General Astronomy (29:61)
Fall 2012
Lecture 14 Notes, October 1, 2012

1 The Equations for Radioisotope Dating

The above equations are a nice exercise in mathematics and give the basic idea of radioisotope dating, However, there is an unwarranted assumption, namely that $N_B(t=0) = 0$. If $N_B(t=0) \neq 0$, then the plot we have shown is invalid for determining the age of formation of a rock.

How do we get around this?

Nucleus B is an isotope of some element. It is the daughter isotope of the radioactive decay

$$A \rightarrow B + X \tag{1}$$

When I analyse a rock, some of the atoms of B come from this decay process, and some were there from the start. I can't tell how much to attribute to the two origins. For any type of physical analysis, a B nucleus is a B nucleus.

The key comes if there is *another isotope* of the same element as B, but which is not a daughter isotope of a radioactive decay reaction. Let's call this isotope C. The number of nuclei N_C of this isotope do not change with time.

The set of equations we have now are

$$N_A(t) = N_{A0}e^{-\alpha t} \tag{2}$$

$$N_B(t) = N_{A0}\left(1 - e^{-\alpha t}\right) + N_{B0} \tag{3}$$

$$N_C(t) = N_C \tag{4}$$

What we can measure is $N_A(t)$, $N_B(t)$, and N_C, and we know α. We don't know N_{A0}, N_{B0}, and t.

Let's divide the top two equations by N_C. Let's also define $f = e^{-\alpha t}$. Our first two equations above then become

$$\frac{N_A(t)}{N_C} = \frac{N_{A0}}{N_C}f \tag{5}$$

$$\frac{N_B(t)}{N_C} = \frac{N_{A0}}{N_C}(1 - f) + \frac{N_{B0}}{N_C} \tag{6}$$

We can rewrite the top equation as

$$\frac{N_{A0}}{N_C} = \frac{1}{f}\frac{N_A(t)}{N_C} \tag{7}$$

and substitute into the second equation to get

$$\frac{N_B(t)}{N_C} = \frac{N_A(t)}{N_C}\left(\frac{1-f}{f}\right) + \frac{N_{B0}}{N_C} \tag{8}$$

This still doesn't seem to be much of a help. We can measure $\frac{N_B(t)}{N_C}$ and $\frac{N_A(t)}{N_C}$, but we don't know f or $\frac{N_{B0}}{N_C}$.

1.1 A more illustrative form of the equation

. Let's try the following change of variables. Let

$$y \equiv \frac{N_B(t)}{N_C} \tag{9}$$

$$x \equiv \frac{N_A(t)}{N_C} \tag{10}$$

$$a \equiv \frac{1-f}{f} \text{which is directly determined by the time} \tag{11}$$

$$b \equiv \frac{N_{B0}}{N_C} \tag{12}$$

Then our equation becomes

$$y = ax + b \tag{13}$$

What is this equation? What does it do for us? Tune in next time.

1.2 Measurements with a Pure Mineral

If we make measurements with a pure mineral, we have one value of x and one value of y. That is not enough to determine the age (governed by the slope), since there is an infinite number of separate slopes and intercepts that could reproduce the data. $\longrightarrow$ Diagram on board

1.3 Measurements with Rocks Having Several Minerals

Usually, a rock will consist of fused pieces of different minerals. These different minerals will have different "affinities" for different elements, and therefore would have had different amounts of (for example) rubidium and strontium when they formed. These different minerals within the same rock presumably solidified at the same time,

so we have a set of different values of (x, y). In this case, we can independently determine the slope and intercept of the linear relationship.
$\longrightarrow$ Diagram on board

2 Rubidium-Strontium Dating on Moon Rocks

The real proof of this technique is that if you make measurements of Sr87, Rb87, and Sr86 in a Moon rock, you should get a linear relationship in our x and y parameters. Let's look at what is seen.

The online figures show the results of a measurement made on a rock returned from the Apollo 17 mission. This rock, sample 72417, came from the lunar highlands.

Let's take the time to look carefully at this neat diagram!

There are two particularly important features.

1. The data adhere very well to the expected linear relationship, giving confidence to the whole procedure.

2. The age of formation of this rock turns out to be 4.47 ± 0.10 billion years (Gyr). That is not only much older than the oldest rocks on Earth (by over a billion years), but is close to what we know to be the age of the solar system.

2.1 Some Notes on the Algebra of the Dating Technique

You may be having a little difficulty really grasping the relation between the age of formation and the slope on this diagram. Let's look at this in more detail.

From above we saw that the slope a is given by

$$a = \frac{1-f}{f} \text{ where} \tag{14}$$

$$f = e^{-\alpha t} \tag{15}$$

Notice that it is only the product αt that matters, not α and t separately. One can get the same slope for two quite different radioisotopes at quite different times. So we can just let $x \equiv \alpha t$ and we have

$$a = \frac{1-e^{-x}}{e^{-x}} \tag{16}$$

Let's consider $a = a(x)$, and see what $a(x)$ looks like
$\longrightarrow$ See plots in online diagrams

With these diagrams (or the corresponding algebraic expressions), one can determine x, or equivalently the age of formation, given a measurement of the slope of a dating diagram.

3 What We Learned from Dating Moon Rocks

The results from dating the Moon rocks are summarized in the slide in the online diagrams and pictures. Take a look!

4 What We Deduce About the Time of Formation of the Craters on the Moon

The rocks in the lunar Maria are very old relative to Earth rocks, 3.2 Gyr and older. Yet there are few craters on the Maria. That means the rate at which craters form on the Moon (and presumably other solar system objects) has been relatively low for the last 3 billion years, an enormous period of geological time.

Some of the terrae regions are "only" about 1 Gyr older. Yet they are very heavily cratered. Without doing any detailed calculations, this indicates that the cratering rate was much higher at the time the terrae rocks formed, and was much lower by the time the Maria formed.

By "cratering rate" we mean a number with units of (number of impacts)/(unit of area)/(unit of time), such as impacts/1000 square kilometer/million years.

There have been two quite different views of how this cratering rate have changed since the formation of the solar system, 4.50 - 4.60 Gyr ago. Both have the ability to explain the overall feature that the terrae are heavily cratered, while the maria are not.

1. From the time of the Apollo landings, when the ages of formation of Moon rocks became available, it has been suggested that the cratering rate has decreased steadily with time. $\longrightarrow$ diagram on blackboard. According to this viewpoint, the objects (asteroids and/or comets) that produced the craters on the Moon and other solar system objects (were the last of the blocks of matter that made up the planets (*planetesimals*).

2. In about the last ten years, a quite different view of the cratering rate has been put forward. According to this viewpoint the "sweepup of the planetesimals" was completed early in the history of the solar system, and the cratering rate

was low for the first 700 million years or so of the Moon's history. Then, from 3.9 - 3.8 Gyr ago it increased by a very large factor for about 100 million years before returning to the low level. According to this viewpoint, the bulk of the craters on the Moon were formed during this *Late Heavy Bombardment*. There are very interesting ideas of what might have caused the *Late Heavy Bombardment*.

At the present, there is not agreement in the astronomical community as to which of these viewpoints is correct.

What is generally done is to assume that the majority of craters throughout the solar system were formed at the same time as the lunar craters, i.e. predominantly prior to 3.2 Gyr ago.