision term. Since \( n_e n_i m_e v_e \sim n_e^{1/2}. \) this term is of lower order in \( m_e \) than the usual inertia terms that would appear on the left-hand side of (4).

The fluid equations are supplemented by the condition of charge neutrality

\[
n_i = n_e + Zn_d.
\]

(5)

Equations (1)–(5) are linearized around a zero-order state that is time independent \((\partial/\partial t = 0)\) and uniform \((\partial/\partial x = 0)\). Then in the zero-order state, (2) and (4) reduce to

\[
-\nu_i n_i m_i \frac{\partial v_i}{\partial x} = 0
\]

and

\[
-kT_i \frac{\partial n_i}{\partial x} - e v_i E = 0
\]

where \( E_0 \) is the (constant) zero-order electric field in the plasma and \( v_{i0} \) and \( v_{e0} \) are the zero-order electron and ion drift velocities respectively. The condition of charge neutrality (5) in zero order reads

\[
n_i0 = n_e0 + Zn_d
\]

(7) becomes

\[
n_i0 = (1 - eZ)n_e0 \quad (9)
\]

where the quantity \( eZ = (n_d/n_i0)Z = Zn_d/(n_e0 + Zn_d) \) is the fraction of the total negative charge per unit volume on dust grains.

Equations (1)–(4) are then linearized and all first-order quantities are taken to have the dependence \( e^{i(Kz - \omega t)} \). Defining \( \xi_i = n_{i1}/n_i0 \) and \( \xi_e = n_e1/n_e0 \), we find from (1) and (3) that

\[
v_{i1} = \frac{\Omega_i}{K} \xi_i \quad (10a)
\]

\[
v_{e1} = \frac{\Omega_e}{K} \xi_e \quad (10b)
\]

where \( \Omega_i = \omega - K\nu_i0 \) and \( \Omega_e = \omega - K\nu_e0 \) are the Doppler-shifted frequencies, in the ion and electron frames, respectively. Equations (10a) and (10b) are used in the linearized form of (2) and (4) to obtain

\[
\xi_i = \frac{i e}{m_i} \left[ \frac{\Omega_i}{\nu_i0} + i\nu_i0 \right] - K^2(\nu_i^2/m_i) E_0
\]

(10c)

\[
\xi_e = \frac{i e}{m_e} \left[ \frac{\Omega_e}{\nu_e0} + i\nu_e0 \right] - K^2(\nu_e^2/m_e) E_0
\]

(10d)

Since the dust grains are taken to be immobile, the first-order dust density \( n_{d1} = 0, \) so that from (5) we have that

\[
n_{i1} = n_{e1}
\]

(10e)

then using (9), \( \xi_{i1} \) and \( \xi_{e1} \) are related as

\[
\xi_{i1} = \xi_{e1} (1 - eZ)
\]

(10f)

Substituting (10f) and (10d) in (10c) yields the dispersion relation.

\[
(1 - eZ) \left[ \Omega_i(\Omega_e - \dot{\omega}_e) - K^2(\nu_i^2/m_i) \right] - K^2(\nu_i^2/m_i) \nu_i0 = 0
\]

(11)

Finally, defining the relative drift velocity

\[
u_0 = v_{i0} - v_{e0}
\]

(12)

(11) can be written as

\[
(1 - eZ) \left[ \Omega_i(\Omega_e - \dot{\omega}_e) - K^2(\nu_i^2/m_i) \right] - K^2(\nu_i^2/m_i) \nu_i0 = 0
\]

(13)

where, for simplicity, we use \( \Omega = \Omega_i \). With

\[
\Omega = \Omega_i + i\gamma
\]

(14)

(13) can be solved to obtain the real frequency (in the ion frame) \( \Omega_i \), and growth rate \( \gamma. \)

Using (14) in (13) and setting the real and imaginary parts equal to zero, we obtain

\[
(1 - eZ) \left[ m_i(\Omega_i^2 - \gamma^2 - \mu_i^2) - K^2(\nu_i^2/m_i) \right] - K^2(\nu_i^2/m_i) = 0
\]

(15a)

\[
\gamma = \frac{1}{2} \mu_i - \frac{1}{2} \frac{m_e}{m_i} \frac{eZ}{1 - eZ} \left[ 1 - \frac{\nu_0}{\Omega_i/K} \right]
\]

(15b)
In general the dispersion relation (13) must be solved numerically, however, (15a) and (15b) are useful for examining conditions near marginal stability ($\gamma \approx 0$).

The validity of (15) can be partially checked by considering the limiting case of $\epsilon Z = 0$, i.e., no dust. In that case we find that (15b) reduces to the growth rate for collisional ion-acoustic waves [19], [20].

### III. RESULTS

In this section numerical solutions to the dispersion relation (13) are obtained. In all of the results that follow, we use the following set of parameters:

- **Ion mass**: $m_i = 40 \times 1.67 \times 10^{-27}$ kg
- **Electron temperature**: $T_e = 2$ eV
- **Electron thermal velocity**: $v_{e,\text{th}} = 5.9 \times 10^5$ m/s
- **Ion temperature**: $T_i = 0.1$ eV
- **Ion thermal velocity**: $v_{i,\text{th}} = 490$ m/s
- **Electron-neutral collision cross section** [21]: $\sigma_{\text{en}} = 2 \times 10^{-20}$ m$^2$
- **Ion-neutral collision cross section** [21]: $\sigma_{\text{in}} = 5 \times 10^{-19}$ m$^2$

These parameters (16a–f) were chosen since they are generally representative of conditions in laboratory dusty plasma experiments. It is convenient to express the neutral atom density in terms of the pressure in millitorr

$$N(\text{m}^{-3}) = 3 \times 10^{19} \ P \ (\text{mtorr}),$$

(17)

The ion and electron collision frequencies then become

$$\nu_i(s^{-1}) = 7.35 \times 10^3 \ P \ (\text{mtorr}),$$

(18a)
$$\nu_e(s^{-1}) = 3.5 \times 10^5 \ P \ (\text{mtorr}),$$

(18b)

The relative drift velocity (12) $u_0$ can then be expressed in terms of the dc electric field $E_0$, and $P$,

$$u_0(\text{m/s}) = -5 \times 10^5 \frac{E_0(\text{V/m})}{P \ (\text{mtorr})},$$

(19)

For normalization purposes it is also convenient to have the ion-acoustic speed

$$C_s = \sqrt{\frac{kT_e + kT_i}{m_i}} = 2.2 \times 10^3 \text{ m/s},$$

(20)

Collecting the various numerical parameters, the dispersion relation can be cast in the following form

$$(1 - \epsilon Z)[\Omega(\Omega + 7.4 \times 10^3 P) - 2.4 \times 10^2 K^2] - 4.79 \times 10^6 K^2 + i4.8P \left(\Omega + 5 \times 10^5 \frac{KE_0}{P}\right)$$

(21)

where $K$ is the wavenumber in m$^{-1}$, $P$ is the neutral pressure in mtorr, $E_0$ is the (steady-state) electric field in V/m, and the quantity $\epsilon Z$ is the fraction of negative charge per unit volume on dust grains (for negatively charged dust $0 \leq \epsilon Z \leq 1$, where, with $\epsilon Z = 1$ all negative charge is on the dust grains, and when $\epsilon Z = 0$ there is no negatively charged dust).

Fig. 1 shows a plot of the solution to the dispersion relation $\Omega$ versus $K$ for $\epsilon Z = 0.5$, $E_0 = 250$ V/m and two neutral gas pressures (10 and 50 mtorr). The negative of the real part of $\Omega$ is given in Fig. 1(a) while the growth rates are shown in Fig. 1(b). The growth rates have been normalized by their respective values at $K = 10^3$ m$^{-1}$ so that their $K$ dependence is more clearly seen. The growth rates at $K = 10^3$ m$^{-1}$ for $P = 10$ and 50 mtorr were $1.5 \times 10^3$ s$^{-1}$, and $6.1 \times 10^3$ s$^{-1}$, respectively. For the case of $P = 50$ mtorr and $K = 100$ m$^{-1}$ ($\lambda \cong 6$ cm), the frequency of the mode in the laboratory frame is $f = 25$ kHz with the wave traveling in the direction of the electron drift, $v_{\text{phase}} = \omega_p/K = -1.6 \times 10^3$ m/s. The reduction in the growth rate with decreasing $K$ is likely due to the fact that as the wavelength $\lambda = 2\pi/K$ increases, the effect of ion-neutral collisional damping becomes more significant. For example, for $P = 50$ mtorr, the mean-free path for ion-neutral collisions $\lambda_m = (N\sigma_m)^{-1} \approx 0.13$ cm, whereas $K = 20$ m$^{-1}$ corresponds to a wavelength of $\sim 30$ cm. For $K \gg 100$ m$^{-1}$, $\lambda_m > \lambda$ so the effects of collisional damping would not be as important as for lower $K$’s. This interpretation is consistent with the result that as $P$ is lowered, the region in $K$ corresponding to the maximum (saturated) growth rate moves to the left, i.e., to longer wavelengths.

Next the effect of increasing the concentration of negatively charged dust is investigated. Fig. 2 shows (a) the normalized phase velocity (in the ion frame), $-\Omega_{\text{pe}}/KC_s$, and (b) the normalized growth rate $\gamma/(\Omega_{\text{pe}})$ for $P = 50$ mtorr, $K = 63$ m$^{-1}$ and $E_0$ values of 250 V/m and 500 V/m. Both
the normalized phase velocity and normalized growth rates increase as the concentration of negatively charged dust, \( \epsilon Z \), increases, thus indicating the destabilizing effect of adding more dust. For \( E_0 = 250 \text{ V/m} \) the damping/growth transition occurs at \( \epsilon Z \approx 0.5 \). The mode is unstable over the entire range of \( \epsilon Z \) at the higher value of \( E_0 = 500 \text{ V/m} \).

The destabilizing effect of increasing \( \epsilon Z \) at a fixed \( \epsilon Z = 0.8 \) is shown in Fig. 3. Both the normalized phase velocity (in the ion frame), (3b) and normalized growth rate, \( \gamma(\epsilon Z) \), increase with increasing \( E_0 \). For these conditions the critical electron field for instability is \( E_c = E_0(\gamma \approx 0) \approx 150 \text{ V/m} \).

The critical electric field, \( E_c \), and normalized wave phase velocity in the ion frame \( \left(-\Omega_r/KC_S\right) \) are shown in Fig. 4, as a function of \( \epsilon Z \) for \( P = 100 \text{ mtorr} \). As the amount of negative dust is increased, it becomes increasingly easier to excite DIA waves in the plasma. In fact in the limit \( \epsilon Z \to 1 \) in which all electrons are removed, \( E_c \to 0 \). For fixed values of \( \epsilon Z = 0.1 \) and 0.9 the dependence of the critical field \( E_c \) on pressure is shown in Fig. 5. Over the range \( 10 \leq P < 10^3 \text{ mtorr} \), \( E_c \) increases linearly with \( P \). For \( P = 100 \text{ mtorr} \), relatively moderate dc electric fields \( \sim 100-500 \text{ V/m} (1-5 \text{ V/cm}) \) are sufficient to excite DIA waves.

The conditions at marginal stability (\( \gamma = 0 \)) can be obtained analytically from the dispersion relation. With \( \gamma = 0 \) in (15a) and (15b), we obtain

\[
\frac{\Omega_r}{K} = \pm \sqrt{\frac{kT_i}{m_i} + \frac{kT_e}{m_e(1-\epsilon Z)}} \equiv \pm C_{S,D} \tag{22a}
\]

\[
\gamma \asymp -\frac{1}{2} \frac{\mu_e}{m_e} \frac{m_e}{m_i} \left(1 - \epsilon Z\right) \left[1 + \left|\frac{u_0}{\pm C_{S,D}}\right|\right]. \tag{22b}
\]

We have used the fact that, \( u_0 = v_i0 - v_{i0} \) is always negative, since \( v_i0 < 0 \) [see (6b)] and write \( u_0 = -|u_0| \). For any possibility of growth (\( \gamma > 0 \)), the term in brackets must
be negative which requires the choice $\Omega_r/K = -C_{SD}$ and $|u_0| > C_{SD}$. The critical drift $|u_{0c}|$ is obtained by setting $\gamma = 0$ in (22b)

$$|u_{0c}| = \left[ 1 + (1 - \varepsilon Z) \frac{m_i}{m_e} \frac{\varepsilon_0}{\varepsilon} \right] \sqrt{\frac{K T_i}{m_e} + \frac{K T_e}{m_i (1 - \varepsilon Z)}}. \tag{23}$$

For example, for $\varepsilon Z = 0.9$, (23) gives $|u_{0c}| \sim 10^6$ m/s $\approx 2 \nu_{te} \tau_{et}$. At higher values of $\varepsilon Z$, smaller relative drift velocities would be required to excite the DIA waves. For comparison, when there is no dust ($\varepsilon Z = 0$), $u_{0c} \approx 6 \nu_{te} \tau_{et}$, about three times higher than in a plasma in which 90% of the negative charge is on dust grains.

Finally, for purposes of comparison, it is instructive to consider the excitation of ion-acoustic waves in a dusty plasma with positively charged grains. This is readily done by replacing $(1 - \varepsilon Z)$ by $(1 + \varepsilon Z)$ in (13). Note, however, that now the quantity $\varepsilon Z = Z n_d / n_0$ can take on any positive value, i.e., $0 \leq \varepsilon Z \leq \infty$, with $\varepsilon Z \rightarrow \infty$ corresponding to the case when there are no positive ions in the plasma. The solution to the dispersion relation for $\varepsilon Z$'s up to 2 and $K = 63$ m$^{-1}$, $P = 50$ mtorr, and $E_0 = 500$ V/m are shown in Fig. 6. Evidently, as the amount of positive dust increases, the growth rate decreases, eventually becoming negative for $\varepsilon Z \approx 1$, indicating that positive dust has a stabilizing effect on current-driven ion-acoustic waves.

IV. DISCUSSION AND CONCLUSION

The current-driven, ion-acoustic instability in a plasma consisting of electrons, ions, massive charged dust grains, and neutral gas atoms has been analyzed. A zero-order electric field $E_0$ is applied to the plasma which, in the presence of the neutral gas, results in zero-order drifts of the electrons and ions in opposite directions. Under certain conditions this relative drift is sufficient to excite ion-acoustic waves. With negative dust relatively modest electric fields are required to excite the instability. For example, with $P = 50$ mtorr and $\varepsilon Z = 0.5$ (half the negative charge in the plasma on dust grains), we find that $E_0$'s $\sim 2.5$ V/cm are sufficient for an instability to occur.

Physically, the effect of negative dust on the instability can be understood by realizing that as more and more electrons become attached to the essentially immobile dust grains, the ion space charge perturbations are "neutralized" to a lesser and lesser degree. In the limit as $\varepsilon Z \rightarrow 0$ (no free electrons), we find then that vanishingly small $E_0$'s will produce instability. For positively charged dust, of course, the opposite is true.

With negative dust relatively modest electric fields are required to excite the instability. For example, with $P = 50$ mtorr and $\varepsilon Z = 0.5$ (half the negative charge in the plasma on dust grains), we find that $E_0$'s $\sim 2.5$ V/cm are sufficient for an instability to occur.
the fluid analysis neglects effects such as Landau damping. In this analysis we have taken $T_e/T_i = 20$ so that the possible effects of Landau damping would not be expected to be significant. However, since the effect of adding negative dust to increase the wave phase velocity (see, e.g., Fig. 4) we expect the wave (Landau) damping to be much less important for the DIA waves.

DIA waves may also be damped due to the mechanisms discussed by Havnes et al. [22] and D’Angelo [23]. The combined effects of these mechanisms have been analyzed by D’Angelo [23]. The so-called “Tromsø damping” [22] is related to the fact that in general the grain charge cannot be treated as constant. For wave periods on the order of the characteristic charging time of the grains, a finite phase shift exists between the wave potential oscillation and the oscillation of the grain charge in the presence of the wave leading to wave damping. However, the “Tromsø damping” would probably not hold here as both the electron and ions are collisional. The “creation damping” mechanism discussed by D’Angelo [23] is due to the loss in average momentum of the ions that results from the continuous injection of new ions that must necessarily occur to replace ions which are lost to the dust gains. D’Angelo [23] showed that for conditions similar to those considered in this paper the creation damping is the dominant mechanism of wave damping. A wave damping rate $\gamma_{\text{CD}}$ due to creation damping [23] can be obtained from $\gamma_{\text{CD}} \approx N a^2 n_d u_i; d_i(1 - e U_0/kT_i)$, where $a$ is the radius of the dust grains and $U_0$ is the potential of each dust grain relative to the plasma. This damping rate must be compared with that due to ion-neutral collisions, $\nu_i/2$ from (22b), or $\gamma_{\text{CD}}/(\nu_i/2) \approx N a^2 n_d u_i; d_i/(N\sigma_{\text{in}} u_i; d_i/2)$. For example, with $a = 5 \times 10^{-6}$ m, $n_d = 10^{10}$ m$^{-3}$, $N = 3 \times 10^{21}$ m$^{-3} (P = 0.1 \text{ torr})$, we find $\gamma_{\text{CD}}/(\nu_i/2) \approx 5 \times 10^{-3}$ so that the damping due to ion-neutral collisions is much more significant than the creation damping. We note, however, that the creation damping becomes increasingly more important at lower neutral pressures or higher dust densities.

Finally, a few comments are in order concerning the possible applications of the results presented here. Obviously our main motivation concerns laboratory dusty plasmas which generally operate at relatively high neutral gas pressures and where it is possible that steady-state electric fields on the order of those required for instability might be present as a natural consequence of the plasma formation process. However, the results presented here may also be relevant in various astrophysical and near-Earth space environments where dusty plasmas coexist with substantial amounts of neutral gas.

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