

General Astronomy (29:61)
Fall 2012
Lecture 13 Notes, September 26, 2012

1 Main Surface Features of the Moon

There are a number of important features of the lunar surface that are visible in a small telescope. It is good to look at a picture of the Moon while discussing these.

- Maria – plural of Mare. Dark, smooth areas on the surface of the Moon. There are several of these on the Moon, and they have Latin names like *Mare Tranquillitatis* where the first Apollo spacecraft landing was.
- Terrae – these are the brighter areas on the Moon, that are higher than the Maria.
- Craters – these are big holes on the surface of the Moon. They are the most distinctive lunar feature. Before the space program in the 1960s, it is almost the case that the Moon was the only object known to possess them. They are now known as a major feature of the solar system. More below.
- Crater rays—a small telescope will show rays coming out of some of the prominent craters, such as Tycho and Kepler (see picture in online diagrams).
- Rilles, Graben, etc – There are features on the Moon that look like old river beds, or straight walls. The most famous example is the Hadley Rille, and another (geologically unrelated) feature is the Aridaeus Rille, that we can see with our telescopes from the roof.

1.1 Craters of the Moon

Amazingly, the debate as to what these are continued until the early 1960s. They range in size from a few feet in diameter to the many tens of kilometers in diameter. There are 5 with diameters greater than 200 km. The diameters of some of the best known (and easiest to find in a small telescope) are: Tycho, 102 km, Copernicus, 107 km, Kepler, 31 km.

By contrast, a prominent terrestrial example, the Barringer crater in Arizona, is only 1.1 kilometer in diameter. Look at some pictures of them in the online notes.

1.2 Craters as impact craters

In the early 1960s, it was finally accepted that the craters of the Moon are *impact craters*, caused by the collisions of objects from outer space with the Moon. The kinetic energy of the moving impactor was converted to, essentially, explosive energy that excavated the hole in the lunar surface. Empirical studies show that at the speeds expected for interplanetary projectiles, the impactor would be 1/3 to 1/5 the diameter of the crater it produces.

In looking at the Moon, the question arises as to when these impacts occurred. We will see the answer to that next time.

2 The Apollo Moon Landings

No discussion of the Moon would be complete without a description of the Apollo program landings that took place between July, 1969 and December 1972. To this day, it represents the only time that human beings traveled to another astronomical object. See the online pictures.

The main scientific result from the Apollo landings was the determination of the age of formation of lunar rocks.

3 Radioisotope Dating of the Age of Formation of a Rock

The age of formation of a rock means how long ago did it solidify into a solid object, with its atoms locked into a lattice structure. Just looking at a rock, it would seem impossible to say.

The method of determining the age of formation from *radioactive isotopes* is extremely important in modern planetary science, so we will take some time to discuss it. Be sure to review your knowledge of what we mean by *atoms, nuclei, and isotopes*.

Let's assume that in a radioactive decay, nucleus A decays into nucleus B,



where X is a particle that conserves something like electrical charge. An real example would be



where e is an electron and ν is a neutrino.

If a rock forms with atoms of type A, then as time goes on the number of A decreases and that of B increases.

Let $N = N(t)$ be the number of nuclei of type A in a rock sample. A basic law of radioactivity says how the number of radioisotopes in a sample changes with time. It is naturally expressed in terms of a derivative in calculus,

$$\frac{dN}{dt} = -\alpha N \quad (3)$$

What is the solution to this equation, $N(t)$? Let's do the following.

$$\frac{dN}{dt} = -\alpha N \quad (4)$$

$$\frac{dN}{N} = -\alpha dt \text{ we can integrate both sides} \quad (5)$$

$$\int_{N_0}^N \frac{dN}{N} = \int_0^t -\alpha dt \quad (6)$$

$$\ln N(t) - \ln N_0 = -\alpha t \quad (7)$$

$$\ln N(t) = \ln N_0 - \alpha t \text{ so} \quad (8)$$

$$N(t) = N_0 e^{-\alpha t} \quad (9)$$

This equation tells us that the number of radioisotopes *decays exponentially with time*. A plot of this function for the specific parameters $N_0 = 1000$ and $\alpha = 0.300$ is shown in the accompanying plots.

A *very important* concept is that of the *half life* of an isotope, that is, the time until the remaining number of nuclei is half the initial number. For radioisotopes (radioactive nuclei) there is an incredible range in the values of the half life.

Let's make it explicit that this equation applies to the "parent isotope" N_A ,

$$N_A(t) = N_{A0} e^{-\alpha t} \quad (10)$$

Every A isotope that decays produces a B isotope, so

$$N_A + N_B = N_{A0} \text{ so} \quad (11)$$

$$N_B(t) = N_{A0} - N_A(t) \quad (12)$$

$$N_B(t) = N_{A0} (1 - e^{-\alpha t}) \quad (13)$$

These equations say that we can determine the age of a rock sample by comparing $N_A = N_A(t)$ and $N_B = N_B(t)$, A plot comparing the two functions is shown in the accompanying diagrams.

4 The Real Case

The above equations are a nice exercise in mathematics and give the basic idea of radioisotope dating, However, there is an unwarranted assumption, namely that $N_B(t = 0) = 0$. If $N_B(t = 0) \neq 0$, then the plot we have shown is invalid for determining the age of formation of a rock.

How do we get around this?

Nucleus B is an isotope of some element. It is the daughter isotope of the radioactive decay



When I analyse a rock, some of the atoms of B come from this decay process, and some were there from the start. I can't tell how much to attribute to the two origins. For any type of physical analysis, a B nucleus is a B nucleus.

The key comes if there is *another isotope* of the same element as B, but which is not a daughter isotope of a radioactive decay reaction. Let's call this isotope C. The number of nuclei N_C of this isotope do not change with time.

The set of equations we have now are

$$N_A(t) = N_{A0}e^{-\alpha t} \quad (15)$$

$$N_B(t) = N_{A0} (1 - e^{-\alpha t}) + N_{B0} \quad (16)$$

$$N_C(t) = N_C \quad (17)$$

What we can measure is $N_A(t)$, $N_B(t)$, and N_C , and we know α . We don't know N_{A0} , N_{B0} , and t .

Let's divide the top two equations by N_C . Let's also define $f = e^{-\alpha t}$. Our first two equations above then become

$$\frac{N_A(t)}{N_C} = \frac{N_{A0}}{N_C} f \quad (18)$$

$$\frac{N_B(t)}{N_C} = \frac{N_{A0}}{N_C} (1 - f) + \frac{N_{B0}}{N_C} \quad (19)$$

We can rewrite the top equation as

$$\frac{N_{A0}}{N_C} = \frac{1}{f} \frac{N_A(t)}{N_C} \quad (20)$$

and substitute into the second equation to get

$$\frac{N_B(t)}{N_C} = \frac{N_A(t)}{N_C} \left(\frac{1 - f}{f} \right) + \frac{N_{B0}}{N_C} \quad (21)$$

This still doesn't seem to be much of a help. We can measure $\frac{N_B(t)}{N_C}$ and $\frac{N_A(t)}{N_C}$, but we don't know f or $\frac{N_{B0}}{N_C}$.

4.1 A more illustrative form of the equation

. Let's try the following change of variables. Let

$$y \equiv \frac{N_B(t)}{N_C} \quad (22)$$

$$x \equiv \frac{N_A(t)}{N_C} \quad (23)$$

$$a \equiv \frac{1-f}{f} \text{ which is directly determined by the time} \quad (24)$$

$$b \equiv \frac{N_{B0}}{N_C} \quad (25)$$

Then our equation becomes

$$y = ax + b \quad (26)$$

What is this equation? What does it do for us? Tune in next time.