Cubical box with 5 sides at \( V = 0 \), one side at \( V = V_0 \), \( x, y, z \), are normalized by the length of the sides.

(Jackson, Section 2.5, with constant potential on the top surface.

\[ F_{nm} := \frac{16 \cdot V_0}{\pi^2} \cdot \left( \frac{1}{N \cdot M} \right) \cdot \sin(N \cdot \pi x) \cdot \sin(M \cdot \pi y) \cdot \sinh\left( \pi \cdot (N^2 + M^2)^{\frac{1}{2}} \cdot z \right) \]

\[ F_{nm} := \frac{16 \cdot V_0 \sin(N \pi x) \sin(M \pi y) \sinh(\pi \sqrt{N^2 + M^2} z)}{\pi^2 N M \sinh(\pi \sqrt{N^2 + M^2})} \quad (1) \]

sum is only over the ODD \( n \) and \( m \) values

\[ N := 2 \cdot n + 1; \quad M := 2 \cdot m + 1; \quad N := 2 \cdot n + 1 \]
\[ M := 2 \cdot m + 1 \quad (2) \]

\[ F_{nm} : \]
\[ V_0 := 1; \quad y := 0.5; \quad x := 0.5; \quad z := 1; \quad V_0 := 1 \]
\[ y := 0.5 \]
\[ x := 0.5 \]
\[ z := 1 \quad (3) \]

Numerical values for different numbers of terms in series

\[ \text{evalf(sum(sum(Fnm, n = 0..1), m = 0..1));} \]
\[ 0.7205061947 \quad (4) \]

\[ \text{evalf(sum(sum(Fnm, n = 0..5), m = 0..5));} \]
\[ 0.8973866100 \quad (5) \]

\[ \text{evalf(sum(sum(Fnm, n = 0..10), m = 0..10));} \]
\[ 1.058590098 \quad (6) \]

\[ \text{evalf(sum(sum(Fnm, n = 0..50), m = 0..50));} \]
\[ 1.012520485 \quad (7) \]

\[ \text{evalf(sum(sum(Fnm, n = 0..100), m = 0..100));} \]
\[ 1.006312943 \quad (8) \]

\[ \text{evalf(sum(sum(Fnm, n = 0..1000), m = 0..1000));} \]
\[ 1.000636077 \quad (9) \]

Plot \( \Phi(0.5, y, 1) \): show Gibbs Phenomenon

\[ V_0 := 1; \quad x := 0.5; \quad z := 1; \quad F_{nm}; \quad V_0 := 1 \]
\[ x := 0.5 \]
\[ z := 1 \quad (10) \]
\[
\frac{16 \sin (0.5 (2 n + 1) \pi) \sin (2 m + 1) \pi y}{\pi^2 (2 n + 1) (2 m + 1)}
\]

\[> \text{plot(sum(sum(Fnm, n = 0..5), m = 0..5), y = 0..1);} \]

\[> \text{plot(sum(sum(Fnm, n = 0..10), m = 0..10), y = 0..1);} \]
> plot(sum(sum(Fnm, n = 0 ..100), m = 0 ..100), y = 0 ..1);
Comments on the Gibbs phenomenon: This occurs when trying to match a discontinuity in boundary conditions, from one boundary to another. In this example, the potential must change from 0 on the side walls of the box to Vo on the top. The numerical solutions exhibit oscillations due to this effect. The effect cannot be eliminated, but by including more terms in the expansion, it is possible, as seen above, to limit the effect to points closer and closer to the boundary.

Plot $\Phi(0.5, y, z = z_0)$

```bash
> Vo := 1; x := 0.5; z := 0.9; plot(sum(sum(Fnm, n = 0..100), m = 0..100), y = 0..1);
> Vo := 1
> x := 0.5
> z := 0.9
```
Vo := 1; x := 0.5; z := 0.5; plot(sum(sum(Fnm, n = 0 .. 100), m = 0 .. 100), y = 0 .. 1);
Vo := 1
x := 0.5
z := 0.5
> \( V_0 := 1 \); \( x := 0.5 \); \( z := 0.2 \); \( plot(sum(sum(Fnm, n = 0 .. 100), m = 0 .. 100), y = 0 .. 1) \);

\( V_0 := 1 \)

\( x := 0.5 \)

\( z := 0.2 \)
Plot \( \Phi(x_0, y, z) \)

```plaintext
with(plots):
V0 := 1; x := 0.5; plot3d(sum(sum(Fnm, n = 0..100), m = 0..100), y = 0..1, z = 0..1, color = z, grid = [25, 25]);
```

\[ V0 := 1 \]
\[ x := 0.5 \]