

General Astronomy (29:61)  
Fall 2012  
Lecture 5 Notes, August 30, 2012

## 1 Precession of the Equinoxes

The Greek philosopher Hipparchus first noted that the right ascension and declination of stars were different in his time than they had been recorded by earlier astronomers.

We now know that this is due to the *precession of the equinoxes*. This is fundamentally due to the precession of the Earth's rotation axis. The idea is illustrated in Figure 1.10. Precession occurs when a rotating object is subject to a torque. The precession period is 25800 years. Since the location of the north celestial pole is changing, the declination of stars will change. Furthermore, since the intersection of the celestial equator with the ecliptic changes as well, the right ascension changes.

The precession of the Earth's rotation axis, together with periodic variations of a number of properties of the Earth's orbit, is believed by many scientists to be responsible for the Ice Ages on Earth.

## 2 Astronomical Fundamentals of Time

Astronomical phenomena help define our idea of time, and the measurement of time is tied inextricably with astronomy. Our fundamental terms for units of time, the day, and month, and the year, all relate to astronomical cycles.

### 2.1 The Day

You might think the definition of the day is ridiculously simple. However, in reality you have to be careful on how you define it. The book defines the day as "*the interval between successive transits of a celestial object*".

The most common (and seemingly only) choice for this celestial object is the Sun. This leads to the definition of the **Solar Day**.

However, you could also choose a star, such as Vega, and measure the day to be the time between successive transits. This is the definition of the **Sidereal Day**. Aren't these the same?

No they aren't. To see why, and estimate the difference, look at Figure 1.11. The crucial point is that the Sidereal Day is the rotation period of the Earth in an *inertial*

*reference frame* defined by the distant stars. The Solar Day is the rotation period in a rotating reference frame in which one axis is defined by the direction from the Earth to the Sun.

At the end of one sidereal day, the Earth has to still turn through a little angle to be back to the direction of the Sun.

## 2.2 Numbers, numbers, numbers

Let's work out the difference between the sidereal day and the solar day. One way of doing this is with the derivation in Equation 1.1 - 1.5. Here's an easier (although not as exact) way of seeing the same thing. Look at Figure 1.1.

After 1 sidereal day, the Earth has advanced in its orbit by an angle given by

$$\Delta\theta = \omega_E P_{sid} \quad (1)$$

Where  $\omega_E$  is the *angular speed* (radians/sec) of the Earth's orbital motion around the Sun ( $\omega_E = \frac{2\pi}{P_E}$ ), and  $P_{sid}$  is the length of the sidereal day.

The time  $\Delta t$  it takes for the Earth to rotate through this angle is (approximately)

$$\Delta\theta \simeq \omega_{sid} \Delta t \text{ so} \quad (2)$$

$$\omega_{sid} \Delta t \simeq \omega_E P_{sid} \quad (3)$$

$$\Delta t \simeq \frac{\omega_E}{\omega_{sid}} P_{sid} \quad (4)$$

$$\Delta t \simeq \frac{P_{sid}}{P_E} P_{sid} \quad (5)$$

To the extent that  $P_{sid}$  and  $P_{sol}$  are the same, this gives the same result as Equation (1.5) of the book, 1/365 of a day, or about 4 minutes.

The reason I used the "nearly equal" sign  $\simeq$  rather than the mathematical identity symbol  $=$  is that my calculation assumed that the Earth stayed put in its orbit while it rotates through the angle  $\Delta\theta$ .

The bottom line of this is the very important result that the sidereal day is 4 minutes shorter than the solar day. This fact provides another way of understanding seasonal variations in the appearance of the night sky.

You should look at the alternative (and more rigorous) derivation of this result on p18 and 19 of the book.

## 2.3 Choice of the Solar Day

We choose to organize our lives using the solar day because human beings are up and around during daylight. However, a major complication arises. *The solar day is*

*not a constant; it varies through the year.*

### 3 The Mean Solar Day and the Equation of Time

If you measured the length of the solar day with a precise clock, you would find that it varies through the year. You would also find that the Sun does not transit at noon every day. By contrast, the length of a sidereal day is vastly more constant. What is going on?

There are two effects responsible for the changing length of the day. To start with, the solar day is given by the sidereal day (the true, inertial rotation period of the Earth), plus “something extra”. That something extra is the time it takes the Earth to rotate through the angle that the Sun has moved to the east during a day. This angle is roughly a degree.

*If the Earth were in a circular orbit, and the obliquity of the ecliptic were zero, then this “something extra” would be constant.* In this case, the solar day would be as constant as the sidereal day. However, those conditions are not true. Let’s see what the consequences are.

1. *Obliquity of the Ecliptic.* The Earth rotates parallel to its equator. Celestial objects move across the sky on diurnal circles parallel to the celestial equator. For this reason, it is eastward component of the Sun’s daily motion relative to the equatorial coordinate system that matters.

The fact that the ecliptic is inclined at  $23.5^\circ$  with respect to the celestial equator means that this eastward daily motion changed through the year. It is a maximum at the time of the solstices, and a minimum at the time of the equinoxes (see Figure in slides and pictures).

2. *Nonuniform Motion of the Sun Along the Ecliptic* On the other hand, even if the obliquity of the ecliptic were zero, the solar day would vary through the year because the daily angular speed of the Sun along the ecliptic changes.

The reason for this is *Keplers’s 2nd Law of Planetary Motion*. We will discuss this in detail in the next chapter. Let’s pick the main features now.

The motion of the Earth about the Sun is not a circle, but an ellipse. The Sun is not at the center of the ellipse, but offset at one of the focuses (see Figure in lecture notes). Kepler’s 2nd Law says that the Earth moves at a small angular speed when it is far from the Sun, and at a high angular speed when it is close. This means we see the Sun shifting against the background stars rapidly when

we are closest to the Sun (*perihelion*), and slowly when furthest from the Sun (*aphelion*).

In practice, both of these effects are present, and cause the length of the solar day to vary in a complicated way through the year.

### 3.1 Mean Solar Day

It would be horribly impractical to run daily affairs according to solar time, or specifically, *apparent solar time*, which is time based on the true position of the Sun in the sky.

For this reason, civil time is based on *Mean Solar Time*, which is that every day is taken to be equal to the average of the solar day during the year. Every day is defined to have the same length, so the length of a minute and hour every day is the same.

While this makes sense, it leads to an interesting astronomical phenomenon called *The Equation of Time*. Remember that the Sun transits at noon *Local Apparent Solar Time*. Because the length of the apparent solar day varies during the year, this can be before or after noon, mean solar time. The difference between the two is called *The Equation of Time*, and is equal to Apparent - Mean Solar Time.

A very nice plot is given in Figure 1.12. This figure shows how much of the The Equation of Time is due to the obliquity of the ecliptic, and how much due to the eccentricity of the Earth's orbit. You can see that the Sun will transit up to 17 minutes early or late relative to apparent solar time (civil time).