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Effect of an external magnetic field on a critical point for phase separation in a dusty plasma

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Abstract
The effect of an external magnetic field on a critical point for phase transitions in a dusty plasma is investigated. It is shown that the ambient magnetic field increases the effective hard core radius of dust particles, which, in turn, would affect a critical point in terms of the Coulomb coupling parameter and the ratio between the inter-dust grain spacing and the dusty plasma Debye radius. The present result may be useful in understanding the phenomenon of liquid–vapor phase transitions in laboratory dusty plasmas that are held in an external magnetic field.

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1. Introduction

The existence of a critical point and the phenomenon of liquid–vapor phase transitions in an unmagnetized complex/dusty plasma has drawn much interest in recent years. Avinash and Shukla [1] were the first to predict theoretically the possibility of the liquid–vapor phase transition in a dusty plasma. Their theory employed a Lenard–Jones-like interaction potential, which is characterized by a short-range repulsive and a weak long-range attractive interaction between two negatively charged dust particles [2]. Khrapak *et al* [3] also carried out a calculation of the interaction between two negatively charged dust particles and reported evidence for a weak attractive potential. The latter was used by them to postulate the existence of a critical point in a complex/dusty plasma [4]. Recently, Avinash [5] provided a mean field theory which shows the existence of a critical point even in the absence of an attractive potential.

In this paper, we present an investigation of the effect of an external magnetic field (which is usually present in many laboratory dusty plasma experiments and in astro-dusty plasmas) on a critical point and liquid–vapor phase transitions in dusty plasmas. The effect of the magnetic field enters through a modification of the ion susceptibility, which affects the Debye–Hueckel repulsive interaction potential [6] and the oscillatory wake-field potential [7] around a charged dust particle. In particular, in an external magnetic field, weakly correlated electrons and ions in dusty plasmas are magnetized, and subsequently, the dielectric susceptibilities of the electrons and ions are altered by the ambient magnetic field. It then turns out that the latter significantly modifies the interaction potential distribution around a charged dust grain that floats at a constant potential in dusty plasmas, in addition to introducing a new screening length (the modified ion sound speed/the ion gyrofrequency) across the external magnetic field direction. Using the modified Debye–Hueckel (MDH) repulsive interaction potential, we here recalculate the attractive interaction potential between two negatively charged dust particles and use that potential in the dust-free energy expression to calculate a critical point for liquid–vapor phase transitions in a dusty plasma with an external magnetic field.

2. Inter-particle potential

We consider an electron–ion–dust plasma in an external $B_0\hat{z}$, where $\hat{z}$ is the unit vector along the z-axis in the Cartesian coordinate system and B_0 the strength of the magnetic field. At equilibrium, we have $n_{i0} = n_{e0} - Z_d n_{d0}$, where n_{j0} is the unperturbed number density of the particle species j (j is e for electrons, i for ions and d for dust particles), and Z_d is the number of electrons residing on a dust grain. The electrostatic interaction potential distribution around a test dust grain with a charge Q is

$$\Phi = \Phi_I + \Phi_{II}, \tag{1}$$

where the MDH potential reads as [6, 7]

$$\Phi_I(r) = \frac{Q}{(1+f)\,r}\exp\left[-r\frac{\left(1 - C_s^2/u_{i0}^2\right)^{1/2}}{\sqrt{1+f}\lambda_d}\right], \tag{2}$$

where $f = \omega_{pi}^2/\omega_{ci}^2$, with $\omega_{pi} = (4\pi n_{i0}e^2/m_i)^{1/2}$, $\omega_{ci} = eB_0/m_i c$, e is the magnitude of the electron charge, m_i the ion mass and c the speed of light in vacuum, while $C_s = (n_{i0}k_B T_e/n_{e0}m_i)^{1/2}$ and u_{i0} are the modified (due to the presence of stationary negative dust grains) ion-acoustic speed [8, 9] and the equilibrium ion streaming speed, respectively. The Boltzmann constant is denoted by k_B. The oscillatory wake potential (OWP) is given as [7]

$$\Phi_{II} \approx A_0\frac{\cos\left(\xi/L_{]]}\right)}{|\xi|},\quad A_0 = \frac{2Q\,(1+\beta)}{(1+f)}, \qquad L_{]]} = \sqrt{1+f}\lambda_d\frac{|v_t - v_0|}{C_d}, \tag{3}$$

where $\beta = C_d^2/(v_t - v_0)^2\left(1 - C_s^2/u_{i0}^2\right)$, $C_d = \omega_{pe}\lambda_d$ is the dust-acoustic speed, λ_d is the electron Debye length and v_t (v_0) is the magnetic field-aligned test dust charge (ion flow) speed. In deducing equation (2), we have assumed that $|z - v_t t| = 0$, so that the MDH potential is isotropic. On the other hand, the OWP is oscillating along the z-direction. Hence, the total interaction potential around a test dust particle is asymmetric.

We comment on the derivation of equation (1), which is sum of the short- and long-range repulsive and attractive potentials, given by equations (2) and (3), respectively. It has been obtained from the Fourier-transformed Poisson's equation, in which the electron density perturbation obeys the Boltzmann law (which is deduced from the balance between the electric force and the electron pressure gradient along the external magnetic field direction, since the electrons rapidly thermalize along the latter which is aligned along the z-axis in the Cartesian coordinate system), while the ion number density perturbation [7] includes the contributions of the ion polarization drift and the magnetic field-aligned streaming ions.

We carry out our calculations in the limit $f \gg 1$, which is valid for a dense dusty plasma in which the ion plasma frequency is much larger than the ion gyrofrequency. In this limit, the amplitude of the wake potential is relatively small and the scale length of the wake oscillations is rather large. Hence, the contribution of the OWP can be safely ignored. In this limit, the total ES potential around a test dust grain is

$$\Phi = \frac{Q}{(1+f)\,r}\exp\left[-\frac{r}{\sqrt{(1+f)}\lambda_d}\right], \tag{4}$$

where we have assumed that $u_{i0} \gg C_s$, which is justified for most of the laboratory dusty plasma conditions.

Next, to calculate the interaction potential between two dust grains with similar charges Q, we make the following space and charge transformations: $r' = r/\sqrt{1+f}$, $Q' = Q/(1+f)^{3/2}$. In the transformed space r', the potential around the transformed charge Q' is

$$\Phi = \frac{Q'}{r'}\exp\left[-\frac{r'}{\lambda_d}\right], \tag{5}$$

which reveals that the Debye–Hueckel potential of a dust charge Q in the presence of the magnetic field is the same as the potential of the transformed charge Q' in the transformed space r'. Clearly, the effect of the magnetic field has been transformed away by suitable space and charge transformations. Furthermore, to calculate the interaction potential between two charges Q that are at a distance r apart, we consider the equivalent problem of two charges Q' placed a distance r' apart and having the Debye–Hueckel potential given by equation (5). As shown by Resendes *et al* [2], the interaction potential in this case consists of two parts: (i) a repulsive part due to the electrostatic repulsion between bare charges of the two dust grains and (ii) an attractive part due to the interaction of the positive sheath of one dust grain and the bare charge of the other dust grain. Following [2], we can easily calculate the interaction potential W in the transformed space r' and then make the transformation $W(Q', r') = W(Q, r)$. We have

$$W = \frac{Q^2}{(1+f)^{3/2}\,r}\left(1 - \frac{r}{2\lambda_d\sqrt{(1+f)}}\right)\exp\left(-\frac{r}{\sqrt{(1+f)}\lambda_d}\right), \tag{6}$$

which exhibits that $W = 0$ at $r_0 = 2\lambda_d\sqrt{(1+f)}$. Since f is greater than 1 for the parameters that are representative of laboratory conditions, the effective hard core radius r_0 of the dust grain increases due to the presence of the external magnetic field.

To calculate the equation of state and a critical point for phase transitions, we consider N_d dust particles with the temperature T_d in a volume V. The inter-dust particle potential W is given by equation (6).

3. Free energy and the equation of state

Following Landau and Lischitz [2, 10], the Helmholtz free energy F of the system can be calculated directly from the inter-particle potential W by the expression

$$F = F_{id} + \frac{T_d N_d^2}{V}C\,(T_d)\,, \tag{7}$$

where $C\,(T_d) = \frac{1}{2}\int(1 - \exp[-W/T_d])\,d^3V$ and F_{id} is the usual ideal gas-like contributions. The integration in the latter expression may be carried out by noting that within the interval $0 \leqslant r \leqslant r_0$, $W/T_d \to \infty$, hence $\exp(-W/T_d) \to 0$. On the other hand, within the interval $r_0 \leqslant r \leqslant \infty$, $W/T_d \leqslant 1$, and hence $(1 - \exp[-W/T_d]) \approx W/T_d$.

Performing the integration, we calculate the free energy F and the dust pressure from the expression $P_d = -\left.\frac{\partial F}{\partial V}\right|_{T_d}$. This gives the equation of state for the dust particles in the presence of the magnetic field as

$$P_d = \frac{N_d T_d}{(V - N_d b)} - a N_d^2, \qquad (8)$$

where

$$b = \frac{2\pi\,(1+f)^{3/2}}{3}\lambda_d^3, \quad a = \frac{3Q^2}{2}\lambda_d^2.$$

We note that the effect of the magnetic field enters through the parameter f. In terms of Γ and κ defined by $\Gamma = Q^2/r_d T_d$, $\kappa = r_d/\lambda_d$, a critical point turns out to be $\Gamma_c = 0.25\,(1+f)$, $\kappa_c = 1.5\,(1+f)^{3/2}$. Since $f > 1$, the critical point increases with an increase of f.

4. Summary and conclusions

To summarize, in this paper we have presented an investigation of the effect of an external magnetic field on a critical point associated with the liquid–vapor phase transitions in dusty plasmas. For this purpose, we utilized the modified (by the magnetic field) short-range repulsive and long-range attractive potentials and calculated the free energy and the equation of state of our dusty plasma in which Boltzmann distributed electrons rapidly thermalize along the magnetic field direction, while magnetized ions are subjected to a polarization drift. It is found that the magnetic field effect enters through the parameter f, which is representative of the ion polarization drift. In a dense dusty plasma with $f \gg 1$, the critical point is shifted to higher value with an increase of f. The knowledge of the critical point is an essential component for the understanding of the phenomenon of liquid–vapor phase transitions in dusty plasmas that are held in an external magnetic field. A dusty plasma device capable of operating with magnetized electrons, ions and charged dust particles will be built at Auburn University, USA under the direction of E Thomas Jr [11]. We hope that future experiments in this magnetoplasma device will verify the theoretical predictions we have made in this paper.

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