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THE ROLE OF FAST ELECTRONS FOR THE CONFINEMENT OF PLASMA BY MAGNETIC CUSPS

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Abstract—The influence of fast primary electrons on the leak width of a plasma in a picket fence or cusp geometry in the presence of a neutral gas background is discussed theoretically. Without primary electrons a leak width of the order of an ion Larmor radius (a_i) is found, whereas with fast primary electrons a leak width of the order of a hybrid width ($2\sqrt{a_i a_e}$) can be obtained. This is in agreement with experimental data in the literature.

1. INTRODUCTION

THE BEHAVIOUR of plasma in a magnetic cusp (compare Fig. 1), in particular the effective leak width of the escaping plasma stream is still a controversial subject (HAINES, 1977). It was generally believed that the leak width is of the order of an ion gyroradius a_i. However, KITSUNESAKI *et al.* (1974) found, in a laser produced plasma, that the leak width is much less than an ion Larmor radius. In 1975 HERSHKOWITZ *et al.* (1975) and LEUNG *et al.* (1976) found in a low density plasma produced by ionizing high energy electrons of about 50 eV, that the leak width was of the order $2\sqrt{a_i a_e}$, where a_i, a_e are the ion and electron Larmor radii, respectively.

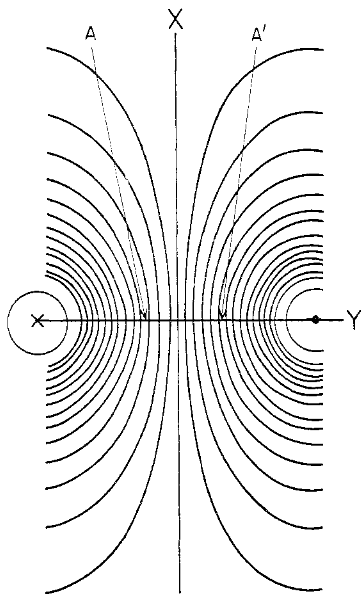

FIG. 1.—Typical magnetic field of a cusp. The centers of the circles are the location of current carrying wires.

This was corroborated by HERSHKOWITZ *et al.* (1979), when very characteristic electrostatic potential structures were measured in a picket fence configuration. Unfortunately, other authors were not able to reproduce these results. KOGOSHI *et al.* (1978) found in a laser produced plasma, a leak width of order a_i, only in the beginning of the experiment the leak width appeared to be smaller. PEHACHEK *et al.* (1980), again in a laser produced plasma, also found only a scaling with a_i. CARTIER (1980) found a_i in the after glow of a plasma, generated by ionizing electrons. CARPENTER (private communication) shielded the primary electrons by a screen so that they could not be present in the cusp regions of his device. Again, a leak width of the order a_i was measured and substantially diminished electrostatic potentials were observed, very much in contrast to the observation of HERSHKOWITZ *et al.* (1979). Recently, in a spindle cusp configuration, MERLINO *et al.* (1982) observed ion leak widths which were much less than a_i and on the order of the hybrid radius. These conflicting results did not find any theoretical explanation until a recent paper by KNORR and WILLIS (1982). They considered a beam of ions and electrons travelling along a homogeneous magnetic field (see Fig. 2). In the absence of an electric field, the effective diameter of the ion beam is a few Larmor radii and thus much larger than the diameter of the electron beam, because the ion Larmor radius is so much larger. If one takes into account the self-consistent electric field the ions will be drawn in, the electrons will be drawn out. In the resulting Poisson equation the ion and electron densities are a strongly non-linear function of the potential itself. The solution of this equation can be obtained numerically or, if some simplifications are introduced, a model equation can even be solved analytically. In both cases it turns out that the half width of the resulting potential trough (compare Fig. 3) scales with the hybrid width $a_h = \sqrt{a_i a_e}$ in the limit of small $\varepsilon = a_e/a_i = \sqrt{m_e\theta_e/m_i\theta_i}$. The hybrid width is thus recognized as a purely electrostatic steady state effect. It is not necessary to invoke any plasma instabilities to explain this phenomenon. On the other hand,

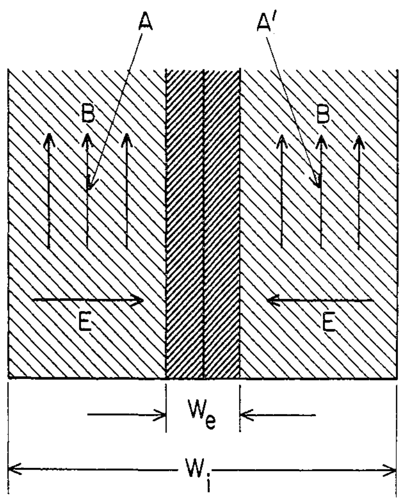

FIG. 2.—Ion and electron beams in a magnetic field. w_i, w_e are the effective widths of the ion and electron beams. A, A′ are the areas where electrons tend to diffuse due to collisions with neutrals.

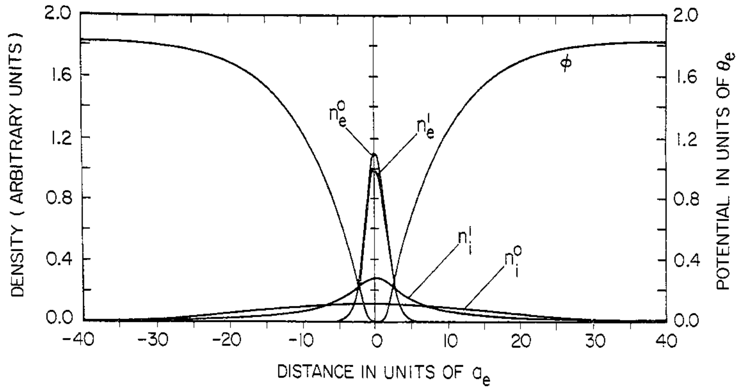

FIG. 3.—Numerical solution of the resulting potential trough, if the self-consistent electric field is taken into account. Curves n_i^0 and n_e^0 are the ion and electron densities if the electric field is switched off. With self-consistent potential $\varphi(x)$ the ion density n_i^1 is pulled in, the electron density n_e^1 is, to a lesser degree, pushed out.

if plasma instabilities indeed exist, they would make the effective leak width larger. Instabilities may very well be present under certain laboratory conditions.

The following discussion is concerned mainly with plasma parameters which are typical of many steady state cusp experiments: plasma densities of 10^8–10^{10} cm^{-3}, electron temperatures $T_e \approx 10\ T_i \approx$ 1–5 eV, magnetic fields of 100–200 G, and neutral pressures in the range of 10^{-5}–10^{-4} torr. The theory, however, should apply to any plasma for which the electrostatic approximation is valid.

2. THE ROLE OF THE FAST ELECTRONS

The electrostatic potential structure obtained by KNORR and WILLIS (1982) can be easily destroyed by placing electrons in those locations (A, A′ in Figs. 1 and 2), which are free from negative charges in the Knorr and Willis theory. In a realistic magnetic field, like that of Fig. 1, such electrons would be trapped on a magnetic field line because of the smallness of their Larmor radii. The electrostatic potential structures would be washed out, to a large degree, the ion beam would expand and scale with the ion Larmor radius.

We claim that this is the case with the experiments which have observed a leak width of the order of an ion Larmor radius and where no large fluctuations indicative of instabilities were observed. On the other hand, such charge compensating electrons were not present in the early phase of the KOGOSHI *et al.* (1978) experiment and in the experiments of HERSHKOWITZ *et al.* (1975); LEUNG *et al.* (1976); HERSHKOWITZ *et al.* (1979).

The essential difference between the experiments finding an ion Larmor radius width and those finding a hybrid width is that in the latter a tenuous population of high energy electrons was present. Indeed (HERSHKOWITZ *et al.*, 1975; LEUNG *et al.*, 1976; HERSHKOWITZ *et al.*, 1979) the primary electrons had an energy of about 50 eV, whereas the plasma electrons had an average energy of about 2 eV. Why does the existence of a high energy electron population correspond closely to the results obtained by KNORR and WILLIS (1982), even though the high energy electrons were not taken into account explicitly?

We suggest the following mechanism: Plasma electrons are diffused by collisions out of the original electron beam into the regions A and A′ in Fig. 2. These become trapped for a long time, compensate the ion charges, and thus annihilate the electric field. However, due to the presence of the fast electrons in that region there will be a collisional energy transfer from the fast to the plasma electrons. The electrons will be energized and will be able to escape from the cusp region.

We describe this process by the following mathematical model. The motion of the plasma electrons out of the center region of Fig. 2 into regions A and A′ can be described in the plane perpendicular to $\mathbf{B}$ by the momentum equation

$$m_e n_e\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + n_e q_e \mathbf{E} + n_e \frac{q_e}{c}\mathbf{v}\times\mathbf{B} - \nu_{en} m_e n_e \mathbf{v}. \tag{1}$$

$\mathbf{v}$ is the average velocity of the plasma electrons, n is their density, and ν_{en} is the electron-neutral collision frequency. The last term describes the loss of momentum due to collisions with the neutral background gas. In the steady state (1) becomes, after dividing by $m_e n_e$,

$$-\nu_{en}\mathbf{v} - \Omega_e(\mathbf{v}\times\mathbf{e}_z) + \frac{q_e}{m_e}\mathbf{E} - \frac{1}{m_e n_e}\nabla p = 0, \tag{2}$$

where $\Omega_e = qB/m_e c$ is the electron gyro frequency and we assumed the magnetic field to point in the z-direction. Assuming a constant electron temperature, (2) can be written in cartesian coordinates as

$$A\cdot\mathbf{v} = \frac{q_e}{m_e}\mathbf{E} - \frac{\theta_e}{m_e n_e}\nabla n_e, \tag{3}$$

where

$$A = \begin{pmatrix} \nu_{en} & \Omega_e \\ -\Omega_e & \nu_{en} \end{pmatrix}.$$

Solving (3) for $\mathbf{v}$, we obtain the flux

$$n_e\mathbf{v} = n\frac{q}{m_e}A^{-1}\cdot\mathbf{E} - \frac{\theta_e}{m_e}A^{-1}\cdot\nabla n, \tag{4}$$

with

$$A^{-1} = \frac{1}{\nu_{en}^2 + \Omega_e^2}\begin{pmatrix} \nu_{en} & -\Omega_e \\ \Omega_e & \nu_{en} \end{pmatrix}.$$

The mobility is defined as

$$\mu = q/m_e A^{-1}$$

and the diffusion coefficient as

$$\boldsymbol{D} = \frac{\theta_e}{m_e}\boldsymbol{A}^{-1}.$$

From these definitions the Einstein relation

$$\boldsymbol{D} = \frac{\theta_e}{q_e}\boldsymbol{\mu}$$

is an immediate consequence. The flux (4) can now be written as

$$\Gamma = n_e \mathbf{v} = n_e \boldsymbol{\mu} \cdot \mathbf{E} - \boldsymbol{D} \cdot \nabla n. \tag{5}$$

A diffusion-like is obtained by inserting the flux into the continuity equation

$$\frac{\partial n_e}{\partial t} = -\nabla(n_e \mathbf{v}) = \nabla \cdot (\boldsymbol{D} \cdot \nabla n) - \nabla \cdot (n_e \boldsymbol{\mu} \cdot \mathbf{E}). \tag{6}$$

In our case we have to add to (6) a sink term $(-\nu_{fs} n_e)$ which takes into account the loss of plasma electrons due to collisions with fast primary electrons. For the stationary case (6) becomes:

$$\nabla \cdot (\boldsymbol{D} \cdot \nabla n_e) - \nabla \cdot (n \boldsymbol{\mu} \cdot \mathbf{E}) - \nu_{fs} n_e = 0. \tag{7}$$

The above arguments are similar to discussions of transport phenomena (compare Krall and Trivelpiece, 1973), with one exception: The electric field in (7) is not an ambipolar field, but the field created by charge separation as discussed in Knorr and Willis (1973). An upper limit of the electric field strength can be estimated, using Knorr and Willis (1973). We obtain

$$\frac{|qE|a_e}{\theta_e} \leqslant 2\left(\frac{\pi}{4}\right)^2 \frac{\theta_i \lambda_D^2}{\theta_e a_i^2}. \tag{8}$$

Returning to equation (7), we would like to estimate the broadening of the plasma electron distribution by diffusion and the electric field. We neglect the variations of $\boldsymbol{D}$, μ and $\mathbf{E}$ and assume that ∇n and $\mathbf{E}$ have one component in the x-direction only. Equation (7) reduces to

$$D_\perp \frac{\partial^2 n_e}{\partial x^2} - \mu_\perp E \frac{\partial n_e}{\partial x} - \nu_{fs} n_e = 0, \tag{9}$$

where

$$D_\perp = \frac{\theta_e}{m_e} \frac{\nu_{en}}{\nu_{en}^2 + \Omega_e^2}$$

and

$$\mu_\perp = \frac{q_e}{m_e} \frac{\nu_{en}}{\nu_{en}^2 + \Omega_e^2}.$$

The mobility $\mu_\perp$ depends on the sign of charge of the species and is negative for electrons. The electric field E is negative in the geometry of Fig. 2. Equation (9) is a differential equations with constant coefficients and the solution is given by

$$n(x) = n_0 e^{-\lambda x/a_e}, \tag{10}$$

where a_e is the electron Larmor radius and

$$\lambda = \left\{\frac{\nu_{fs}}{\nu_{en}}\left[1 + \left(\frac{\nu_{en}}{\Omega_e}\right)^2\right] + \frac{1}{4}\left(\frac{qEa_e}{\theta_e}\right)^2\right\}^{1/2} - \frac{1}{2}\frac{|qEa_e|}{\theta_e}. \tag{11}$$

It is evident that the density distribution is broadened, which corresponds to a decreasing λ, if the electric field E grows from zero to infinity. To see the relative importance (or unimportance) of the electric field term, we compare the first and second term in the square root of equation (11). According to TRUBNIKOV (1965) and BOOK (1977), ν_{fs}, the collision frequency between fast and slow electrons, is given by

$$\nu_{fs} = 7.7 \cdot 10^{-6} n_f \varepsilon^{-3/2} \lambda_{ee'}, \tag{12}$$

where ε and n_f are the energy in eV and the density in cm^{-3} of the fast electrons and $\lambda_{ee'}$ is the Coulomb logarithm, given for $\theta_{ef} > 10$ eV by

$$\lambda_{ee'} = 24 - \ln(n_{ef}^{1/2}/\theta_{ef}).$$

The collision frequency between plasma electrons and neutrals is given by

$$\nu_{en} = n_n v_{th,ep} \sigma_{ne} = 4.19 \cdot 10^7 n_n(\mathrm{cm}^{-3}) \theta_e^{1/2}(\mathrm{eV}) \sigma_{ne}(\mathrm{cm}^2), \tag{13}$$

where n_n is the density of neutral atoms (in most cases argon), $v_{th,ep}$ is the thermal velocity of plasma electrons, and σ_{ne} is the collision cross section of electrons with neutrals.

We use the following set of parameters, which correspond closely to ongoing experiments:

$n_n = 3.5 \cdot 10^{11}\ \mathrm{cm}^{-3}$

$n_{ep} = 4.5 \cdot 10^9\ \mathrm{cm}^{-3}$

$n_{ef} = 1.0 \cdot 10^8\ \mathrm{cm}^{-3}$

$\theta_{ep} = 3$ eV

$\theta_i = 0.3$ eV

$\theta_{ef} = 60$ eV

$B = 125$ G (in cusp)

$m_i = 1840 \cdot 40\ m_e$ (argon)

$\sigma_{ne} = 0.84 \cdot 10^{-15}$ cm^2 (for scattering of plasma electrons with neutrals, compare Section 3)

$\sigma_f^i = 3\pi a_0^2$ (for ionizing collisions of 50 eV electrons with argon atoms. Bohr radius $a_0 = 5.29 \cdot 10^{-9}$ cm).

With these data we obtain:

$$\lambda_D = 1.9 \times 10^{-2} \text{ cm},$$

$$\nu_{en} = 2.13 \cdot 10^4 \text{ s}^{-1},$$

$$\nu_{fs} = 3.1 \cdot 10^1 \text{ s}^{-1},$$

$$\nu_{fs}/\nu_{en} = 1.46 \cdot 10^{-3},$$

$$\nu_{en}/\Omega_e = 0.98 \cdot 10^{-5},$$

and, using equation (8),

$$\frac{qEa_e}{\theta_e} \leqslant 2.2 \cdot 10^{-4}.$$

We realize that in equation (11) the electric field terms are small and can be neglected, as well as the term $(\nu_{en}/\Omega_e)^2$. Equation (11) reduces to

$$\lambda = \sqrt{\frac{\nu_{fs}}{\nu_{en}}}.$$

The characteristic width of the diffused plasma electron density profile is

$$a_e/\lambda = \left(\sqrt{\nu_{en}/\nu_{fs}}\right) a_e.$$

This length has to be compared with the characteristic width of the potential trough as obtained by Knorr and Willis (1982) ($a_n = 2\sqrt{a_i a_e}$). If

$$\left(\sqrt{\nu_{en}/\nu_{fs}}\right) a_e \lesssim 2\sqrt{a_i a_e},$$

or

$$\gamma \equiv \frac{\nu_{en} a_e}{4\nu_{fs} a_i} \lesssim 1 \tag{14}$$

a hybrid width will be observed in an experiment. On the other hand, if $\gamma \gg 1$, electrons will be able to diffuse and compensate for the space charge of the ions. The potentials will be sharply reduced and the leak width will widen from a hybrid width to the width of an ion Larmor radius. For our data, we obtain $\gamma = 2.0$ and we should observe a hybrid width, which is indeed the case.

Conversely, we can write (14) as a condition on the density ratio between fast electrons and neutral gas, which allows us to obtain a hybrid width. With $\Lambda_{ee'} = 18$ and using (12) and (13), we obtain

$$\gamma = 1.47 \cdot 10^{-6} \frac{n_n}{n_f} \frac{\theta_e \theta_f^{3/2}}{\mu^{1/2} \theta_i^{1/2}},$$

where the temperatures are measured in eV and $\mu = m_i/m_{\text{proton}}$. Equation (14) becomes

$$n_f/n_n \gtrsim 1.47 \cdot 10^{-6} \frac{\theta_e \theta_f^{3/2}}{\mu^{1/2} \theta_i^{1/2}}. \tag{15}$$

For argon and temperatures quoted above

$$n_f/n_n \gtrsim 6 \cdot 10^{-4}. \tag{16}$$

Equation (14) indicates why the influence of diffusion is minor, even though the ratio of collision frequencies ν_{en}/ν_{fs} is large. This ratio is multiplied by the ratio of the Larmor radii, which is quite small. In other words, the numerous collisions with neutrals displace the electrons by the distance of a Larmor radius. In order to broaden the profile effectively, the electron has to diffuse a distance of a hybrid radius and over that distance it has a good chance of being eliminated by a fast electron.

The addition of the fast electrons has a twofold effect: By kicking out plasma electrons, they reduce the leak width. On the other hand, because they are part of the negative charge, the fast electrons also tend to increase the leak width.* In our example, the fast electrons were about 2% of the plasma electrons. In the plasma channel of about 10 cm length, one in 1000 fast electrons will make an ionizing collision. After the collision the fast electron will still have about 35 eV energy and will continue to the wall. The remaining ion–electron pair will not considerably change the charge balance in the plasma stream channel.

3. COLLISION WITH NEUTRALS

The cross section σ_{en} is usually given in references (TRUBNIKOV, 1965; BOOK, 1977) as

$$\sigma_{en} \sim 4 \cdot 10^{-15} \text{ cm}^2.$$

However, at energies less than 10 eV, the magnitude of σ_{ne} for argon is depressed

* This effect has been studied by K. Clark and will be reported elsewhere.

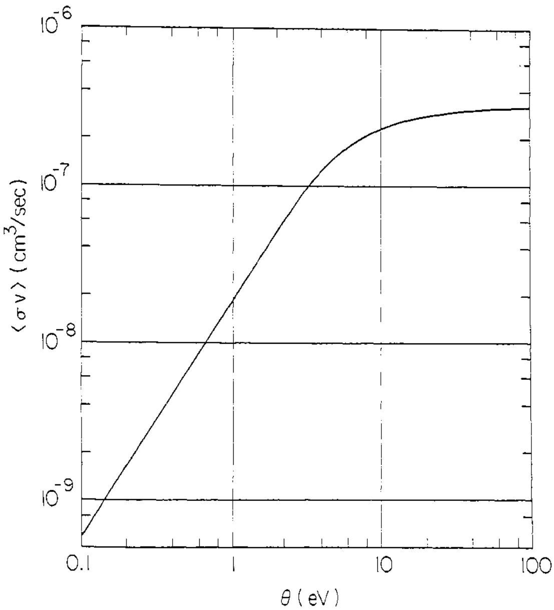

FIG. 4.—The rate $\langle \sigma\ v \rangle$ for momentum transfer for electron–argon atom collisions as a function of the electron temperature θ.

due to the Ramsauer–Townsend effect (FROST and PHELPS, 1964). FERREIRA and RICARD (1983) give an analytical interpolation curve which approximates well the measurements of Frost and Phelps and others. In order to obtain the collision frequency $\nu_{en} = n\langle \sigma_{en} v \rangle$, one has to average the cross section over a Maxwellian. As this was not available in the literature, we show the results in Fig. 4. If one writes

$$\langle \sigma_{en} v \rangle = \overline{\sigma_{en}} v_{e,\mathrm{th}}$$

one obtains in our case (plasma electron temperature equals 2 eV)

$$\overline{\sigma_{en}} = 0.84 \cdot 10^{-15}\ \mathrm{cm}^2.$$

This value is smaller than the one quoted in TRUBNIKOV (1965) and BOOK (1977) by a factor of almost 5.

4. CONCLUSIONS

We study the leakage of a plasma through a picket fence with and without the presence of fast 50 eV primary electrons. Taking into account collisions of the plasma electrons with the neutral gas background as the electrons stream through the cusp, we find that without primary electrons, enough plasma electrons will diffuse out and diminish the electrostatic potentials, which are responsible for a hybrid leak width. Indeed, a leak width of an ion Larmor radius has been found in many experiments. If, however, fast primary electrons are present, the diffusion of plasma electrons will be inhibited and we find a leak width of the order of a hybrid width. Such leak widths have been measured in some experiments where fast electrons were present. This model is consistent with the experimental observations of MERLINO *et al.* (1982) in which a clearly defined hybrid leak width could be obtained only at sufficiently low pressures when the primary concentration was relatively high.

Thus, we have shown that in a steady state device, collisional processes may be ultimately responsible for the conservation of the reduced hybrid leak width, the central parameter being the density ratio n_n/n_f of neutral particles to fast primary electrons. Using typical experimental values, we find that the model we have presented can indeed explain the discrepencies in the various experiments.

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REFERENCES

BOOK D. L. (1977) NRL Memorandum Report 3332 and NRL Plasma Formulary.
CARPENTER R. T., private communcation.
CARTIER S. (1980) Master's Thesis, Department of Physics and Astronomy, The University of Iowa, Iowa City, Iowa.
FERREIRA C. M. and RICARD A. (1983) *J. appl. Phys.* **54**, 2261.
FROST L. S. and PHELPS A. V. (1964) *Phys. Rev. A* **136**, 1538.
HAINES M. G. (1977) *Nucl. Fusion* **17**, 811.
HERSHKOWITZ N., LEUNG K. N. and ROMESSER T. (1975) *Phys. Rev. Lett.* **35**, 277.
HERSHKOWITZ N., SMITH J. R. and KOZIMA H. (1979) *Physics Fluids*, **22**, 122.
KITSUNEZAKI A., TANIMOTO A. and SEKIGUCHI T. (1974) *Physics Fluids* **17**, 1895.
KNORR G. and WILLIS D. (1982) *Z. Naturforsch.* **37a**, 780.
KOGOSHI S., SATO K. N. and SEKIGUCHI T. (1978) *J. Phys. D* **11**, 1057.
KRALL N. A. and TRIVELPIECE A. W. (1973) *Principles of Plasma Physics*, Chapter 6, McGraw Hill, New York.
LEUNG K. N., HERSHKOWITZ N. and MACKENZIE K. R. (1976) *Physics Fluids* **19**, 1046.
MERLINO R., HUDEK D. and BOSCH R. (1982) *Bull. Am. phys. Soc.* **27**, 1000 (and to be published).
PEHACEK R. E., GREIG J. R., RALEIGH M., KOOPMAN D. W. and DESILVA A. W. (1980) *Phys. Rev. Lett.* **45**, 256.
TRUBNIKOV B. A. (1965) *Reviews of Plasma Physics*, (edited by M. A. LEONTOVICH) Vol. 1. Consultants Bureau, New York.