Electron and ion inertia effects on current-driven collisional dust acoustic, dust ion acoustic, and ion acoustic instabilities

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Ion acoustic waves can be excited by electrons drifting relative to the ions in a plasma. This relative drift can be produced, in a collisional plasma, by a static electric field. In the analysis of this instability, which occurs for frequencies well below the ion plasma frequency, the electron inertia term in the momentum equation is typically neglected. A similar assumption has been employed in the investigation of the dust ion acoustic instability in a collisional dusty plasma. In the study of the collisional current driven dust acoustic instability, both the electron and ion inertial terms were neglected. It is shown here that the inclusion of the appropriate inertia terms can result in significant differences in the growth rates for these instabilities. The cases studied, including the inertia terms, led to a decrease in the critical electric fields necessary to excite the current driven dust acoustic, dust ion acoustic, and ion acoustic instabilities. © *2005 American Institute of Physics*.
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In this Brief Communication the effects of electron inertia on the current-driven dust ion acoustic and ion acoustic instabilities and the effects of electron and ion inertia on the current-driven dust acoustic instability are examined. Collisional ion acoustic instabilities occur in weakly ionized plasmas which contain an equilibrium electric field which produces a relative drift of electrons and ions.

This problem was first considered by Self,[1] Sundaram and Kaw,[2] and Kaw.[3] In these works, the electron inertia terms in the electron fluid equation were neglected based on the argument that since the waves are of low frequency, a modified Boltzmann relation was appropriate for the electrons. The same approximation was also applied to the problem of current-driven ion cyclotron waves in a collisional plasma.[4] Subsequently, a similar approach for the electron dynamics in a collisional dusty plasma was adopted by Merlino[5] in analyzing the current-driven dust ion acoustic instability. Annou,[6] Ostrikow *et al.*,[7] and Tribeche and Zerguini,[8] using the same approximation for the electrons, extended the fluid analysis of Merlino[5] to include the effects of dust charge fluctuation. D'Angelo and Merlino[9] also investigated the excitation of the current-driven dust acoustic instability in a collisional plasma using a fluid model in which both the electron and ion inertia terms were neglected in the momentum equations for the electrons and ions.

It will now be shown that the inclusion of the electron inertia terms in the momentum equations can lead, under certain conditions, to significant differences in the growth rate of current-driven dust ion acoustic waves. In particular, we find that the inclusion of the electron inertia terms tends to reduce the critical electric field for instability, as compared to the case in which the electron inertia is neglected. For the case of the current-driven dust acoustic instability, a similar result is obtained when both the electron and ion inertia terms are retained in the fluid equations. We provide numerical solutions to the dispersion relations to illustrate these points.

We start by obtaining the general dispersion relation for long wavelength (compared to the relevant Debye length) acoustic modes (dust acoustic, DA, and dust ion acoustic, DIA) in a collisional dusty plasma. This dispersion relation, of course, also contains the current-driven ion acoustic, IA, instability. The analysis follows directly that given in Ref. 9 with the difference that the ion and electron inertia terms are now included in the momentum equations as well as finite dust temperature.

We consider a three species plasma containing ions, electrons and negatively charged dust particles of mass m_d and charge number Z. The equilibrium state is uniform and time independent with a dc electric field E_0 which produces zero-order particle drifts u_{i0}, u_{e0}, and u_{d0} for the ions, electrons, and dust, respectively. The species temperatures are T_i, T_e, and T_d. All three species obey the continuity, momentum equations, and the condition of charge neutrality

$$\frac{\partial n_j}{\partial t} + \frac{\partial (n_j u_j)}{\partial x} = 0, \tag{1}$$

$$n_j \delta_j m_j \left(\frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x} \right) + kT_j \frac{\partial n_j}{\partial x} - q_j n_j E = - \nu_{jn} n_j m_j u_j, \tag{2}$$

$$n_i = n_e + Z n_d, \tag{3}$$

where $j=(i,e,d)$ refers to ions, electrons and dust, respectively. The ion, electron, and dust charges are given by $q_i = e$, $q_e = -e$, and $q_d = -eZ$. ν_{jn} is the collision frequency between species j and the neutrals. The quantity δ_j in Eq. (2) is inserted to allow comparison with the cases in which the inertia terms are neglected. For calculations in which the inertia terms are included, $\delta_i = \delta_e = 1$. To investigate the effect of neglecting the electron inertia term, we set $\delta_e = 0$. For the dust acoustic case, in which both the electron and ion inertia are neglected, $\delta_i = \delta_e = 0$. For all cases $\delta_d = 1$ for the dust fluid.

The zero-order state is defined by the equations

TABLE I. Values of appropriate plasma parameters used in the numerical solution to the dispersion relation for the (a) dust acoustic DA, (b) dust ion acoustic DIA, and (c) ion acoustic IA, instabilities.

Parameter		DA	DIA	IA
Ion mass (kg)	m_i	4.7×10^{-26}	6.7×10^{-26}	3.3×10^{-26}
Neutral atom mass (kg)	m_n	4.7×10^{-26}		
Dust mass (kg)	m_d	1.0×10^{-12}		
Ion temperature (K)	T_i	300	1160	1160
Neutral atom temperature (K)	T_n	300		
Dust temperature (K)	T_d	300		
Electron temperature (K)	T_e	23 000	23 000	23 000
Dust radius (μm)	a	5.0		
Dust charge number	Z	40 000		
Dust to ion mass ratio n_{d0}/n_{i0}	ε	1.25×10^{-5}		
Fraction of negative charge on dust	εZ	0.5	0.8	
Ion-neutral cross section (m^2)	$\sigma_{\rm in}$	5.0×10^{-20}	5.0×10^{-19}	5.0×10^{-19}
Electron-neutral cross section (m^2)	$\sigma_{\rm en}$	1.0×10^{-20}	2.0×10^{-20}	1.0×10^{-20}
Neutral atom density (m^{-3})	N	3.0×10^{21}	1.5×10^{21}	1.0×10^{19}
Wave number (m^{-1})	K	1000	63	100

$$u_{j0} = \frac{q_j}{m_j \nu_{jn}} E_0, \tag{4a}$$

$$n_{i0} = n_{e0} + Z n_{d0}. \tag{4b}$$

With $\varepsilon = n_{d0}/n_{i0}$ we obtain from Eq. (4b)

$$n_{e0} = (1 - \varepsilon Z) n_{i0}, \tag{5}$$

where the quantity εZ is the fraction of the negative charge on dust grains, so that in the absence of dust $\varepsilon Z = 0$.

The dispersion relation is obtained by linearizing Eqs. (1)–(3), using the zero-order Eqs. (4) and (5) and assuming that all first-order quantities vary as $e^{i(Kx-\omega t)}$,

$$\frac{1}{-K^2\kappa T_i + m_i\Omega_i(\delta_i\Omega_i + i\nu_{in})} + \frac{1-\varepsilon Z}{-K^2\kappa T_e + m_e\Omega_e(\delta_e\Omega_e + i\nu_{en})} + \frac{\varepsilon Z^2}{-K^2\kappa T_d + m_d\Omega_d(\Omega_d + i\nu_{dn})} = 0 \tag{6}$$

where K is the wave number, ω the complex wave angular frequency, $\Omega_j = \omega - K u_{j0}$ is the angular frequency in the frame of reference of species $j, j = (i, e, d)$, and κ is the Boltzmann constant. The collision frequencies for ion and electron collisions with the neutrals are

$$\nu_{in} = N\sigma_{in} c_i, \tag{7a}$$

$$\nu_{en} = N\sigma_{en} c_e, \tag{7b}$$

where σ_{in} and σ_{en} are the cross sections for ion-neutral and electron-neutral collisions, N is the density of the neutral atoms, and $c_i = \sqrt{\kappa T_i/m_i}$ and $c_e = \sqrt{\kappa T_e/m_e}$ are the ion and electron thermal speeds. The dust-neutral collision frequency is given by[9]

$$\nu_{dn} = \frac{4 m_n N a^2 c_n}{m_d}, \tag{8}$$

where $c_n = \sqrt{\kappa T_n/m_n}$ is the thermal speed of the neutrals with temperature T_n and mass m_n, a is the radius, and m_d is the mass of the dust particles.

Dust acoustic instability. The dispersion relation, Eq. (6) was solved numerically for the complex angular wave frequency $\Omega_d = \Omega_{d,r} + i\gamma$, where $\Omega_{d,r}$ is the real frequency in the dust frame and γ is the growth rate, using the set of plasma parameters appropriate to a laboratory discharge plasma listed in column (a) of Table I. Figure 1 shows a plot of the real frequency $\Omega_{d,r}$ and growth rate γ of the dust acoustic instability as a function of the zero-order electric field. The solid curves are for the case in which the ion and electron inertia terms are retained ($\delta_i = \delta_e = 1$) while the dashed curves show, for comparison, the corresponding real frequency and growth rate when the ion and electron inertia terms are both neglected ($\delta_i = \delta_e = 0$). The critical electric field ($\gamma \approx 0$) for wave growth is about a factor of 3–4 lower for the case in which the ion and electron inertia terms are retained as compared to the case in which the inertia terms are neglected. The same conclusion holds if only the ion inertia term is retained ($\delta_i = 1$) while the electron inertia term is omitted ($\delta_e = 0$), indicating that the ion inertia has a much larger effect on the critical electric field than the electron inertia.

Dust ion acoustic instability. For the analysis of the dust ion acoustic instability (see, e.g., Ref. 5) the last term in Eq. (6) is eliminated since the dust is treated simply as an immobile ($m_d \to \infty$) background of negative charge, and set $\delta_i = 1$. The resulting dispersion relation dispersion is given by

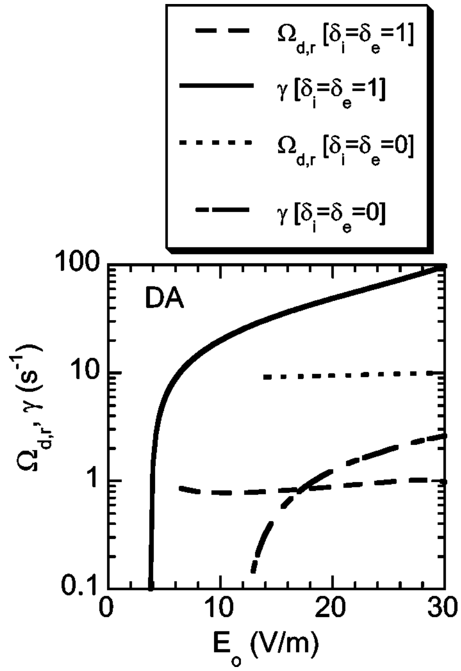

FIG. 1. Real frequency (in the dust frame) $\Omega_{d,r}$ and growth rate γ for the current driven dust acoustic instability as a function of the applied electric field. $\delta_i=\delta_e=1$ is for the case in which the ion and electron inertia terms are retained and $\delta_i=\delta_e=0$ is for the case in which the inertia terms are neglected. The values of the plasma parameters used to obtain this graph are listed in column "DA" of Table I.

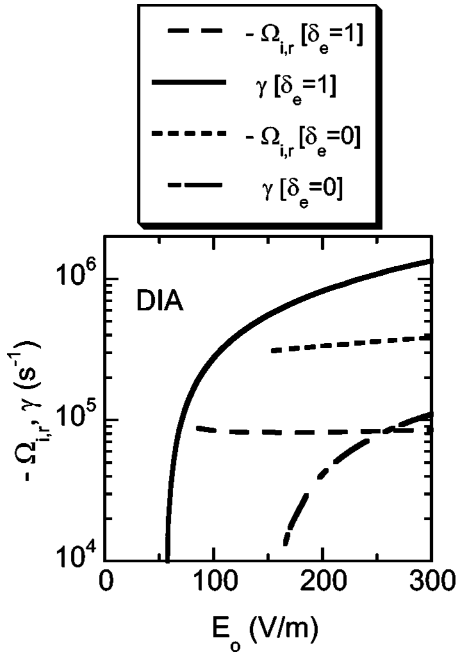

FIG. 2. The negative of the real frequency (in the ion frame) $\Omega_{i,r}$ and growth rate γ of the current-driven dust ion acoustic instability as a function of the applied electric field. $\delta_e=1$ is the case in which the electron inertia terms are included and $\delta_e=0$ correspond to the case in which the electron inertia terms are neglected. The values of the plasma parameters used to obtain this graph are listed in column "DIA" of Table I.

$$\frac{1}{-K^2\kappa T_i + m_i\Omega_i(\Omega_i + i\nu_{in})} + \frac{1-\varepsilon Z}{-K^2\kappa T_e + m_e\Omega_e(\delta_e\Omega_e + i\nu_{en})} = 0. \tag{9}$$

The dispersion relation was solved numerically for the complex angular frequency $\Omega_i=\Omega_{i,r}+i\gamma$, where $\Omega_{i,r}$ is the real part of the angular frequency in the ion frame and γ is the growth rate for the set of parameters listed in column (b) of Table I. Figure 2 shows a plot of the real frequency $\Omega_{i,r}$ (in the ion frame) and growth rate γ as a function of the zero-order electric field E_0. For this case there is roughly a factor of 3 reduction in the critical electric field and more than an order of magnitude increase in the growth rate when the electron inertia is included.

Ion acoustic instability. Given the results of including the appropriate inertia terms in the analyses of the dust acoustic and dust ion acoustic instabilities, it seems appropriate to reconsider the even more basic problem of current-driven *ion acoustic* waves in dust-free plasma. The dispersion relation for collisional ion acoustic waves can be obtained from Eq. (6) by setting $\varepsilon Z=0$ and $\delta_i=1$:

$$\frac{1}{-K^2\kappa T_i + m_i\Omega_i(\Omega_i + i\nu_{in})} + \frac{1}{-K^2\kappa T_e + m_e\Omega_e(\delta_e\Omega_e + i\nu_{en})} = 0. \tag{10}$$

Numerical solutions of the dispersion relation for the complex wave angular frequency $\Omega_i=\Omega_{i,r}+i\gamma$, where $\Omega_{i,r}$ is the real frequency in the ion frame and γ is the growth rate, are shown in Fig. 3 using the set of parameters listed in column (c) of Table I. Again, for comparison, the results for the case in which the electron inertia is neglected ($\delta_e=0$), are also shown. For the particular parameters chosen, which are typical for laboratory plasmas, the effect of the electron inertia is to reduce the critical electric field by about an order of magnitude and increase the growth rate by more than two orders of magnitude. By varying the parameters K and N over a large range of values, we found that the general thrust of the results shown in Fig. 3 was borne out. The inclusion of electron inertia makes the collisional ion acoustic instability easier to excite.

We note that it is possible to find combinations of parameters in which the effect of including the inertia terms produces less of a difference than that seen in the examples provided here. For example with the following set of parameters: $m_i=m_n=3.3\times10^{-26}$ kg, $m_d=4.2\times10^{-15}$ kg, $a=1.0$ μm, $Z=5000$, $\varepsilon=10^{-4}$, $T_i=T_n=1000$ K, $T_d=300$ K, $T_e=20\,000$ K, $N=10^{19}$ m^{-3}, $\sigma_{\rm in}=5\times10^{-19}$ m^2, $\sigma_{\rm en}=1\times10^{-20}$ m^2, and $K=100$ m^{-1}, we found that the real frequencies and critical electric fields ($\cong$0.1 V/m) for the dust acoustic instability were about the same in both cases, although the growth rate was more than an order of magnitude higher for the case in which the electron and ion inertia terms were retained.

Although the general, qualitative conclusions of Refs. 5 and 9 are not altered, the present results show that the inclusion of the electron inertia terms in the electron momentum equation for the dust ion acoustic instability and the ion and electron inertia terms for the dust acoustic instability, can

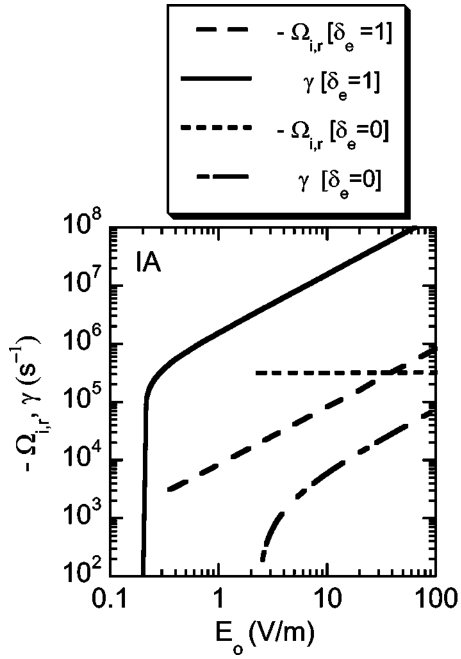

FIG. 3. The negative of the real frequency (in the ion frame) $\Omega_{i,r}$ and growth rate γ of the current-driven ion acoustic instability as a function of the applied electric field. $\delta_e=1$ is the case in which the electron inertia terms are included and $\delta_e=0$ correspond to the case in which the electron inertia terms are neglected. The values of the plasma parameters used to obtain this graph are listed in column "IA" of Table I.

lead to significant differences in the growth rates and critical electric fields. Apparently, this conclusion also holds for the current-driven ion acoustic instability considered by Kaw over 30 years ago.[3]

These results seem surprising since one might expect that electron inertia effects would make little difference when dealing with waves with frequencies well below the ion plasma frequency for the case of the ion acoustic waves and below the dust plasma frequency for the dust acoustic waves. An additional justification offered for the neglect of the inertia terms in Eq. (2) is that they are of higher order in m_e (or m_i) compared to the collision term on the right-hand side, since the collision frequency [Eq. (7)] is $\sim m_e^{-1/2}$ and thus the collision term is of order $m_e^{1/2}$. The collisional ion acoustic instability is driven by the relative drift of electrons and ions. The waves are damped due to ion-neutral collisions but electron-neutral collisions are destabilizing as first pointed out by Kaw.[3] A perturbation in the ion charge density will be partially neutralized by electrons which are attracted to regions of excess positive charge. Electrons colliding with neutrals will be impeded from fully neutralizing the ion space charge, thus the instability is enhanced by electron-neutral collisions. The electron inertia may have a similar effect in reducing their ability to neutralize regions of excess positive charge. That electron inertia can play a role similar to plasma resistivity has been pointed out previously.[10]

ACKNOWLEDGMENT

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