PHYSICS OF PLASMAS **13**, 012901 (2006)

Generation of "spiky" potential structures associated with multiharmonic electrostatic ion-cyclotron waves

Su-Hyun Kim and Robert L. Merlino
Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa 52242

Gurudas I. Ganguli
Plasma Physics Division, Naval Research Laboratory, Washington, DC 20375

(Received 11 August 2005; accepted 14 November 2005; published online 10 January 2006)

The production of coherent, "spiky" electrostatic potential structures, similar to the spiky electric-field structures that have been observed in the Earth's auroral region, is investigated in a magnetized Q machine plasma. These structures are associated with coherent multiharmonic electrostatic ion-cyclotron (EIC) waves in a current-free plasma having a localized region of ion flow shear (ion flow parallel to $\mathbf{B}$ with a gradient in the direction transverse to $\mathbf{B}$). A multiharmonic EIC spectrum is detected in the region of ion flow shear when broadband electrostatic white noise is applied to the plasma column from an antenna. The time series of the potential fluctuations exhibit spiky structures with repetition rates near the ion-cyclotron frequency. Spiky structures can arise from linear combinations of phase-locked multiharmonic EIC waves when the amplitudes of the harmonic components are comparable to that of the fundamental EIC mode. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2148919]

I. INTRODUCTION

A ubiquitous feature of electric fields observed on satellites in the Earth's auroral region is their spiky, repetitive nature. These spiky electric-field structures appear as either unipolar or bipolar pulses in high-resolution time domain wave forms of the potential difference between pairs of spheres deployed from the spacecraft. Time domain wave forms of three different hydrogen-cyclotron wave events observed with the S3-3 satellite showed examples of both narrow spectral features at a frequency just above the local hydrogen-cyclotron frequency ($\Omega_{\mathrm{H}}{}^{+}$) and spiky, bipolar structures with a repetition frequency just above $\Omega_{\mathrm{H}}{}^{+}$. The latter were interpreted as "steepened" ion-cyclotron waves.[1] Measurements on the Polar Satellite, which traverses the southern auroral region at altitudes of about 6000 km, showed bipolar structures in the parallel electric field in conjunction with spikes in the perpendicular electric field that occurred with an average repetition rate of 1.2 $\Omega_{\mathrm{H}}{}^{+}$.[2] Data obtained from the Fast Auroral Snapshot (FAST) satellite in the upward current northern auroral region showed a multiharmonic electrostatic ion-cyclotron (EIC) spectrum with corresponding spiky structures in both the perpendicular and parallel electric-field wave forms.[3,4] Spiky bipolar structures in the parallel electric-field signals were associated with regions of inhomogeneous intense upward ion flows with a spatial dependence consistent with a transverse shear $dv_{i,\|}/dx_{\perp} \approx 1.3\ \Omega_{\mathrm{O}}{}^{+}$, where $v_{i,\|}$ is the ion flow speed along the B field, $x_{\perp}$ is the coordinate transverse to B, and $\Omega_{\mathrm{O}}{}^{+}$ is the local oxygen gyrofrequency.[4] The wave experiment on the Swedish Viking satellite frequently detected solitary bipolar structures in the potential difference measurements on probes separated along the magnetic field.[5] It was noted that the frequently observed large amplitude EIC waves may form the seeds for the solitary wave development.[6] Measurements using the wideband plasma-wave receiver located on the four-cluster spacecraft at $4.5-6.5R_E$ showed both bipolar and tripolar electric-field structures as they crossed magnetic-field lines that map into the auoral zone.[7]

Gavrishchaka *et al.*[4] and Ganguli *et al.*[8] have shown theoretically that parallel ion flows with transverse shear (ion flow shear) can generate a multimode spectrum of EIC waves even in the absence of an electron drift along B (field-aligned current). Unlike current-driven EIC waves in which the critical electron drift required to excite higher harmonics increases with harmonic number, the critical ion flow shear is approximately independent of the harmonic number. Thus a number of higher harmonics can be simultaneously excited. The plasma equilibrium considered in these studies corresponded to that encountered in the ionosphere where the entire ion population was found to be drifting along the magnetic field. Lakhina showed that a multiharmonic ion-cyclotron instability can also be driven by velocity shear of a hot ion beam embedded in a thermal ion background.[9] Using a fluid treatment, Merlino[10] showed that electrostatic waves with frequencies $\sim\Omega_{ci}$ propagating at large angles to the magnetic field can be excited in a plasma by ion flow shear. The results showed that in the presence of a density gradient with a scale length $\ell_n \sim \rho_i$, the ion gyroradius, the instability could be excited even with negligible shear.

The presence of a multiharmonic spectrum is a critical factor in understanding the origin of coherent electric-field structures, since as Ganguli *et al.*[8] have argued, a linear superposition of spontaneously generated multimode EIC waves can be the seed that leads directly to the formation coherent electric-field structures. If the linear combinations last long enough for the phases to get locked due to nonlinear processes, they can develop into coherent structures. The nonlinear properties of the shear-driven EIC waves were studied using a particle-in-cell (PIC) code.[4] An understand-

1070-664X/2006/13(1)/012901/6/$23.00

ing of how these coherent electric-field structures are generated is related to the question of how electrons that produce the visible aurora get accelerated parallel to the geomagnetic field.

The effects of parallel ion flow shear on the excitation of EIC waves have been studied previously in laboratory experiments.[11–15] Agrimson *et al.*[11] showed that ion flow shear has an effect on the EIC wave excitation in the presence of electron drift along B. In the presence of ion flow shear and electron current, a multiharmonic EIC spectrum was observed. When electron current was present but without ion flow shear, only the fundamental EIC mode was excited, and with considerably smaller amplitude. Teodorescu *et al.*[12] (see also Koepke *et al.*[13]) reported the generation of a broadband multiharmonic spectrum of EIC waves in a laboratory plasma in which shear in the parallel ion flow and electron drift along B (current) were present. Laser-induced fluorescence (LIF) measurements were made to determine the profile of the parallel ion drift velocity, $v_{i,\|}$ and the corresponding value of the velocity shear, $dv_{i,\|}/dx_{\perp}$. The propagation characteristics of the EIC waves were also fully determined. The results verified the theoretical model of multiharmonic EIC wave excitation due to inverse ion-cyclotron damping.[4,8]

The experiments[11–13] discussed above all contained both ion flow shear and magnetic field-aligned electron drift. One of the important results of the theory,[4,8] however, is that ion flow shear can excite multiharmonic ion-cyclotron waves, even in the absence of magnetic-field-aligned electron drift. Our focus then in subsequent experiments[15] was to isolate and study the effects of ion flow shear on EIC wave excitation in a plasma *with no electron current*. Electrostatic waves with frequencies slightly above the ion-cyclotron frequency or its harmonics were launched into the plasma from an antenna. The waves were observed to grow in amplitude in the region of ion flow shear, demonstrating the effect of shear alone without any current. When broadband electrostatic waves were launched into the plasma, selective amplification at the cyclotron fundamental frequency and several harmonics was observed in the region of ion flow shear. In our previous experiments[11,15] no measurement of the ion flow shear parameter or wave propagation characteristics was made.

The purpose of the present experiment was to study the formation of spiky potential structures and their relation to multiharmonic EIC waves in a *currentless* plasma with inhomogeneous parallel ion flow. We show that the spiky time series are the result of a linear superposition of phase-locked multiharmonic EIC waves, consistent with the theoretical and numerical results of Ganguli *et al.*[8] Examples of the time series of ion-cyclotron waves excited in a plasma with parallel ion flow shear and electron drift were shown by Teodorescu *et al.*[12] and Koepke *et al.*[13] For the case in which the shear was negligible, the time series was nearly sinusoidal, whereas for the case in which shear was present, the time series was less sinusoidal.

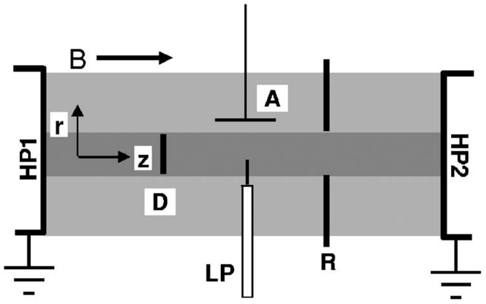

FIG. 1. Schematic diagram of the double-ended Q machine. A current-free cesium (Cs^+) plasma is formed on two electrically grounded hot plates (HP1 and HP2). The ring and disk (D) electrodes used to collect ions from each source and create an annular region of parallel ion flow with transverse shear. A strip antenna (A) is used to launch electrostatic waves into the plasma. Plasma parameters are monitored with a Langmuir probe (LP). The (r,z) coordinate system used for reporting the measurements of physical quantities is shown.

II. EXPERIMENTAL SETUP

The experiment was performed in a double-ended Q machine,[16] shown schematically in Fig. 1. A Cs^+ plasma was formed by contact ionization of cesium atoms on two 6-cm-diam tantalum hot plates (HP) maintained at ~2000 K which also emit thermionic electrons. Both hot plates are electrically grounded, so that no current (relative electron drift) is driven through the plasma. The plasma is confined radially by a uniform magnetic field in the range of 0.2–0.4 T. Typical plasma densities are $\sim 10^{10}\ \mathrm{cm}^{-3}$, with electron and ion temperatures, $T_e \approx T_i \approx 0.2$ eV. Both electrons and ions are magnetized and the plasma is collisionless. The hot plate sources are operated under electron-rich conditions in which a potential drop of ~3–4 V is present in a sheath at each grounded hot plate. The ions are accelerated into the plasma by this potential drop, acquiring a flow energy ~3–4 eV. A profile of the floating potential, V_f, of a Langmuir probe (within a few T_e of the plasma potential) which was scanned across the plasma column is shown in the lower plot of Fig. 2(a). Note that over the central portion of the plasma, where the experimental measurements were carried out, there is a negligible radial (transverse to B) electric field.

To produce a plasma having parallel ion flow with transverse shear, a metal ring of 8 cm od and 2.3 cm id was placed at one plasma cross section, and a metal disk of 2.2 cm diameter was placed at another cross section, as shown in Fig. 1. The ring and disk were separated axially by 88 cm. The ring and disk were both biased at ~−4 V to collect all ions flowing to them, so that between the ring and disk, the central core contains mainly plasma flowing from HP2 and the outer portion contains mainly plasma flowing from HP1. The boundary between the inner and outer plasma is then a region of ion flow shear. Measurements of the ion flux profiles from each plasma source using a double-sided Langmuir probe were previously reported for this configuration.[17] The measurements showed that there was some "mixing" of the oppositely directed flows near the boundary between the inner edge of the ring and the edge of the disk. The scale length of the flux gradient, however, was

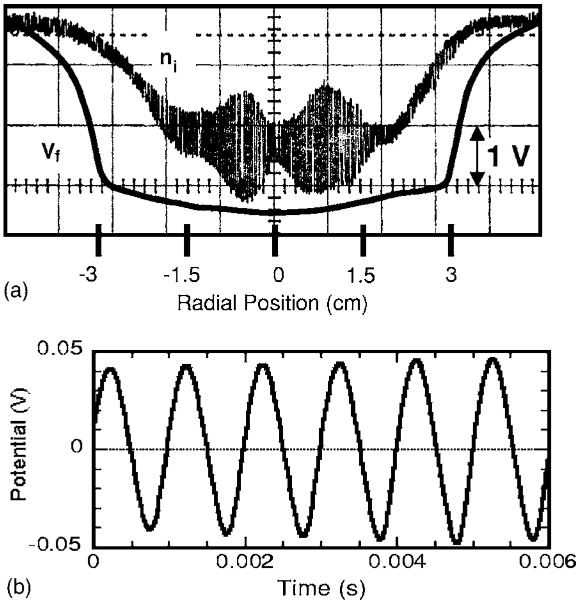

FIG. 2. (a) Radial profiles of the ion density, n_i, and Langmuir probe floating potential (very close to the plasma potential), V_f. The base line for both plots is at the top of the plot. The peak ion density (at $r=0$) is 2.5×10^{10} cm^{-3}. The large low frequency ($\sim$1 kHz) fluctuations seen on the ion density profile are due to the parallel velocity shear (D'Angelo) instability. (b) Time series of the low-frequency D'Angelo instability.

significantly less than the scale length of the density profile. When the bias on the ring and disk was raised to >-2 V, the ions were reflected from the ring and disk, resulting in no net flow or shear. The presence of velocity shear could also be inferred by observing the very low-frequency ($\sim$1 kHz) fluctuations due to the parallel velocity shear instability (also known as the D'Angelo instability), discussed theoretically by D'Angelo[18] and observed experimentally by D'Angelo and von Goeler.[19] Figure 2(a) shows the radial profile of the ion density, n_i, with the large amplitude low-frequency fluctuations [shown expanded in time in Fig. 2(b)] that mark the radial locations of velocity shear. The radial extent of the low frequency oscillations provides a measure of the width of the shear region as $\Delta x \approx 7-8$ mm$\approx(3-4)\rho_i$, where ρ_i is the ion gyroradius. Assuming that the difference in flow velocity across the shear layer corresponds to twice the ion drift acquired at the hot plate, we estimate the shear parameter at $B=0.3$ T as $S=(\Omega_{ci})^{-1}(d\nu_{i,\|}/dx)\approx(\Omega_{ci})^{-1}(\Delta\nu_{i,\|}/\Delta x)$, where Ω_{ci} is the ion gyrofrequency and $\nu_{i,\|}$ is the ion drift speed along the magnetic field. With $\Omega_{ci}=2.2\times10^5$ s^{-1}, $\Delta\nu_{i,\|}\approx 2\nu_{i,\|}=2(2\times10^3$ m/s), and $\Delta x\approx 4\rho_i=4(2\times10^{-3}$ m), we find that $S\sim 1$.

III. EXPERIMENTAL RESULTS

A strip antenna (see Fig. 1) 5 cm long and 1 cm wide, oriented with its normal perpendicular to the magnetic field was used to launch electrostatic waves in the ion-cyclotron frequency range, $f \gtrsim nf_{ci}$, into the plasma.[15] The amplitude of the potential fluctuations of the EIC wave was measured at several radial positions in the plasma cross section coincident with the center of the antenna. A factor of $\sim$2 increase in the wave amplitude was observed in the regions of ion flow shear as compared to the case in which ion flow shear was not present.[15] This result was obtained using wave frequencies corresponding to the fundamental EIC mode and several harmonics.

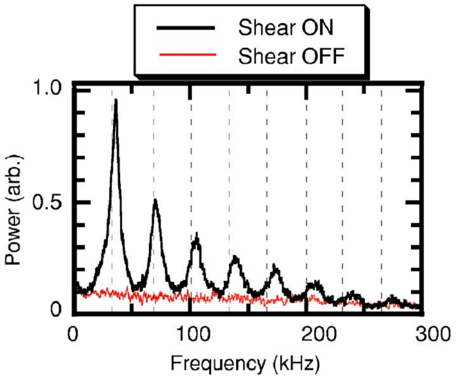

FIG. 3. (Color online) Power spectrum (thick line) of the floating potential oscillations of a probe located in the region of velocity shear ($r \approx 0.75$ cm), when a broadband signal was applied to the antenna. The thin line is the corresponding spectrum obtained when the parallel ion flow shear was OFF. The vertical lines correspond to multiples of the cesium ion-cyclotron frequency.

A multiharmonic spectrum of EIC waves was produced by applying a broadband white-noise signal (random noise extending up to about 1 MHz) to the antenna. A probe was located in the region of velocity shear to record the potential fluctuations. Figure 3 shows the power spectra of the potential fluctuations for the cases in which the ion flow shear was ON or OFF. When the ion flow shear was ON, a multiharmonic spectrum corresponding to the simultaneous excitation of the fundamental EIC mode and seven harmonics was observed. When the ion flow shear was OFF, no EIC modes were present in the spectrum. We note in Fig. 3 that the amplitudes of the first five harmonics are all within an order of magnitude of the amplitude of the fundamental EIC mode.

Since detailed wave propagation measurements within the narrow shear layer were not possible in this study, a quantitative comparison of the multiharmonic EIC waves observed in this experiment with those predicted in Ref. 8 could not be made. However, the observation of a multiharmonic EIC spectrum with harmonic amplitudes comparable to that of the fundamental *in the region of ion flow shear* is certainly consistent with the inverse ion-cyclotron damping mechanism of Ganguli *et al.*[8] As pointed out by Teodorescu *et al.*[12] the observation of a multiharmonic spectrum is an important demonstration of reduced ion-cyclotron damping since in the usual (current-driven) case without shear, the cyclotron damping increases with increasing harmonic number, which would inhibit the excitation of harmonics.

The time series of the EIC potential fluctuations corresponding to the spectrum in Fig. 3 is shown in Fig. 4 (heavy full line). (The oscillations due to the low-frequency D'Angelo instability have been filtered out in Figs. 3 and 4.) This time series exhibits large spiky features with a repetition rate $\gtrsim\Omega_{ci}/2\pi$. Examples of EIC potential time series obtained for various values of the magnetic field are shown in Figs. 5(a)–5(c). All time series show spiky, bipolar struc-

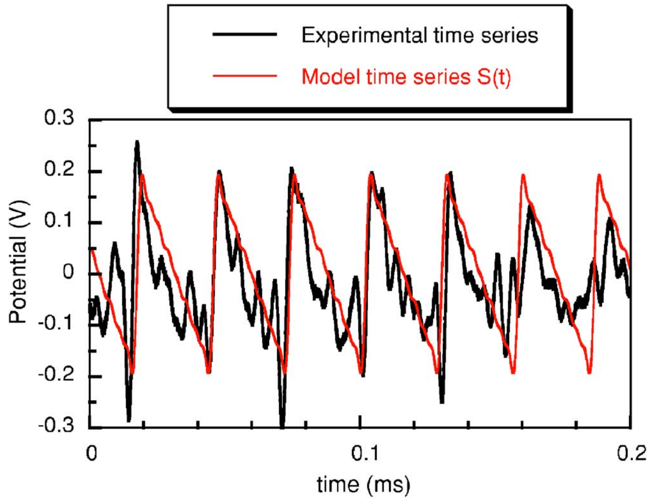

FIG. 4. (Color online) Heavy full line: experimentally observed "spiky" time series of the electrostatic floating potential fluctuations (in V) associated with the EIC spectrum shown in Fig. 3. The largest peaks are separated in time by the period of the fundamental ion-cyclotron mode. Thin line: a model time series resulting from the linear combination of sinusoidal EIC waves with amplitudes corresponding to the spectrum in Fig. 3, with zero relative phase factors. The amplitude of the model time series has been approximately normalized to the experimental time series.

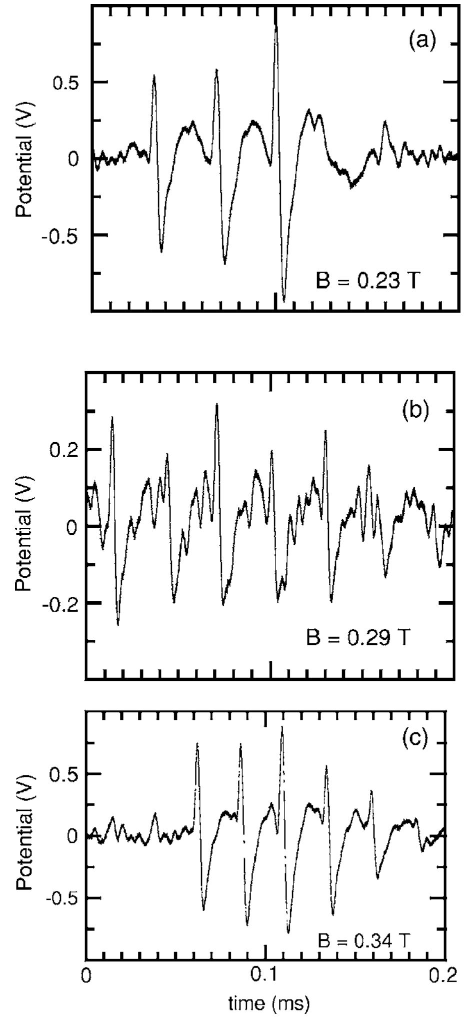

FIG. 5. EIC potential wave forms observed for (a) $B=0.23$ T, (b) $B=0.29$ T, and (c) $B=0.34$ T.

tures, with repetition rates just above the fundamental cyclotron frequency, Ω_{ci}.

Measurements of the time separation (Δt) between the spiky features were obtained for many wave forms of the type shown in Figs. 5(a)–5(c) for several values of the magnetic field. A plot showing the dependence of the time separation (Δt) on the magnetic-field strength is given in Fig. 6. The solid line in Fig. 6 is the ion-cyclotron period, $2\pi/\Omega_i$. The time separation is clearly determined by the *period* of the fundamental ion-cyclotron mode, which is slightly below the ion-cyclotron period.

IV. DISCUSSION

As pointed out by Temerin *et al.*,[1] harmonics can be generated in the linear Vlasov theory of EIC waves. This can occur as a result of the current-driven instability of Drummond and Rosenbluth,[20] but usually requires a very large electron drift (which are not typically observed) since the critical drift velocity increases with harmonic number. On the other hand, the inverse ion-cyclotron damping mechanism of Ganguli *et al.*[8] can explain the simultaneous appearance of multiple cyclotron harmonics since the critical value of the parallel ion flow shear for destabilizing the nth harmonic is nearly independent of the harmonic number n. Teodorescu *et al.*[12] showed a comparison of EIC wave excitation for the case in which there was a negligible shear in the plasma and for the case in which a much larger shear was present. For their negligible shear case [Fig. 2(a) of Ref. 11] only one harmonic within 1.5 orders of magnitude of the fundamental was excited, while in their intermediate shear case there were 11 harmonics within 1.5 orders of magnitude of the fundamental. In our previous[15] and present experiments in which there was no electron current, the simultaneous appearance of multiple cyclotron harmonics has been connected to the presence of ion flow shear in two ways. First, we show that there is no amplification of the EIC waves when there is no shear (see Fig. 3). Second, the physical location of the observed EIC wave amplification coincides with the region of ion flow shear (see Fig. 2 of Ref. 14).

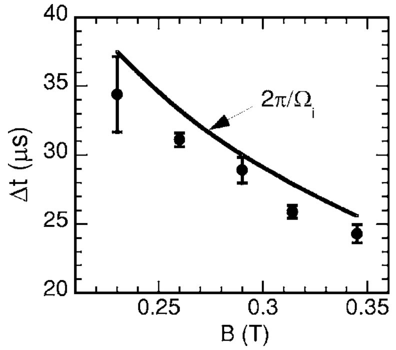

FIG. 6. Dependence of the time separation (Δt) of the large peaks in the spiky time series of multiharmonic EIC waves (of the type shown in Fig. 5) on the magnetic-field strength. The solid line shows the values of the ion-cyclotron period over the range of magnetic fields investigated.

A multiharmonic EIC spectrum in itself, however, would not produce coherent spiky potential or electric-field structures. As Temerin *et al.*[1] and Ganguli *et al.*[8] pointed out, linear combinations of the harmonics must persist long enough for the phases to get locked due to nonlinear processes and develop into spiky coherent structures. To illustrate the formation of spiky time series structures, a linear combination of multiharmonic EIC modes was formed at an arbitrary spatial point in the plasma. The resulting model time series, $S(t)=\Sigma_n A_n \sin(n\omega_0 t+\varphi_n)$, where A_n and φ_n are the amplitudes and relative phases of the harmonics, corresponding to the spectrum in Fig. 3, is shown in Fig. 4 (thin line). This model time series was obtained using the measured amplitudes of the fundamental and seven harmonics taken from Fig. 3, with $\omega_0=2\pi(35.5\text{ kHz})$, assuming $\varphi_n=0$ for each component. The model time series, $S(t)$, reproduces the general morphology of the observed time series shown in Fig. 3(a). $S(t)$ cannot capture all of the details of the experimental time series since it takes each mode as a pure sine wave whereas in the actual spectrum (Fig. 3) the spectral features have a finite frequency spread. The formation of spiky structures is not reproduced with the model time series if the amplitudes of the harmonics are significantly lower than that of the fundamental. For example, if the harmonic amplitudes are reduced by an order of magnitude relative to the fundamental, the resulting model time series is nearly sinusoidal. The model time series was also utilized to study the effects of the relative phases of the mode. For example, when random phases (between 0 and 2π) were assigned to the modes in the model time series, the resulting linear combination was quite different morphologically from the actual time series.

The generation of spiky structures in both the electrostatic potential and electric field was also observed in the numerical particle-in-cell code of Gavrishchaka *et al.*[4] and Ganguli *et al.*,[8] and arise when several coherent EIC waves simultaneously grow and saturate in amplitude.

V. SUMMARY AND CONCLUSIONS

We have demonstrated experimentally that spiky electric potential structures with peaks separated by ion-cyclotron time scales can be produced by a linear combination of multiharmonic electrostatic ion-cyclotron waves. The formation of spiky structures requires that the amplitudes of the EIC harmonics be comparable to that of the fundamental and are phase locked.

There have been many theoretical attempts to model these structures in terms of nonlinear waves (see, e.g., Refs. 1 and 21). These approaches generally try to describe the evolution of a single linear EIC wave as it grows nonlinearly into a finite amplitude wave. The solutions that are obtained may resemble the observed wave forms, but this only implies that a nonlinear state is possible. This approach does not clarify the chain of physical events that leads to the formation of these nonlinear structures. In addition, they cannot explain why in some laboratory experiments the fundamental cyclotron mode remains sinusoidal even at very high amplitudes.[22]

The mechanism discussed here, which was proposed by Ganguli *et al.*,[8] takes as its starting point the generation of a multiharmonic spectrum of EIC waves due to parallel ion flow shear, and proceeds to show the formation of spiky potential structures self-consistently, thereby clarifying the causal relationship between the physical processes that leads to their formation. The identification of the nonlinear processes that act to ensure the necessary phase locking of the modes is beyond the scope of this work, and remains as an important aspect of the problem that needs to be explored further.

The multiharmonic EIC spectrum was observed in a region of plasma with parallel ion flow shear when broadband electrostatic noise was applied to the plasma. This result points out the possibility that EIC waves may be generated when broadband noise produced in one plasma region propagates into another plasma region where inhomogeneous ion flows may be present.

ACKNOWLEDGMENTS

We thank M. Miller for technical support in carrying out the experiments and N. D'Angelo and F. Skiff for useful discussions.

This work at the University of Iowa was supported by the National Science Foundation and The U.S. Department of Energy. The work at the Naval Research Laboratory was supported by the Office of Naval Research.

[1] M. Temerin, M. Woldorff, and F. S. Mozer, Phys. Rev. Lett. **43**, 1941 (1979).

[2] F. S. Mozer, R. E. Ergun, M. Temerin, C. A. Cattell, J. Dombeck, and J. Wygant, Phys. Rev. Lett. **79**, 1281 (1997).

[3] R. E. Ergun, C. W. Carlson, J. P. McFadden, F. S. Mozer, G. T. Delory, W. Peria, C. C. Chaston, M. Temerin, R. Elphic, R. Strangeway, R. Pfaff, C. A. Cattell, D. Klumpar, E. Shelley, W. Peterson, E. Moebius, and L. Kistler, Geophys. Res. Lett. **25**, 2025 (1998).

[4] V. V. Gavrishchaka, G. I. Ganguli, W. A. Scales, S. P. Slinker, C. C. Chaston, J. P. McFadden, R. E. Ergun, and C. W. Carlson, Phys. Rev. Lett. **85**, 4285 (2000).

[5] R. Boström, G. Gustaffson, B. Holback, G. Holmgren, H. Koskinen, and P. Kintner, Phys. Rev. Lett. **61**, 82 (1988).

[6] H. E. J. Koskinen, P. M. Kintner, G. Holmgren, B. Holback, G. Gustafsson, M. Andre, R. Lundin, Geophys. Res. Lett. **14**, 459 (1987).

[7] J. S. Pickett, S. W. Kahler, L.-J. Chen, R. L. Huff, O. Santolík, Y. Khotyaintsev, P. M. E. Décréau, D. Winningham, R. Frahm, M. L. Goldstein, G. S. Lakhina, B. T. Tsurutani, B. Lavraud, D. A. Gurnett, M. André, A. Fazakerley, A. Balogh, and H. Rème, Nonlinear Processes Geophys. **11**, 183 (2004).

[8] G. Ganguli, S. Slinker, V. Gavrishchaka, and W. Scales, Phys. Plasmas **9**, 2321 (2002).

[9] G. S. Lakhina, J. Geophys. Res. **92**, 12,161 (1987).

[10] R. L. Merlino, Phys. Plasmas **9**, 1824 (2002).

[11] E. P. Agrimson, N. D'Angelo, and R. L. Merlino, Phys. Lett. A **293**, 260 (2002).

[12] C. Teodorescu, E. W. Reynolds, and M. E. Koepke, Phys. Rev. Lett. **89**, 105001 (2002).

[13] M. E. Koepke, C. Teodorescu, E. W. Reynolds, C. C. Chaston, C. W. Carlson, J. P. McFadden, and R. E. Ergun, Phys. Plasmas **10**, 1605 (2003).

[14] E. Agrimson, S.-H. Kim, N. D'Angelo, and R. L. Merlino, Phys. Plasmas **10**, 3850 (2003).

[15] S.-H. Kim, E. Agrimson, M. J. Miller, N. D'Angelo, and R. L. Merlino, Phys. Plasmas **11**, 4501 (2004).

[16] R. W. Motley, *Q Machines* (Academic, New York, 1975).

[17] J. Willig, R. L. Merlino, and N. D'Angelo, J. Geophys. Res. **102**, 27,249 (1997).

[18] N. D'Angelo, Phys. Fluids **8**, 1748 (1965).

[19] N. D'Angelo and S. von Goeler, Phys. Fluids **9**, 309 (1966).

[20] W. E. Drummond and M. N. Rosenbluth, Phys. Fluids **5**, 1507 (1962).

[21] P. K. Chaturvedi, Phys. Fluids **19**, 1064 (1976); P. K. Shukla and S. G. Tagare, Phys. Rev. A **30**, 2118 (1984); H. L. Rowland and P. J. Palmadesso, J. Geophys. Res. **92**, 299 (1987); P. K. Shukla and L. Stenflo, Ann. Geophys. **16**, 889 (1998); D. Jovanovic and P. K. Shukla, Phys. Rev. Lett. **84**, 4373 (2000); R. V. Reddy, G. S. Lakhina, S. V. Singh, and R. Bharuthram, Nonlinear Processes Geophys. **9**, 25 (2002); J. F. McKenzie, J. Plasma Phys. **70**, 533 (2004).

[22] The first measurement of the amplitude of an EIC wave by R. W. Motley and N. D'Angelo [Phys. Fluids **6**, 296 (1965)], showed a *sinusoidal* wave of amplitude, $\Delta n/n \sim 50\%$.