| Name |  |
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| Date |  |
| Partner(s) |  |
| Grade |  |
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## PARALLAX MEASUREMENT

## Introduction

Parallax is a term that refers to the apparent shift in location of an object against the distant background caused by a change in the observer's position. Through the use of some simple geometry, you can use the apparent shift and the change in the observer's position to estimate the distance to an object. In this exercise, we will do just that and use parallax to determine the distance to a familiar object. The instruments used to measure the parallax angle will be the same Theodolites which were used in a previous exercise. It is important to note that this technique is the same as what astronomers use to determine the distances to stars (stellar parallax). Although the measurements are mathematically identical, the parallaxes measured in astronomy are much smaller than those we will be dealing with today.

The object you will be observing today is the Jefferson Building. After determining the distance to the Jefferson Building by parallax, we will look up the "ground truth" value from a map, and see how close our result is.

## Part 1 - Distance to the Jefferson Building via Parallax

The geometry of the situation is shown in the Figure 1 below. Positions 1, 2, and 3, and 4 represent the locations of the four Theodolites. For this experiment, you need to make measurements with a pair of Theodolites i.e., you need to make measurements with 1 and 3 , or 2 and 4 . The analog of this in the stellar parallax case would be like pretending that the two Theodolite positions (say 1 and 3) would correspond to the location of the Earth at different times of the year.


FIGURE 1 - OBSERVING SETUP
In this experiment, the object we use as the very remote reference (like distant stars in stellar parallax measurements) is "The Tree", which was found and measured in the first experiment of the semester.

### 1.1 Procedure

a. Choose a set of Theodolites, either 1 and 3 or 2 and 4 . Write down the set you have chosen in the first column of the table below. Go to either one of the Theodolites (pick one with the minimum mob scene) for your first set of measurements. Record your value in column 2 of the table below.
b. Measure the azimuth angles of "The Tree" and the upper right (the northwest) corner of the Jefferson Building. Call these numbers A1 and A2, respectively, and write down your measurements in columns 3 and 4 of the data table.
c. Write down the difference DA1 = A1-A2 in column 5. Angle DA1 is actually the angle of the corner of the Jefferson Building with respect to our "reference line" to the distance point on the horizon.
d. Repeat steps (a) - (c) for the other Theodolite. Call this difference DA2 $=A 1-A 2$ for the second Theodolite.

If the Jefferson Building were very far away (a distance much greater than the distance between the two Theodolites) DA1 and DA2 would be identical. Since the distance to the Jefferson Building is measurable, DA1 and DA2 will be different. Calculate your value as DDA = DA1 - DA2. The quantity DDA is the parallax of the Jefferson Building. Put the answer in column 10.

[^0]TABLE 1 - THEODOLITE MEASUREMENTS


| Num | A1 | A2 | DA1 | Num | A1 | A2 | DA2 | D. | DDA |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |

## 1.2 - Estimates of the Error in the Measurement

A fundamental concept in measurements in the physical sciences is that of errors. Every measurement has an associated error. The term error does not mean that some mistake was made that makes the number complete nonsense. Rather, this term means that there is an uncertainty associated with the measurement. Some measurements are good to $25 \%$, others good to $1 \%$, and sometimes there are precise measurements that are exact to within a tiny fraction of $1 \%$.

The subject of how to estimate the errors associated with a measurement is a large subject in its own right. In this exercise, we will adopt a much simpler approach to estimating the error, but one which will give us an approximate idea of the uncertainty associated with our measurements.

Repeat the procedure described above two more times. For the first repeat, use the second set of spaced Theodolites (in other words, if you used 1 and 3 in your first set of measurements, use 2 and 4 the next time). For the third and final set of measurements, go back to the set you started with. Enter all the data in the table below.

TABLE 2 - FOLLOW-UP MEASUREMENT 1


| NuM | A1 | A2 | DA1 | NuM | A1 | A2 | DA2 | DDA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

TABLE 3 - FOLLOW-UP MEASUREMENT 2


| Num | A1 | A2 | DA1 | Num | A1 | A2 | DA2 | D. | DDA |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |

When you are done with the three measurements, calculate the average value, called the mean. Calculate the mean by using the table below. As a measure of the uncertainty, calculate the maximum difference of a measurement of DDA from the mean. To put a name on this variable, we call it MxD. Put it next to the value for the mean of DDA. The significance of the error is that the true value for the parallax of the Jefferson Building could be anywhere between DDA - MxD to DDA + MxD.

TABLE 4 - MEAN AND MAXIMUM DIFFERENCE

| DDA (TRIAL 1) |  |  |
| :--- | :--- | :--- |
| DDA (TRIAL 2) |  | MxD |
| DDA (TRIAL 3) |  |  |
| MEAN |  |  |

Your percentage error is the maximum difference divided by the mean value, $\mathrm{PE}=\mathrm{MxD} / \mathrm{DDA}$. Calculate your value of PE and record it below.

TABLE 5 - PERCENTAGE ERROR

| MxD | Mean DDA | PE |
| :--- | :--- | :--- |
|  |  |  |

## 1.3 - Calculation of the Distance to the Jefferson Building

The relationship between the distance to an object $r$, the size $s$ of the "baseline" on which the observer moves, and the parallax is:

$$
D D A=s / r
$$

To use this formula, it is necessary that DDA is a small angle (say less than 10 degrees), and DDA must be expressed in radians. Remember that one radian is 57.3 degrees. In the exercise above, we measured the parallax in degrees, so we must divide by 57.3 to get the angle in radians. The distance to the object is then given by

$$
r=s / D D A_{\text {radians }}
$$

To make this calculation, we need to know s, the distance between the Theodolites. Measure the distance between the Theodolites by using the fact that the distance between the round posts on the railing is 7.5 ft . Count the number of posts between the Theodolites, and multiply by the separation given above. It is recommended to express the distance in meters, which is the metric unit of length. One meter $=3.28$ feet. Write down the result here.

## Separation between Theodolites



A final correction which must be made is that s given above is the distance perpendicular to the direction to the object which is being measured. To get the perpendicular distance, multiply your value for the separation above by cosine (B) where $B=180$ - Azimuth Angle of "The Tree".
(This is because the building runs essentially East-West, "The Tree" should be at an azimuth of 180 degrees to make it a good reference. Since it is not, we need to correct for this.)

## Angle (B)

Corrected Separation ( $\mathrm{s}_{\text {corrected }}$ )


In the space below, or on the facing page, calculate the distance to the Jefferson Building using the formula above and the described corrections. Give the range of acceptable distances allowed by the errors. Show your work. Give the answer in the space indicated below.

# Distance to Building <br> Error Range 

|  |
| :--- |
|  |

Get a map of downtown lowa City from the internet or phone book (be sure to get one with a scale; travel.yahoo.com, maps.google.com are just a few of many that do), and measure the distance from the northwest corner of Jefferson Building to the middle of the south side of Van Allen Hall. Give the number below.

## MAP SOURCE

## DISTANCE FROM MAP

$\square$

In the space below, briefly comment on the degree to which the two values are, or are not in agreement. Are they consistent with each other?


[^0]:    Don't forget your units!

