General Astronomy (29:61)
Fall 2012
Lecture 10 Notes, September 17, 2012

1 Just one more thing . . . Orbital Elements

1.1 How many numbers do we need to specify an orbit?

How many parameters to we need to uniquely determine an orbit? From what we have discussed so far, and as far as the discussion in nearly all physics books, the answer is two: a the semimajor axis and e the eccentricity (see accompanying diagram).

We have already seen that orbital motion stays in a plane (remember what the argument for that was?)

However, if we are dealing with real astronomical orbits, it is clear we need more numbers. There is an infinite number of orbits that would have the same a and e and have the Sun at one focus. They could all have different angles of orientation with respect to each other.

Let's start selecting the other orbital parameters we need. We will begin by specifying the plane of the ecliptic as the reference plane. Let's make a list of them, and check the accompanying figure for a graphical definition.

- The inclination ι. This is the angle of intersection of the plane of the orbit (of the other planet or comet) with the plane of the ecliptic (see diagram). For the major planets, these angles are less than 10°, and except for Mercury, are less than 10°.

- The longitude of the ascending node, Ω. This angle tells how the line of nodes between the orbit and the plane of the ecliptic is oriented relative to the stars (Ω is measured relative to the Vernal Equinox)

- The Argument of Periapsis ω (what a great name, eh?). This angle tells how the orbit is oriented relative to the plane that contains it.

- The time of perihelion passage T. This doesn't say anything about the properties of the orbit, but does tell us where the planet is in its orbit, which is generally something we want to know.

All together, then, there are 6 orbital elements that must be specified to tell us where a solar system object is in space.

2 The Earth-Moon System

Why do we speak about the "Earth-Moon System". Because the Earth's moon is anomalously large compared to its planet. We'll see this later in the semester. This seems to indicate that there was something special about the processes of formation of the Moon.

2.1 Basic Facts about the Moon

Because the Moon is the closest astronomical object, we know a lot about it. Let's begin by looking at as we can see it in a small telescope (see picture in notes and diagrams for this lecture).

Because it is so close, we have very good information on its properties.

- radius = 1737 km (27 % that of Earth)
- mass = 7.35×10^{22} kg
- You can see a lot of features on the face of the Moon; we will discuss these when we talk about the "geology" of the Moon.
- Even with a small telescope, you can see that there is no atmosphere on the Moon. This is an important difference between the Earth and the Moon.

2.2 The Orbit of the Moon

Naked eye observations of the Moon show it moving from night to night against the background stars. Let's talk about its orbit. The overall properties of its orbit are shown in a diagram in the associated plots and diagrams for this lecture.

The semimajor axis is 384,000 km, but the distance of the Moon from the focus of its orbit ranges from 356,400 to 406,700 km. So the eccentricity of the orbit is 0.055.

To use some of the other orbital elements we have discussed, the *inclination* of the Moon's orbit is 5.1°. For this reason, the trajectory of the Moon across the sky is perceptibly offset from the ecliptic. There are two points on the ecliptic, called the nodes, where the Moon's orbit intersects the ecliptic.

Since the distance of the Moon changes, the angular size of the Moon changes during the month as well. There is a total 10 % change from smallest to largest. This fact is responsible for the fact that some eclipses are "annular" rather than total (more discussion later).

2.3 The Two Body Problem and the Center of Mass

It's time to discuss exactly what happens when two objects interact via a gravitational force (the 2 body problem). Let's let r(t) be the distance from the center of one to the center of the other. This does indeed trace out an ellipse, as we have seen. However, *both* objects move in response to the gravitational force exerted by the other.

Each object undergoes an elliptical motion around the *center of mass* (CM; see accompanying diagram). The distance from the center of mass to M is

$$r_1 = \left(\frac{m}{M+m}\right) r \tag{1}$$

and the distance from the center of mass to m is

$$r_2 = \left(\frac{M}{M+m}\right) r \tag{2}$$

What this means is that, during the month, the Earth is also moving back and forth in space around the center of mass of the Earth-Moon system. Strictly speaking, it is the center of mass of the Earth-Moon system that goes in an elliptical orbit around the center of mass of the solar system.

2.4 The Month and the Orbit of the Moon

See Section 4.4 of the textbook. From night to night we see the Moon moving against the background stars, approximately on ecliptic (except for the 5.1° offset due to the inclination of the Moon's orbit). This has been noted since prehistory. We now know that is due to the Moon's orbital motion around the CM of our system.

Associated with this motion against the background stars, there is a change in the phases of the Moon from *New Moon* to *First Quarter Moon*, to *Full Moon*, to *Third Quarter Moon*, and then back to New Moon. This is explained in Figure 4.10 of your textbook, or in the accompanying diagrams.

The period for this orbital motion of the Moon is the basis for probably the oldest unit of time, the month. The time for the Moon to complete an orbit with respect to the stars is called the *Sidereal Month*, and has a duration of 27.32 days.

The other definition of the month is the time to go through all of the phases and end up with the same phase. This is called the *Synodic Month* and is equal to 29.53 days. Ask yourself why these two values for the month would be different. You should be able to figure it out.

2.5 Synchronous rotation of the Moon

One of the most striking features of the Moon is that we only see one side of, that facing the Earth. Until the Space Age, no one had seen the back side of the Moon. That is because it is our closest example of the important astrophysical process of *synchronous rotation*. The rotation period of the Moon equals its orbital sidereal period. This is a consequence of the tides raised by the Earth on the Moon. Synchronous rotation occurs throughout the solar system, and astrophysics in general.

2.6 Librations

Even though one hemisphere is always pointed towards the Earth, we nonetheless see more than 50 % of the Moon's surface. This is due to the phenomenon of *librations*.

An illustration of librations is shown for the September 2, 2007 Astronomy Picture of the Day under "lunations". There are three types of librations, clearly described on p100 and 101 and illustrated in Figures 4.11, 4.12, and 4.13. These librations can be summarized as follows.

- Diurnal libration .... a parallax effect
- Libration in longitude ... a consequence of Kepler's 2nd Law
- Libration in latitude ... a consequence of the fact that the Moon's rotation axis is not perpendicular to its orbital plane, but is closer to being perpendicular to the plane of the ecliptic.