Name $\qquad$
Date

## Lab Partner(s)

## Project Score

$\square$

## Parallax Measurements

## Project level: Intermediate

Project goals: Students will use angular measurements to calculate parallax angles and determine the distance to a familiar object.

## 1 Introduction

PARALLAX is a term that refers to the apparent shift in location of an object against the distant background caused by a change in the observer's position. Through the use of some simple geometry (see Figure 1), you can use the apparent shift and the change in the observer's position to measure the distance to an object.


## FIGURE 1 - ILLUSTRATION OF PARALLAX

This equation only works when the Reference object, in this case the radio tower, is much farther away than the object you are trying to measure. In addition, the angles $\delta \theta 1$ and $\delta \theta 2$ must be small, say under 10 degrees or so.

In this exercise, we will do just that and use parallax to determine the distance to a familiar object. The instruments used to measure the parallax angle will be the same THEODOLITES which were used in a previous exercise. It is important to note that this technique is the same as what astronomers use to determine the distances to stars (STELLAR PARALLAX). Although the measurements are mathematically identical, the parallaxes measured in astronomy are much smaller than those we will be dealing with today.


FIGURE 2 - WESTERN SKYLINE FROM VAN ALLEN HALL
The object you will be observing today is Gilmore Hall. Your teaching assistant can help you pick it out; in the above picture, it is between the First Congregational Church and the School of Business. We will use a red chimney on the roof of Gilmore Hall as a reference marker. After determining the distance to Gilmore Hall by parallax, we will look up the "ground truth" value from a map, and see how close our result is.

## 2 Distance to the Gilmore hall via Parallax

The geometry of the situation is shown in Figure 3. Substitute Gilmore Hall for "Hospital Tower"; the figure was made for a previous version of this laboratory exercise. Positions 1,2 , and 3 , and 4 represent the locations of the four Theodolites. For this experiment, you need to make measurements with a pair of Theodolites i.e., you need to make measurements with 1 and 3 , or 2 and 4 . The analog of this in the stellar parallax case would be like pretending that the two Theodolite positions (say 1 and 3) would correspond to the location of the Earth at different times of the year.

In this experiment, the object we use as the very remote reference (like distant stars in stellar parallax measurements) is a radio tower visible just to the left of Gilmore Hall. There are two such radio towers; choose the leftmost of the two. Your lab instructor will point the tower out to you.

### 1.1 Procedure

1. Choose a set of Theodolites, either 1 and 3 or 2 and 4 . Go to either one of the Theodolites (pick one with the minimum mob scene) for your first set of measurements. Record your pair selection where indicated.
2. Measure the azimuth angles of the radio tower and Gilmore Hall with the Theodolite. Call these numbers $\theta 1$ and $\theta 2$, respectively, and write down your measurements in the space provided.
3. Write down the difference $\delta \theta 1=\theta 1-\theta 2$ in next column.
4. Move to the second Theodolite in your pair. Again, measure the azimuth angles and calculate the difference. Call this difference $\delta \theta 2=\theta 1-\theta 2$ for the second Theodolite.


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FIGURE 3 - OBSERVING SETUP
If Gilmore Hall were very far away (a distance much greater than the distance between the two Theodolites) $\delta 01$ and $\delta \theta 2$ would be identical. Since the distance to Gilmore Hall is measurable, $\delta \theta 1$ and $\delta \theta 2$ will be different. Calculate your value as $\Delta \theta=\delta \theta 1-\delta \theta 2$. The quantity $\Delta \theta$ is the parallax of Gilmore Hall. Record your result in the $\Delta \theta$ column of the table.

## Don't forget your units!

## Question 1:

Record your angle measurements for a pair of Theodolites

Pair


| Num | $\theta 1$ | $\theta 2$ | $\delta \theta 1$ | Num | $\theta 1$ | $\theta 2$ | $\delta \theta 2$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  | $\Delta \theta$ |

### 1.2 Estimates of the Error in the Measurement

A fundamental concept in measurements in the physical sciences is that of errors. Every measurement has an associated error. The term error does not mean that some mistake was made that makes the number complete nonsense. Rather, this term means that there is an uncertainty associated with the measurement. Some measurements are good to $25 \%$, others good to $1 \%$, and sometimes there are precise measurements that are exact to within a tiny fraction of $1 \%$.

The subject of how to estimate the errors associated with a measurement is a large subject in its own right. In this exercise, we will adopt a much simpler approach to estimating the error, but one which will give us an approximate idea of the uncertainty associated with our measurements.

Repeat the procedure described above two more times. For the first repeat, use the second set of spaced Theodolites (in other words, if you used 1 and 3 in your first set of measurements, use 2 and 4 the next time). For the third and final set of measurements, go back to the set you started with. Enter all the data in the table below.

## Question 2:

Record your angle measurements for the second pair of Theodolites


| Num | $\theta 1$ | $\theta 2$ | $\delta \theta 1$ | Num | $\theta 1$ | $\theta 2$ | $\delta \theta 2$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |

## Question 3:

Repeat you angle measurements for the first pair of Theodolites


| Num | $\theta 1$ | $\theta 2$ | $\delta \theta 1$ | Num | $\theta 1$ | $\theta 2$ | $\delta \theta 2$ |  | $\Delta \theta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |

When you are done with the three measurements, calculate the average value, called the MEAN. As a measure of the uncertainty, calculate the maximum difference of a measurement of $\Delta \theta$ from the mean. To put a name on this variable, we call it $\mathrm{M} \theta$. The significance of the error is that the true value for the parallax of Gilmore Hall could be anywhere between $(\Delta \theta-M \theta)$ to $(\Delta \theta+M \theta)$.

Question 4:
Calculate the mean parallax angle.

| $\Delta \theta($ Trial 1) |  |
| :---: | :--- |
| $\Delta \theta$ (Trial 2) |  |
| $\Delta \theta$ (Trial 3) |  |
| $\Delta \theta$ Mean |  |

Question 5:
What is the maximum difference between your individual measurements and the mean value you calculated in Question 4?
$\square$

## Question 6:

Your percentage error is the maximum difference divided by the mean value, $\mathrm{PE}=\mathrm{M} \theta / \mathrm{D} \theta$ (MEAN). Calculate your value of PE and record it below.
$\square$

### 1.3 CALCULATION OF THE Distance to Gilmore hall

The relationship between the object distance $D$, the baseline size $d$ (how much the observer moves), and the parallax is:

$$
\Delta \theta=\mathrm{d} / \mathrm{D}
$$

To use this formula, it is necessary that $\Delta \theta$ is a small angle (say less than 10 degrees), and $\boldsymbol{\Delta \theta}$ must be expressed in radians. Remember that one radian is 57.3 degrees. In the exercise above, we measured the parallax in degrees, so we must divide by 57.3 to get the angle in radians. The distance to the object is then given by:

$$
\mathrm{d}=\mathrm{D} \times \frac{\Delta \theta}{57.30 \operatorname{deg}}
$$

To make this calculation, we need to know d, the distance between the Theodolites. You can calculate d by using the fact that the distance between the cylindrical posts on the railing is 7.5 ft . Count the number of posts between the Theodolites, and multiply by given separation.

It is recommended to express the distance in meters, which is the metric unit of length (1 meter $=3.28$ feet).

## Question 7:

Calculate the distance between the Theodolites using the method described in the text.


You now have everything you need to calculate the distance to Gilmore Hall.

## Question 8:

In the space provided, calculate the distance to Gilmore Hall. Give the range of acceptable distances allowed by the errors. Show your work.
Error Range $\square$

## Question 9:

Get a map of downtown lowa City from the internet or phone book (be sure to get one with a scale; travel.yahoo.com, maps.google.com are just a few of many that do), and measure the distance from Gilmore Hall to the middle of the west end of Van Allen Hall. Record your findings below.

| Map Source |  |
| :---: | :---: |
| Distance from Map |  |

## Question 10:

In the space below, briefly comment on the degree to which the two values are, or are not in agreement. Are they consistent with each other?

