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Current-driven dust-acoustic instability in a collisional plasma

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Abstract. **The excitation of the dust-acoustic instability in a collisional dusty plasma is investigated. For conditions similar to those of recent laboratory experiments with neutral gas pressures of ~ 100 mTorr, it is found that zero-order electric fields $E_0 \gtrsim 10\ \mathrm{V\,m^{-1}}$ are sufficient for the growth of perturbations with centimeter wavelengths. Much larger wavelengths generally require larger values of E_0. Free electrons in the dusty plasma have a stabilizing effect, which can be very pronounced at the longest wavelengths. Copyright © 1996 Elsevier Science Ltd**

1. Introduction

Dusty plasmas are a common occurrence in the universe (e.g. Spitzer, 1978). They are found in nebulas, planetary magnetospheres, comet environments, as well as in the Earth's upper atmosphere where the "noctilucent clouds" are observed.

Waves in dusty plasmas have been studied theoretically by a number of workers, beginning with the work of Bliokh and Yarashenko (1985), dealing with waves in Saturn's rings. Among more recent contributions are those of de Angelis *et al.* (1988), Rao *et al.* (1990), D'Angelo (1990), Shukla (1992), Baruthram and Shukla (1992), Havnes *et al.* (1992), Melandsø *et al.* (1993), Rosenberg (1993), Chow and Rosenberg (1995), and Winske *et al.* (1995). In most of these wave studies the assumption was made that the charge on the dust grains remains constant in time, even in the presence of the waves. This means that negatively charged dust grains are treated as very massive negative ions of large charge. The variation of the grain charge in the presence of the waves has been analyzed most exhaustively by the Tromsø group (Havnes *et al.*, 1992; Melandsø *et al.*, 1993).

Several of the wave modes in dusty plasmas that have been studied both theoretically and experimentally (e.g. Barkan *et al.*, 1995a, 1996) are simply modifications of standard modes in normal plasmas, whose properties are altered by the presence of (generally negatively) charged dust grains assumed to be so massive that they constitute a fixed and constant background. One dusty plasma mode, however, has also been investigated, which involves in an essential manner the dust grain dynamics. It gives rise to extremely low frequency waves, since the negatively charged dust grains providing the inertia, are so much more massive than normal negative ions. It has been named the "dust-acoustic" mode by Rao *et al.* (1990), who first analyzed it theoretically and noted that it is very similar to the so-called "slow mode" investigated by D'Angelo *et al.* (1966) for plasmas containing negative ions, when allowance is made for the much larger (negative) charge and mass of the dust grains.

Experimentally, extremely low frequency waves, involving the grain dynamics, were observed by Chu *et al.* (1994) and later interpreted by D'Angelo (1995) as dust-acoustic waves. More recently Barkan *et al.* (1995b) reported the observation of the dust-acoustic mode in a dusty plasma in which the negative dust grains were indefinitely "levitated" by the electric fields of a double layer. In both experiments the pressure of the neutral gas was relatively high, up to ~ 100 mTorr.

The excitation of the dust-acoustic mode by ions and electrons streaming through the dust grains has been analyzed theoretically by Rosenberg (1993), who more recently has also investigated the possible effect of collisions of ions and electrons with neutral gas molecules, using Krook type collision terms in the Vlasov equation (Rosenberg, 1996).

In the present paper the situation is considered in which the system consists of four components, i.e. ions, electrons, negative dust grains and neutral gas molecules, the first three being acted upon by a steady (zero-order) electric field which causes the dust grains and the electrons to stream in opposite direction to the positive ions. For a situation similar to the two experiments mentioned above, the critical electric field is calculated which will excite the dust-acoustic mode, and, in addition, both the frequency

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and the growth rate of the mode are predicted. The theory is presented in Section 2, while Section 3 contains some concluding remarks.

2. Theory

Consider a dusty plasma in which n_+, n_e, n_d and N represent the positive ion, electron, negatively charged dust grain and gas molecule density, respectively. The ions and the electrons have temperatures T_+ and T_e, while the dust grains are taken to be cold ($T_d = 0$). The symbols v_+, v_e and v_d stand for the velocities of the ions, electrons and dust, respectively, and m_+, m_e and m_d for their masses. The neutral gas is taken to be at rest. All dust grains are assumed to have the same size and to be spheres of radius a. The quantity eZ represents the magnitude of each grain charge. Collisions with the neutral gas occur with collision frequencies ν_+, ν_e and β for the ions, the electrons and the charged grains, respectively. E indicates the electric field in the plasma. The equations describing the behavior of the three charged components are as follows:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_d) = 0 \quad (1a)$$

$$\frac{\partial n_+}{\partial t} + \frac{\partial}{\partial x}(n_+ v_+) = 0 \quad (1b)$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_e) = 0 \quad (1c)$$

$$n_d m_d \frac{\partial v_d}{\partial t} + n_d m_d v_d \frac{\partial v_d}{\partial x} + eZn_d E = -\beta n_d m_d v_d \quad (1d)$$

$$\kappa T_+ \frac{\partial n_+}{\partial x} - en_+ \mathrm{E} = -\nu_+ n_+ m_+ v_+ \quad (1e)$$

$$\kappa T_e \frac{\partial n_e}{\partial x} + en_e E = -\nu_e n_e m_e v_e \quad (1f)$$

$$n_+ = Zn_d + n_e. \quad (1g)$$

Equations (1a)–(1c) are the continuity equations for the charged components, while equations (1d)–(1f) are the momentum equations. Since we will be dealing with waves of extremely low frequency, the inertia of the dust is retained in equation (1d), but the ion and electron inertia terms are neglected in equations (1e) and (1f). Equation (1g) represents the condition of charge neutrality, which is taken to be satisfied not only in the unperturbed (zero-order) state but also in the presence of very low frequency waves (see, e.g. D'Angelo, 1995).

The zero-order state is one in which the plasma is steady and uniform ($\partial/\partial t = 0 = \partial/\partial x$), the electric field, E_0, is constant and the velocities of the three charged components are also constant. We find:

$$eZE_0 = -\beta m_d v_{d0} \quad (2a)$$

$$eE_0 = \nu_+ m_+ v_{+0} \quad (2b)$$

$$eE_0 = -\nu_e m_e v_{e0} \quad (2c)$$

$$n_{+0} = Zn_{d0} + n_{e0}. \quad (2d)$$

Following standard procedure equations (1a)–(1g) are linearized and the first-order quantities are assumed to vary in space and time as $e^{i(Kx-\omega t)}$. With $\xi_d = n_{d1}/n_{d0}$, $\xi_+ = n_{+1}/n_{+0}$, $\xi_e = n_{e1}/n_{e0}$, $\Omega_d = \omega - Kv_{d0}$, $\Omega_+ = \omega - Kv_{+0}$, $\Omega_e = \omega - Kv_{e0}$, we find, from equations (1a) to (1c), that:

$$v_{d1} = \frac{\Omega_d}{K}\xi_d \quad (3a)$$

$$v_{+1} = \frac{\Omega_+}{K}\xi_+ \quad (3b)$$

$$v_{e1} = \frac{\Omega_e}{K}\xi_e. \quad (3c)$$

Similarly, from equations (1d) to (1f), we obtain:

$$(\Omega_d + i\beta)v_{d1} + i\frac{eZ}{m_d}E_1 = 0 \quad (4a)$$

$$iKC_+^2\xi_+ - \frac{e}{m_+}E_1 + \nu_+ v_{+1} = 0 \quad (4b)$$

$$iKC_e^2\xi_e + \frac{e}{m_e}E_1 + \nu_e v_{e1} = 0 \quad (4c)$$

where $C_+^2 = \kappa T_+/m_+$ and $C_e^2 = \kappa T_e/m_e$. With $\epsilon = n_{d0}/n_{+0}$, we obtain from equation (1g)

$$\xi_+ = \epsilon Z\xi_d + (1-\epsilon Z)\xi_e \quad (5)$$

which expresses the condition of charge neutrality in first order.

Introducing next the quantities

$$\mu_+ = v_{+0} - v_{d0}$$

and

$$\mu_e = v_{e0} - v_{d0}$$

which represent the zero-order drifts of the ions and of the electrons relative to the dust, and noting that $\Omega_+ = \Omega_d - K\mu_+$ and $\Omega_e = \Omega_d - K\mu_e$, we obtain from equations (3) and (4):

$$\xi_d = -i\frac{eZ}{m_d} \cdot \frac{K}{\Omega_d(\Omega_d + i\beta)} \cdot E_1 \quad (6a)$$

$$\xi_+ = \frac{e}{m_+} \cdot \frac{K}{iK^2C_+^2 + \nu_+(\Omega_d - K\mu_+)} \cdot E_1 \quad (6b)$$

$$\xi_e = -\frac{e}{m_e} \cdot \frac{K}{iKC_e^2 + \nu_e(\Omega_d - K\mu_e)} \cdot E_1. \quad (6c)$$

Substituting equations (6a)–(6c) into equation (5), the dispersion relation is obtained, which reads:

$$\frac{1}{-K^2\kappa T_+ + im_+\nu_+(\Omega_d - K\mu_+)} + \frac{\epsilon Z^2}{m_d\Omega_d(\Omega_d + i\beta)} + \frac{1-\epsilon Z}{-K^2\kappa T_e + im_e\nu_e(\Omega_d - K\mu_e)} = 0. \quad (7)$$

As checks on the validity of equation (7), consider first the case in which β, ν_+, ν_e and E_0 are all equal to zero. In that case we obtain that

$$\frac{\omega^2}{K^2} = \frac{\kappa T_+}{m_d} \epsilon Z^2 \frac{1}{1+(1-\epsilon Z)\frac{T_-}{T_e}} \quad (8)$$

which, for $T_d = 0$, is the dispersion relation given in D'Angelo (1995).

Next examine the case in which all electrons are attached to the dust grains and no free electrons are present, i.e. $1-\epsilon Z = 0$. We find from equation (7):

$$m_d\Omega_d(\Omega_d + i\beta) + \epsilon Z^2[-K^2\kappa T_+ + im_+ v_+(\Omega_d - K\mu_+)] = 0. \quad (9)$$

With $\Omega_d = \Omega_d^{(r)} + i\gamma$, where $\Omega_d^{(r)}$ is the real part of Ω_d and γ is the growth rate, equation (9) can easily be separated into

$$\Omega_d^{(r)^2} - \epsilon Z^2 \frac{\kappa T_+}{m_d} K^2 - \gamma(\beta+\gamma) - \epsilon Z^2 \frac{m_+}{m_d} v_+ \gamma = 0 \quad (10)$$

and

$$\gamma = -\frac{1}{2}\beta - \frac{1}{2}\epsilon Z^2 \frac{m_+}{m_d} v_+ \left(1 - \frac{\mu_+}{\Omega_d^{(r)}/K}\right). \quad (11)$$

equation (11) is the result obtained in the past (e.g. Kaw, 1973) for resistive ion-acoustic instabilities and it shows that, if the drift μ_+ of the light component relative to the heavy component is large enough (i.e. $\mu_+ > \Omega_d^{(r)}/K$), collisions of the light component with neutral gas molecules have a destabilizing effect. From equation (10) it is seen that near marginal stability ($\gamma \approx 0$) the real part of the dispersion relation reduces to equation (8) above, since now $1-\epsilon Z = 0$.

After these preliminary checks, the dispersion relation equation (7) was utilized to obtain values of the critical zero-order electric field in the general case of a dusty plasma containing all four components ($1-\epsilon Z \neq 0$), and to predict the wave phase velocity and the growth rate.

To this end the proper expression for the coefficient β in the dust momentum equation, equation (1d), related to the drag on the dust grains by the neutral molecules, had to be specified. For the case in which the mean-free-path for molecule–molecule collisions is large compared to the size of the dust grain, the velocity of the dust grains is much less than the gas thermal velocity, and the gas molecules are specularly reflected on the dust surface, detailed calculations performed, e.g. by Baines *et al.* (1965) give

$$\beta \simeq \frac{4m_N N a^2 v_{th,N}}{m_d} \quad (12)$$

where m_N, N and $v_{th,N}$ are the mass, the density and the thermal velocity of the gas molecules (see also Epstein (1924)).

As far as the collision frequencies v_+ and v_e are concerned, they are given by the usual expressions

$$v_+ = N\sigma_+ C_+ \quad (13)$$

and

$$v_e = N\sigma_e C_e \quad (14)$$

where σ_+ and σ_e are the positive ion–molecule and electron–molecule collision cross sections, respectively.

Table 1. Set of parameters

$m_+ = 4.7 \times 10^{-26}$ kg
$m_N = 4.7 \times 10^{-26}$ kg
$m_e = 9.1 \times 10^{-31}$ kg
$m_d = 1 \times 10^{-12}$ kg
$\kappa T_+ = 1.6 \times 10^{-20}$ J
$\kappa T_N = 2 \times 10^{-21}$ J
$\kappa T_e = 4.8 \times 10^{-19}$ J
$a = 5 \times 10^{-6}$ m
$N \simeq 3 \times 10^{21}$ m^{-3}
$\sigma_+ \simeq 5 \times 10^{-20}$ m^2
$\sigma_e = 1 \times 10^{-20}$ m^2
$v_+ = 8.8 \times 10^4$ s^{-1}
$v_e = 2.2 \times 10^7$ s^{-1}
$\beta = 2$ s^{-1}
$Z = 4 \times 10^{4a}$

[a]Note: for any particular experimental situation, the possible reduction of Z due to "close packing" of dust grains (Whipple *et al.*, 1985) should be taken into account.

Calculations were performed for the set of parameters given in Table 1, which should be generally representative of the experimental conditions referred to in the Introduction (Chu *et al.*, 1994; Barkan *et al.*, 1995b).

Figure 1 shows the critical electric field (in V m^{-1}) vs. the wave number (in m^{-1}), for three different values of the quantity ϵZ. With $\epsilon Z = 1$ no free electrons are present in the plasma, with $\epsilon Z = 0.5$ the negative charge present per unit volume is equally divided between free electrons and dust grains, and with $\epsilon Z = 0.2$ the free electrons amount to 80% of the total negative charge. Over the full range of the values of K of Fig. 1, the wavelength, λ, is $\geq \lambda_{in}$, where λ_{in} is the ion-neutral mean-free-path.

For a value of $K = 10^3$ m^{-1} (corresponding to a wavelength of 0.6 cm, comparable to those reported in the experiments of Chu *et al.* (1994) and Barkan *et al.* (1995b)), Fig. 2 shows the variation of the critical electric field with the parameter ϵZ. The magnitudes of these values of E_{crit} (~ 10 V m^{-1}) correspond to zero-order ion drifts, $v_{+0} \approx 3.8 \times 10^2$ m s^{-1}, about a factor two smaller than the ion thermal speed. Evidently, (as seen also in Fig. 1) as the percentage of free electrons is reduced, it becomes progressively easier to excite the dust-acoustic mode. The corresponding variation of the wave phase velocity in the

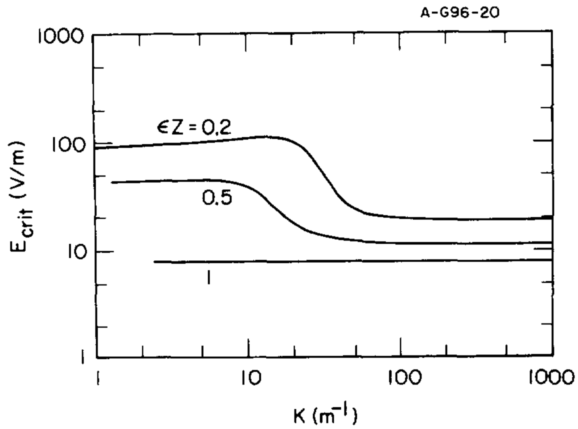

Fig. 1. The critical electric field, in V m^{-1}, vs. the wave number, K, in m^{-1} for $\beta = 2$ s^{-1} and $\epsilon Z = 1$, 0.8 and 0.2

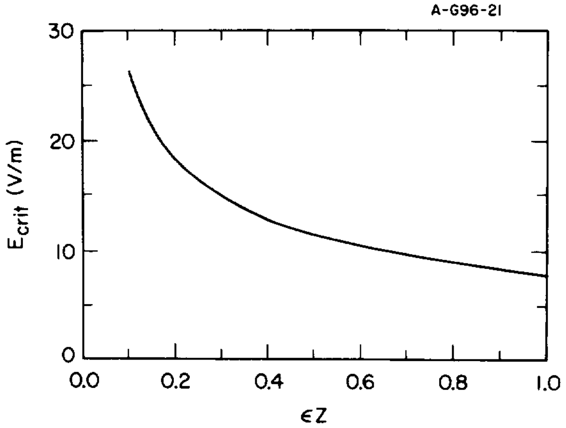

Fig. 2. The critical electric field, in $\mathrm{V\,m^{-1}}$, for $K = 10^3\,\mathrm{m^{-1}}$ and $\beta = 2\,\mathrm{s^{-1}}$, vs. ϵZ

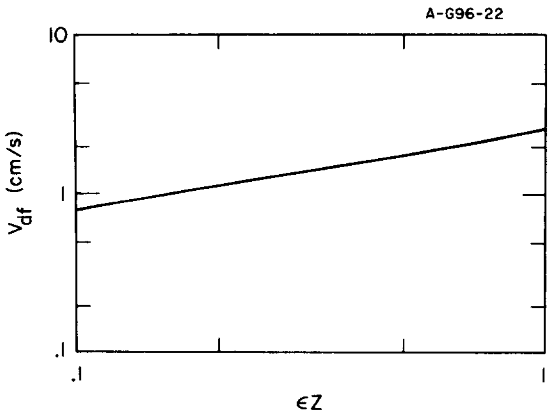

Fig. 3. The wave phase velocity in the dust frame, in $\mathrm{cm\,s^{-1}}$, at $E_0 = E_{crit}$ (i.e. $\gamma = 0$), vs. ϵZ ($K = 10^3\,\mathrm{m^{-1}}$, $\beta = 2\,\mathrm{s^{-1}}$)

dust frame, $\Omega_d^{(r)}/K$, vs. the ϵZ parameter, for $E = E_{crit}$ (i.e. $\gamma = 0$), is shown in Fig. 3. Note the $v_{phase} \propto \sqrt{\epsilon Z}$ relation, which is also expected from equation (8) for our set of parameters.

For $K = 10^3\,\mathrm{m^{-1}}$, Figs 4 and 5 show the wave frequency in the dust frame, $\Omega_d^{(r)}$, and the growth rate, γ, as functions of the zero-order electric field. There is no wave growth in this case for $E \lessapprox 11.5\,\mathrm{V\,m^{-1}}$, whereas γ increases linearly with E_0, at least up to $E_0 \sim 20\,\mathrm{V\,m^{-1}}$. The wave frequency

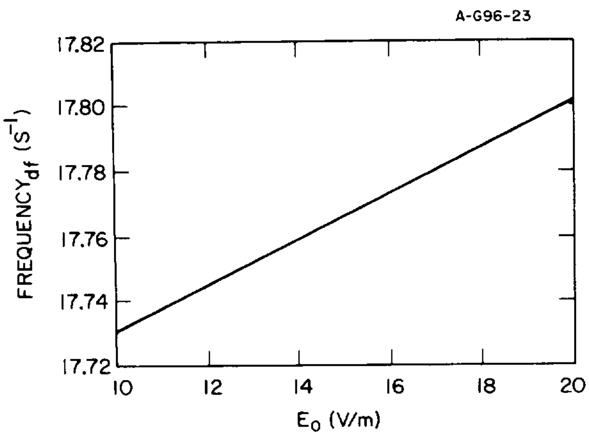

Fig. 4. The wave (angular) frequency in the dust frame, in $\mathrm{s^{-1}}$, vs. the electric field, in $\mathrm{V\,m^{-1}}$, for $K = 10^3\,\mathrm{m^{-1}}$, $\beta = 2\,\mathrm{s^{-1}}$ and $\epsilon Z = 0.5$

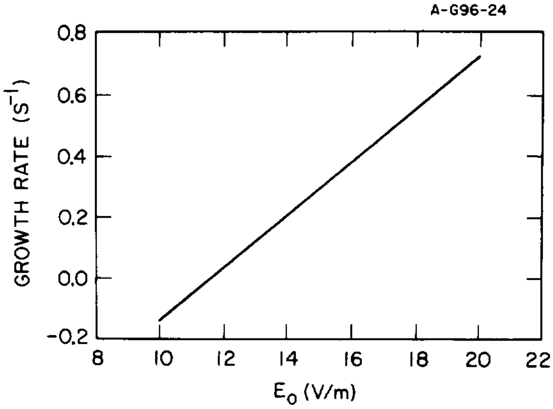

Fig. 5. The growth rate, in $\mathrm{s^{-1}}$, vs. the electric field, in $\mathrm{V\,m^{-1}}$, for the same conditions of Fig. 4

shows only a minor variation with E_0, in this range of electric fields.

As seen from Figs 3 and 4, the wave phase velocity in the frame of the dust is positive, namely in the same direction of the applied zero-order electric field, which is also the direction of the zero-order ion flow. However, when this velocity is converted into a phase velocity in the laboratory frame, one finds that ω_r/K (as distinguished from $\Omega_d^{(r)}/K$) is negative, due to the zero-order drift of the dust grains.

We have assumed in our calculations that the neutral gas is at rest in the laboratory frame, so that $v_{d0} = -(eZ/\beta m_d)E_0$ (see equation (2a)). It may be possible, however, that either because of a drag by a moving neutral gas or, perhaps, by positive ions moving under the action of the zero-order electric field, the quantity v_{d0} is different in the laboratory frame from that given by equation (2a). For instance, it may be possible for v_{d0} to be nearly zero, in which case, of course, the dust frame phase velocity is the same as the laboratory phase velocity. In the presence of an electric field $E_0 \approx 11.5\,\mathrm{V\,m^{-1}}$ (Fig. 5) the gas flow required to keep the dust grains stationary would be approximately

$$v \approx \frac{eZE_0}{4m_N N a^2 v_{th,N}} \approx 2\times 10^{-2}\,\mathrm{m\,s^{-1}} \simeq 2\,\mathrm{cm\,s^{-1}}.$$

A gas flow of this magnitude is quite likely in some experimental situations similar to that of Barkan *et al.* (1995a).

As far as the effect of the ion drag on dust grains is concerned, it can be estimated that, at plasma densities $n \lessapprox 10^8$–$10^9\,\mathrm{cm^{-3}}$, its magnitude is probably several percent of the neutral gas drag.

Finally, it may be interesting to speculate as to why, for $\epsilon Z \neq 1$, the E_{crit} vs. K curves of Fig. 1 indicate that, for any given ϵZ, the E_{crit} at small K is substantially larger than E_{crit} at large K. The transition between the two regimes seems to occur at values of K on the order of β/C_d, where C_d is the dust-acoustic speed. For $\beta = 2\,\mathrm{s^{-1}}$ and $C_d \approx 2\,\mathrm{cm\,s^{-1}}$ (see Fig. 3 for $\epsilon Z = 0.5$), $\beta/C_d \approx 100\,\mathrm{m^{-1}}$. The quantity C_d/β is the distance the dust-acoustic wave travels in a dust damping time $1/\beta$. At large K, where C_d/β is larger than the wavelength, one may expect the effect of damping to be less important, and thus E_{crit} to be smaller, than at the smaller values of K. To see whether

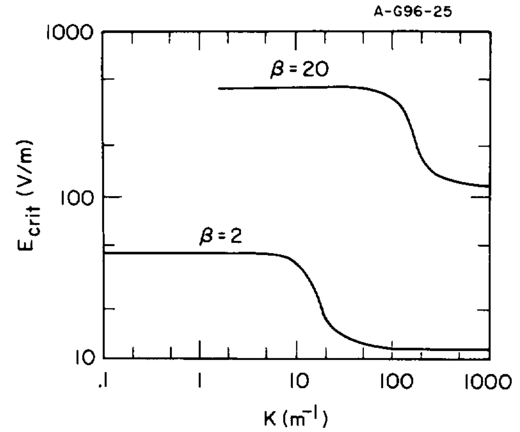

Fig. 6. The critical electric field, in $V m^{-1}$, vs. K, in m^{-1}, for $\epsilon Z = 0.5$ and for two different values of the parameter β, i.e. $\beta = 2$ and $20 s^{-1}$

there may be some substance to this speculation, Fig. 6 compares curves of E_{crit} vs. K, for $\epsilon Z = 0.5$, obtained for two different values of β, namely $\beta = 2$ and 20. It appears that at $\beta = 20$ not only are the E_{crit} values larger, as expected from the increased damping, but also that the transition from the high-K to the low-K regime occurs at values of K about one order of magnitude larger than for $\beta = 2$.

3. Conclusions

The dust-acoustic instability has been analyzed for a dusty plasma consisting of four different components, namely positive ions, electrons, negatively charged dust grains and neutral gas molecules. A zero-order electric field, E_0, imposed on the plasma produces zero-order drifts of each charged component relative to the others and to the neutral gas. As expected for a resistive-type instability, in the presence of frequent collisions among the various species, the electric field, E_0, may be large enough for dust-acoustic waves to be excited. For the situation examined in this paper, which should be generally representative of those of recent experiments, the critical electric field is on the order of $0.1 V cm^{-1}$. The free electrons in the plasma appear to have a generally stabilizing effect, since the most unstable situations occur at values of ϵZ close to unity. The phase velocity of the waves, in the frame of reference of the dust, is on the order of a few $cm s^{-1}$, comparable to observed values.

One may ask to what extent the fluid equations used in Section 2 are applicable to experimental situations such as those of Chu *et al.* (1994) and Barkan *et al.* (1995b), in which the coupling parameter for dust grains, Γ, is probably somewhere between unity and 10^2. The coupling parameter, Γ, is the ratio of the average (unshielded) Coulomb potential energy to the average kinetic energy, i.e. $\Gamma = (e^2Z^2)/(4\pi\epsilon_0 \cdot d \cdot \kappa T_d)$, where d is the average distance between neighboring dust grains (see, e.g. Ichimaru, 1993). A discussion of this point can be found in a very recent paper by Wang and Bhattacharjee (1996), to which the interested reader is referred.

Finally, equations (1d)–(1f) do not include the effects of either viscosity or dust–dust collisions, which should perhaps be considered in future investigations. One may expect that both would have a stabilizing effect and require larger critical electric fields than found in this paper.

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