Electron and ion inertia effects on current-driven collisional dust acoustic, dust ion acoustic, and ion acoustic instabilities

Robert L. Merlino and Nicola D’Angelo

Department of Physics and Astronomy, The University of Iowa, Iowa City, Iowa 52242

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Ion acoustic waves can be excited by electrons drifting relative to the ions in a plasma. This relative drift can be produced, in a collisional plasma, by a static electric field. In the analysis of this instability, which occurs for frequencies well below the ion plasma frequency, the electron inertia term in the momentum equation is typically neglected. A similar assumption has been employed in the investigation of the dust ion acoustic instability in a collisional dusty plasma. In the study of the collisional current driven dust acoustic instability, both the electron and ion inertial terms were neglected. It is shown here that the inclusion of the appropriate inertia terms can result in significant differences in the growth rates for these instabilities. The cases studied, including the inertia terms, led to a decrease in the critical electric fields necessary to excite the current driven dust acoustic, dust ion acoustic, and ion acoustic instabilities. © 2005 American Institute of Physics. [DOI: 10.1063/1.1889445]

In this Brief Communication the effects of electron inertia on the current-driven dust ion acoustic and ion acoustic instabilities and the effects of electron and ion inertia on the current-driven dust acoustic instability are examined. Collisional ion acoustic instabilities occur in weakly ionized plasmas which contain an equilibrium electric field which produces a relative drift of electrons and ions.

This problem was first considered by Sel’t,1 Sundaram and Kaw,2 and Kaw.3 In these works, the electron inertia terms in the electron fluid equation were neglected based on the argument that since the waves are of low frequency, a modified Boltzmann relation was appropriate for the electrons. The same approximation was also applied to the problem of current-driven ion cyclotron waves in a collisional plasma.4 Subsequently, a similar approach for the electron dynamics in a collisional dusty plasma was adopted by Merlino5 in analyzing the current-driven dust ion acoustic instability. Annou,6 Ostrikow et al.,7 and Tribeche and Zerguini,8 using the same approximation for the electrons, extended the fluid analysis of Merlino5 to include the effects of dust charge fluctuation. D’Angelo and Merlino9 also investigated the excitation of the current-driven dust acoustic instability in a collisional plasma using a fluid model in which both the electron and ion inertia terms were neglected in the momentum equations for the electrons and ions.

It will now be shown that the inclusion of the electron inertia terms in the momentum equations can lead, under certain conditions, to significant differences in the growth rate of current-driven dust ion acoustic waves. In particular, we find that the inclusion of the electron inertia terms tends to reduce the critical electric field for instability, as compared to the case in which the electron inertia is neglected. For the case of the current-driven dust acoustic instability, a similar result is obtained when both the electron and ion inertia terms are retained in the fluid equations. We provide numerical solutions to the dispersion relations to illustrate these points.

We start by obtaining the general dispersion relation for long wavelength (compared to the relevant Debye length) acoustic modes (dust acoustic, DA, and dust ion acoustic, DIA) in a collisional dusty plasma. This dispersion relation, of course, also contains the current-driven ion acoustic, IA, instability. The analysis follows directly that given in Ref. 9 with the difference that the ion and electron inertia terms are now included in the momentum equations as well as finite dust temperature.

We consider a three species plasma containing ions, electrons and negatively charged dust particles of mass \( m_d \) and charge number \( Z \). The equilibrium state is uniform and time independent with a dc electric field \( E_0 \) which produces zero-order particle drifts \( u_{i0}, u_{e0}, \) and \( u_{d0} \) for the ions, electrons, and dust, respectively. The species temperatures are \( T_i, T_e, \) and \( T_d \).

All three species obey the continuity, momentum equations, and the condition of charge neutrality

\[
\frac{\partial n_j}{\partial t} + \frac{\partial (n_j u_j)}{\partial x} = 0, \tag{1}
\]

\[
n_j \frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x} + kT_j \frac{\partial n_j}{\partial x} - q_j n_j E = - v_{jm} n_j m_j u_j, \tag{2}
\]

\[
n_i = n_e + Z n_d, \tag{3}
\]

where \( j = (i, e, d) \) refers to ions, electrons and dust, respectively. The ion, electron, and dust charges are given by \( q_i = +e, q_e = -e, \) and \( q_d = -eZ \). \( v_{jm} \) is the collision frequency between species \( j \) and the neutrals. The quantity \( \delta_j \) in Eq. (2) is inserted to allow comparison with the cases in which the inertia terms are neglected. For calculations in which the inertia terms are included, \( \delta_i = \delta_e = 1 \). To investigate the effect of neglecting the electron inertia term, we set \( \delta_e = 0 \). For the dust acoustic case, in which both the electron and ion inertia are neglected, \( \delta_i = \delta_e = 0 \). For all cases \( \delta_i = 1 \) for the dust fluid.

The zero-order state is defined by the equations
where $s$ atoms, and $c_i$ is the density of the neutral atoms, and $c_e = \sqrt{kT_e/m_e}$ is the thermal speed of the ions with temperatures $T_i$ and $T_e$, and $m_i$ and $m_e$ are the ion and electron mass, respectively.

The dispersion relation is obtained by linearizing Eqs. (1)-(3), using the zero-order Eqs. (4) and (5) and assuming that all first-order quantities vary as $e^{i(kz-\omega t)}$:

$$
\frac{1}{\nu_{in}} = \frac{q_j}{m_j} E_0, \\
\nu_{en} = \frac{4m_e Na^2 c_e}{m_d},
$$

(4a)

(4b)

(4c)

where $\nu_{in}$ is the ion-neutral collision frequency, $\nu_{en}$ is the electron-neutral collision frequency, $q_j$ is the charge of species $j$, $m_j$ is the mass of species $j$, $E_0$ is the electric field, $Na$ is the ion density, and $c_e = \sqrt{kT_e/m_e}$ is the thermal speed of the neutrals with temperature $T_e$ and mass $m_e$, $a$ is the radius, and $m_d$ is the mass of the dust particles.

**Dust acoustic instability.** The dispersion relation, Eq. (6) was solved numerically for the complex angular frequency $\tilde{\omega} = \omega - i\gamma$, where $\omega$ is the real frequency in the dust frame and $\gamma$ is the growth rate, using the set of plasma parameters appropriate to a laboratory discharge plasma listed in column (a) of Table I. Figure 1 shows a plot of the real frequency $\Omega_{d,r}$ and growth rate $\gamma$ of the dust acoustic instability as a function of the zero-order electric field. The solid curves are for the case in which the ion and electron inertia terms are retained ($\delta_i = \delta_e = 1$) while the dashed curves show, for comparison, the corresponding real frequency and growth rate when the ion and electron inertia terms are both neglected ($\delta_i = \delta_e = 0$). The critical electric field ($\gamma = 0$) for wave growth is about a factor of 3–4 lower for the case in which the ion and electron inertia terms are retained as compared to the case in which the inertia terms are neglected. The same conclusion holds if only the ion inertia term is retained ($\delta_i = 1$) while the electron inertia term is omitted ($\delta_e = 0$), indicating that the ion inertia has a much larger effect on the critical electric field than the electron inertia.

**Dust ion acoustic instability.** For the analysis of the dust ion acoustic instability (see, e.g., Ref. 5) the last term in Eq. (6) is eliminated since the dust is treated simply as an immobile $(m_d \rightarrow \infty)$ background of negative charge, and set $\delta_i = 1$. The resulting dispersion relation is given by

where $c_n = \sqrt{kT_n/m_n}$ is the thermal speed of the neutrals with temperature $T_n$ and mass $m_n$, $a$ is the radius, and $m_d$ is the mass of the dust particles.

**Table I. Values of appropriate plasma parameters used in the numerical solution to the dispersion relation for the (a) dust acoustic DA, (b) dust ion acoustic DIA, and (c) ion acoustic IA, instabilities.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>DA</th>
<th>DIA</th>
<th>IA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ion mass (kg)</td>
<td>$m_i$</td>
<td>$6.7 \times 10^{-26}$</td>
<td>$3.3 \times 10^{-26}$</td>
</tr>
<tr>
<td>Neutral atom mass (kg)</td>
<td>$m_n$</td>
<td>$4.7 \times 10^{-26}$</td>
<td>$4.7 \times 10^{-26}$</td>
</tr>
<tr>
<td>Dust mass (kg)</td>
<td>$m_d$</td>
<td>$1.0 \times 10^{-12}$</td>
<td>$5.0 \times 10^{-12}$</td>
</tr>
<tr>
<td>Ion temperature (K)</td>
<td>$T_i$</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Neutral atom temperature (K)</td>
<td>$T_n$</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Dust temperature (K)</td>
<td>$T_d$</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Electron temperature (K)</td>
<td>$T_e$</td>
<td>23000</td>
<td>23000</td>
</tr>
<tr>
<td>Dust radius ($\mu$m)</td>
<td>$a$</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Dust charge number</td>
<td>$Z$</td>
<td>40000</td>
<td>40000</td>
</tr>
<tr>
<td>Dust to ion mass ratio $n_d/n$,</td>
<td>$\epsilon$</td>
<td>$1.25 \times 10^{-5}$</td>
<td>$1.25 \times 10^{-5}$</td>
</tr>
<tr>
<td>Fraction of negative charge on</td>
<td>$\epsilon Z$</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Dust</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ion-neutral cross section (m$^2$)</td>
<td>$\sigma_{in}$</td>
<td>$5.0 \times 10^{-20}$</td>
<td>$5.0 \times 10^{-20}$</td>
</tr>
<tr>
<td>Electron-neutral cross section (m$^2$)</td>
<td>$\sigma_{en}$</td>
<td>$1.0 \times 10^{-20}$</td>
<td>$1.0 \times 10^{-20}$</td>
</tr>
<tr>
<td>Neutral atom density (m$^{-3}$)</td>
<td>$N$</td>
<td>$3.0 \times 10^{21}$</td>
<td>$1.5 \times 10^{21}$</td>
</tr>
<tr>
<td>Wave number (m$^{-3}$)</td>
<td>$K$</td>
<td>1000</td>
<td>63</td>
</tr>
</tbody>
</table>
part of the angular frequency in the ion frame and 

\[ E \]

order electric field

Table I. Figure 2 shows a plot of the real frequency

V

s

plex wave angular frequency

Numerical solutions of the dispersion relation for the com-

sion relation for collisional ion acoustic waves can be

glected. The values of the plasma parameters used to obtain this graph are

are listed in column “DA” of Table I.

The dispersion relation was solved numerically for the com-

sion of the electron inertia terms in the electron momentum

\[ K \]

\[ T \]

\[ m \]

\[ e \]

\[ Z \]

\[ 1 \]

\[ - K^2 \kappa T_e + m_e \Omega_e (\Omega_e + iv_{en}) \]

\[ 1 - eZ \]

\[ - K^2 \kappa T_e + m_e \Omega_e (\delta_e \Omega_e + iv_{en}) \]

= 0. (9)

The dispersion relation was solved numerically for the complex angular frequency \( \Omega = \Omega_{i,r} + i \gamma \), where \( \Omega_{i,r} \) is the real part of the angular frequency in the ion frame and \( \gamma \) is the growth rate for the set of parameters listed in column (b) of Table I. Figure 2 shows a plot of the real frequency \( \Omega_{i,r} \) (in the ion frame) and growth rate \( \gamma \) as a function of the zero-order electric field \( E_0 \). For this case there is roughly a factor of 3 reduction in the critical electric field and more than an order of magnitude increase in the growth rate when the electron inertia is included.

**Ion acoustic instability.** Given the results of including the appropriate inertia terms in the analyses of the dust acoustic and dust ion acoustic instabilities, it seems appropriate to reconsider the even more basic problem of current-driven ion acoustic waves in dust-free plasma. The dispersion relation for collisional ion acoustic waves can be obtained from Eq. (6) by setting \( \epsilon Z = 0 \) and \( \delta_i = 1 \):

\[ 1 \]

\[ - K^2 \kappa T_i + m_i \Omega_i (\Omega_i + iv_{in}) \]

\[ 1 - K^2 \kappa T_e + m_e \Omega_e (\delta_i \Omega_i + iv_{en}) \]

= 0. (10)

Numerical solutions of the dispersion relation for the complex wave angular frequency \( \Omega = \Omega_{i,r} + i \gamma \), where \( \Omega_{i,r} \) is the real frequency in the ion frame and \( \gamma \) is the growth rate, are shown in Fig. 3 using the set of parameters listed in column (c) of Table I. Again, for comparison, the results for the case in which the electron inertia is neglected (\( \delta_e = 0 \)) are also shown. For the particular parameters chosen, which are typical for laboratory plasmas, the effect of the electron inertia is to reduce the critical electric field by about an order of magnitude and increase the growth rate by more than two orders of magnitude. By varying the parameters \( K \) and \( N \) over a large range of values, we found that the general thrust of the results shown in Fig. 3 was borne out. The inclusion of electron inertia makes the collisional ion acoustic instability easier to excite.

We note that it is possible to find combinations of parameters in which the effect of including the inertia terms produces less of a difference than that seen in the examples provided here. For example with the following set of parameters: \( m_i = m_n = 3.3 \times 10^{-26} \text{ kg}, m_e = 4.2 \times 10^{-15} \text{ kg}, \)

\( a = 1.0 \mu m, Z = 5000, \epsilon = 1, T_i = T_n = 1000 \text{ K}, T_d = 300 \text{ K}, \)

\( T_e = 20000 \text{ K}, N = 10^{19} \text{ m}^{-3}, \sigma_n = 5 \times 10^{-19} \text{ m}^2, \sigma_e = 1 \times 10^{-20} \text{ m}^2, \text{ and } K = 100 \text{ m}^{-1} \), we found that the real frequencies and critical electric fields (\( \approx 0.1 \text{ V/m} \)) for the dust acoustic instability were about the same in both cases, although the growth rate was more than an order of magnitude higher for the case in which the electron and ion inertia terms were retained.

Although the general, qualitative conclusions of Refs. 5 and 9 are not altered, the present results show that the inclusion of the electron inertia terms in the electron momentum equation for the dust ion acoustic instability and the ion and electron inertia terms for the dust acoustic instability, can
lead to significant differences in the growth rates and critical electric fields. Apparently, this conclusion also holds for the current-driven ion acoustic instability considered by Kaw over 30 years ago.3

These results seem surprising since one might expect that electron inertia effects would make little difference when dealing with waves with frequencies well below the ion plasma frequency for the case of the ion acoustic waves and below the dust plasma frequency for the dust acoustic waves. An additional justification offered for the neglect of the inertia terms in Eq. (2) is that they are of higher order in \( m_e \) (or \( m_i \)) compared to the collision term on the right-hand side, since the collision frequency [Eq. (7)] is \( \sim m_e^{-1/2} \) and thus the collision term is of order \( m_e^{1/2} \). The collisional ion acoustic instability is driven by the relative drift of electrons and ions. The waves are damped due to ion-neutral collisions but electron-neutral collisions are destabilizing as first pointed out by Kaw.3 A perturbation in the ion charge density will be partially neutralized by electrons which are attracted to regions of excess positive charge. Electrons colliding with neutrals will be impeded from fully neutralizing the ion space charge, thus the instability is enhanced by electron-neutral collisions. The electron inertia may have a similar effect in reducing their ability to neutralize regions of excess positive charge. That electron inertia can play a role similar to plasma resistivity has been pointed out previously.10

**ACKNOWLEDGMENT**

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**FIG. 3.** The negative of the real frequency (in the ion frame) \( \Omega_{ir} \) and growth rate \( \gamma \) of the current-driven ion acoustic instability as a function of the applied electric field. \( \delta_e = 1 \) is the case in which the electron inertia terms are included and \( \delta_e = 0 \) correspond to the case in which the electron inertia terms are neglected. The values of the plasma parameters used to obtain this graph are listed in column “IA” of Table I.