ELECTROSTATIC ION-CYCLOTRON WAVES DRIVEN
BY PARALLEL VELOCITY SHEAR

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ABSTRACT

Using a fluid treatment, it is shown that electrostatic waves with angular frequencies ~ \( \omega_{ci} \) (ion cyclotron frequency) propagating at large angles to the ambient magnetic field can be excited in a magnetized plasma by perpendicular shear in the magnetic field aligned plasma flow. The role of density gradients in determining the stability of shear-driven EIC modes is also considered.

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In 1965 D’Angelo\textsuperscript{1} presented a fluid treatment of the Kelvin-Helmholtz (KH) instability in a fully ionized plasma in a magnetic field. This instability\textsuperscript{2} is excited by perpendicular shear ($\partial v_{oz}/\partial x$) in the magnetic field aligned ion flow velocity, $v_{oz}(x)$, where $x$ is the coordinate transverse to $B$. The analysis of Ref. 1 used the continuity and momentum equations for the ions and assumed a Boltzmann equilibrium for the electrons, with equal electron and ion temperatures, $T_e = T_i$. The zero order state included a magnetic field aligned ion flow with a transverse velocity gradient and a non-uniform density distribution in the $x$ direction with e-folding length $\lambda = -(1/n)(\partial n/\partial x)$. The usual linearization procedure led to a fourth order (in the wave angular frequency) dispersion relation. This dispersion relation was investigated only for the case of the low frequency ($\omega \ll \omega_{ci}$, where $\omega_{ci} = eB/m_i$ is the ion-cyclotron frequency) KH instability in which the fourth order term was negligible.

In this Brief Communication we re-examine the full fourth order dispersion relation and demonstrate that parallel velocity shear can lead to the excitation of ion cyclotron modes with $\omega \sim \omega_{ci}$, in the absence of any field aligned currents (relative electron-ion drift). The purpose of presenting these results was to show that a purely fluid mechanism (velocity shear) can give rise to an EIC instability in a plasma. The resonant excitation of EIC modes in a (uniform) plasma in which the electrons drift relative to the ions was originally analyzed using the Vlasov equation by Drummond and Rosenbluth\textsuperscript{3}. Recently, Gavrishchaka et al\textsuperscript{4}, using a kinetic analysis, investigated the effect of parallel velocity shear on the excitation of current-driven ion acoustic and EIC modes. They found that the presence of shear can, in some cases, drastically reduce the necessary critical drift velocities for the excitation of these modes. They also showed that even in cases in which there was no relative electron drift (current-free case) perpendicular shear in the parallel ion flow could lead to the excitation of multiple ion-cyclotron harmonics. Their Vlasov-based calculations, however did not include the effect of a density gradient in the zero-order state which, as we will show can have an important effect of the stability of the EIC modes.
As a starting point we re-derived the dispersion relation for the general case in which $T_e \neq T_i$, which can be expressed in the following normalized form

$$\zeta^4 - [1 + (\alpha_y^2 + \alpha_z^2)(1 + \tau)]\zeta^2 + [\Lambda \alpha_y (1 + \tau)]\zeta - S \alpha_y \alpha_z (1 + \tau) + \alpha_z^2 (1 + \tau) = 0 \quad (1)$$

where $\zeta = \Omega/\omega_{ci}$, $a_y = k_y \rho_i$, $a_z = k_z \rho_i$, $\tau = T_e/T_i$, $\Lambda = \lambda \rho_i$ and shear parameter, $S = (1/\omega_{ci})(\partial v_{oz}/\partial x)$. $\Omega = \omega - k_y v_{oy} - k_z v_{oz}$, where $\omega$ is the wave angular frequency, $v_{oy}$ and $v_{oz}$ are the ion flow velocities in the $y$ and $z$ directions, $k_y$ and $k_z$ are the wavenumbers in the directions perpendicular and parallel to $B$, respectively, and $\lambda$ is the inverse density gradient scale length. Eq. 1 is identical to D’Angelo’s Eq. 14 with $T_e \neq T_i$. If one neglects the $\zeta^4$ term in (1) one obtains the excitation conditions for the KH instability.

Fig. 1 shows numerical solutions of Eq. 1 for the real and imaginary parts of $\zeta$ for $\tau = 1$ (solid) and $\tau = 5$ (dashed), corresponding to EIC modes with $\Omega \approx \omega_{ci}$. Here we have taken $a_y = -0.5$, $a_z = 0.1$ ($k_y/k_z = 5$, propagation angle of 79° to $B$) and $\Lambda = 0.25$. For these conditions the critical shear values, $S_c$, (marginal stability) for excitation of the EIC modes are $S_c \approx 3.5$ and $S_c \approx 2.5$ for $\tau = 1.0$ and $\tau = 5.0$, respectively. The dependence of the critical shear on $\Lambda$ (normalized inverse density gradient scale length) is shown in Fig. 2. A large density gradient tends make the mode more unstable. For $\Lambda \gtrsim 0.7$, $S_c \to 0$, i.e., even a small shear is sufficient to produce the instability. Note also in Fig. 1 that the real frequency for this mode is $\sim \omega_{ci}$ but may be somewhat above or below $\omega_{ci}$. This is in contrast to the current-driven EIC mode of Drummond and Rosenbluth\(^3\) which has a real frequency always $> \omega_{ci}$.

The fluid calculations presented here indicate that relatively large values of the shear parameter, $S$, are required for EIC mode excitation, particularly for the cases of moderate density gradients. Large shear parameters are likely to be present in plasmas which contain localized ion beams. We can write $S = (1/\omega_{ci})(\partial v_{oz}/\partial x) \approx (1/\omega_{ci})(\Delta v_{oz}/\Delta x)$, with $\Delta x$ being the typical scale length of the velocity shear. If $\Delta x \approx \rho_i$ (ion gyroradius)
then $S \approx \Delta v_{oz}/C_i$, where $C_i$ is the ion thermal velocity. The presence of localized energetic ion beams could give rise to large shear parameters necessary to excite EIC instability. Additional calculations also indicate that the shear values required for excitation of the EIC modes would be lowered if the growth rates are maximized with respect to the $k_y/k_z$ parameter.

We note that the model used here does not include several effects (e.g. ion viscosity, cyclotron damping) that might have a bearing on the stability of shear-driven EIC modes.

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REFERENCES

2This instability is also referred to as the parallel velocity shear instability or the D’Angelo mode.
FIGURE CAPTIONS

Fig. 1 Real and imaginary parts of the normalized wave frequency vs the shear parameter, S. Here $a_y = -0.5$, $a_z = 0.1$, $\Lambda = 0.25$. Solid curves: $\tau = 1$, dashed curves: $\tau = 5.0$.

Fig. 2 Critical values of the shear parameter for excitation of EIC modes vs $\Lambda$. Same conditions of Fig. 1 with $\tau = 1.0$. 
Fig. 1
τ = 1.0

Fig. 2