Kepler’s Laws of Planetary Motion

• The orbits of planets (and everything else) are ellipses, with the Sun at one focus.
• A line from the Sun to a planet sweeps out equal areas in equal intervals of time
• The semimajor axes and orbital periods are related by the Harmonic Law

A question (?)

Is Kepler’s 3rd Law, $a^3 = P^2$ only due to the fact that the planets further out have a longer orbit to complete in a period?
Kepler’s Laws are even more general than orbits in the solar system…they govern orbits throughout the universe, like those of stars at the center of the Milky Way galaxy

http://www.eso.org/public/videos/eso0846h

Applications of Kepler’s Laws: variations in the opposition of Mars

Another application of Kepler’s 1st Law:

the orbit of the Earth’s Moon.

How long does it take for a trip to Mars?
The length of a trip to Mars
For Earth: \( a = 1.00 \text{ au} \)
For Mars: \( a = 1.52 \text{ au} \)
Semimajor axis = \( 2.52/2 = 1.26 \text{ au} \)
\[
\begin{align*}
  a^2 & = P^2 \quad (1) \\
  P^2 & = a^2 = (1.26)^2 = 2.00 \quad (2) \\
  P & = \sqrt{2.00} = 1.41 \text{ years} \quad (3)
\end{align*}
\]
We only want half of a period (Earth to Mars, not Mars to Earth) so time = \( 1.41/2 = 0.707 \) years = 258 days (8.5 months)

Next topic: where do Kepler’s Laws come from

What is the deeper significance of them? Why is nature that way?

Isaac Newton...beginning of modern physics

Newton’s Laws of Motion...vocabulary

Newton’s description of dynamics, or the laws governing the motion of the planets, relied on the development of kinematics, which is the mathematical language that describes motion of objects. Here are some terms which are important in kinematics:

- speed is the rate at which you are moving. It has units of meters/sec. The speed doesn’t depend on the direction you are going.
- velocity is a mathematical quantity called a vector; it has both magnitude and direction. The magnitude of velocity is the speed. However, the velocity is also a direction as well. The velocities corresponding to moving east at 50 mph is different from moving north at 50 mph.
- acceleration is also a vector. The acceleration is the amount the velocity changes, divided by the time interval over which this change occurs. In terms of equations, we have

\[
\text{acceleration} = a = \frac{\text{change in velocity}}{\text{change in time}} = \frac{V_f - V_i}{t_f - t_i} \quad (1)
\]
Newton's Laws of Motion

Acceleration occurs if the speed of an object changes, while the direction of motion stays the same, if the speed stays constant while the direction of motion changes, or if both the speed and the direction change.

Newton's Laws of Motion

With the kinematic definitions above, we are ready to state Newton's Laws:

1. An object in motion remains in motion with the same velocity, unless acted on by an external force. An object at rest remains at rest unless acted on by an external force.

2. If an object with a mass $m$ is acted upon by an external net force $F$, it accelerates according to the law

$$F = ma, \quad a = \frac{F}{m}$$

3. "To every action, there is an opposite and equal reaction." That definition sounds odd, but a more useful definition is: If an object A exerts a force on object B, object B also exerts a force on A, which is equal in magnitude, but opposite in direction to the first force.

Demonstrations of Newton's Laws of Motion

What does this have to do with solar system objects? Or astronomy?
The nature of Gravity: gravity holds the solar system together

Gravity is an attractive force between two objects because they have mass

\[ F = \frac{GMm}{r^2} \]

So what does this have to do with the motion of the planets?

Centripetal acceleration and central force