General Astronomy (29:61)
Fall 2012
Lecture 16 Notes, October 8, 2012

1 The Concept of a Spectrum

An important piece of scientific vocabulary to mention is that of the *spectrum*. As we have seen, light carries energy. We can sort out the light from an object according to wavelength (or frequency) and then express the amount of energy received per unit time in each wavelength "bin". When we make a plot of the energy received per unit time as a function of wavelength, we refer to a spectrum. An instrument that measures a spectrum of light is a spectrometer or spectrograph. We will work with some spectrometers in the lab portion of the class later in the semester.

2 The Interaction of Light with Atoms and Molecules

2.1 The Hydrogen Atom

Hydrogen is an extremely important element in astronomy. You learned in HS chemistry that it is the simplest element, with a nucleus of one proton (sometimes the nucleus also has a neutron) and one electron orbiting the nucleus.

It is important in astronomy because the universe likes hydrogen. It doesn't contribute a lot of mass to the Earth; the most conspicuous presence of hydrogen is in water in the oceans. However, the most massive planets, Jupiter and Saturn, are primarily hydrogen by mass. The Sun is even more so. 75 % of the mass of the Sun is in the form of hydrogen, and 24 % of the remainder is helium, the 2nd simplest element.

Think of the hydrogen atom. There is an electrostatic force between them, and a corresponding potential energy $U(r)$, where r is the distance between the proton and electron. There is also kinetic energy K , as the electron and proton move about the center of mass (since the proton is so much more massive than the electron, this kinetic energy is mainly that of the electron).

The total energy of the hydrogen atom, E, is just the sum of these two,

$$E = K + U \tag{1}$$

This sort of sounds like the Earth and Sun in the solar system. However, there is a *huge difference*. In the solar system, the energy of a planet (or comet, or asteroid) can take on any value. In the case of a hydrogen atom, the energy is *quantized*; it can take on only certain values. The realization that this was the case was the big push towards the development of quantum mechanics.

The derivation of this is given on pages 112 and 113, leading to equations 5.10 and 5.12. This result is called the *Bohr Atom*, after the physicist Niels Bohr who (presumably) first derived it. Many physicists don't think it should be taught at any level because it is not right.

The crucial ingredients of the derivation are as follows.

1. The potential energy between the proton and electron is given by

$$U(r) = -\frac{e^2}{4\pi\epsilon_0 r} \tag{2}$$

   where $\epsilon_0 = 8.8542 \times 10^{-12}$ is the *permittivity of free space.* The kinetic energy is given by

$$K = \frac{1}{2}mv^2 \tag{3}$$

2. We assume the orbits are circular, so $\vec{v}$ is perpendicular to $\vec{r}$
3. We substitute the expression for the angular momentum into the expression for the kinetic energy
4. Require that the total energy is a minimum for the specified angular momentum (this is a condition for circular orbits
5. Invoke Planck's discovery, that the angular momentum is quantized, $L = n\frac{h}{2\pi}$

The result of this exercise is we find two things.

First, the orbital radii must be quantized, or take on only discrete values, as given by Equation 5.8

$$r_n = \frac{4\pi\epsilon_0\hbar^2 n^2}{e^2 m} \tag{4}$$

Second, the energy is also quantized, with the following expression

$$E_n = -\left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{m}{2\hbar^2 n^2} \tag{5}$$

If we plug in numbers for the fundamental physical constants, we get the following expressions, given by equations 5.8 and 5.13 of the textbook,

$$r_n = 5.29 \times 10^{-11} n^2 \text{ m} \tag{6}$$

$$E_n = \frac{-13.6}{n^2} \text{ eV} \tag{7}$$

The energy level diagram of hydrogen shown in Figure 5.2 is one of the most important diagrams in all of astronomy. Although it would be stupid to memorize it, you should thoroughly understand its content.

As was discussed in class on Friday, when electrons make a *transition* from one energy level to another, the photon energy must be equal to the difference in the starting and ending energy levels. This is the reason why light emitted and absorbed by atoms and molecules occurs at certain distinct wavelengths. We call these wavelengths *spectral lines*.

2.2 Other Atoms

Other atoms have similar energy levels to the hydrogen atoms. In other words, only certain energies exist. Light is emitted or absorbed by these atoms only at certain wavelengths.

An example is shown in the online figures for carbon, which is an important element in astronomy. The arrows give the transitions that can occur with the emission or absorption of a photon. You might wonder what is the significance of the separate sets of energy level diagrams for "singlets" and "triplets". In a few words, the ordering of the energy level diagrams in multielectron atoms are more complicated that for hydrogen, and involve the possible combinations of angular momentum of the several electrons in such an atom.

3 Transitions in Atoms, Radiative and Non-Radiative

When an atom changes from one energy state to another, it either takes energy (if you move up the energy level diagram) or gives off energy (if you move down the energy level diagram). This energy has to come from, or go to somewhere. Sometime, the energy of a photon is involved, and sometime it is not. Let's look at the possibilities. You should look at Figures 5.3, 5.4, 5.5, and 5.6.

Here is a list of the most important processes.

- photon absorption or photoexcitation of the atom (Figure 5.3a)
- collisional excitation of the atom (5.3b)
- photon emission or photodeexcitation of the atom. (Spontaneous emission), (Figure 5.4a of book)
- stimulated emission of radiation (basis of lasers) (Figure 5.4b of book)

- Collisional deexcitation of an atom (Figure 5.4c)
- photoionization, in which absorption of a photon breaks an atom into an electron and an ion (Figure 5.5a)
- collisional ionization, in which the kinetic energy of a colliding particle (atom, electron, etc) is sufficient to ionize an atom (Figure 5.5b)
- radiative recombination, in which an electron with finite energy recombines with an ion to form a neutral atom. The excess energy is taken off in the form of photon energy (Figure 5.6)

In the case of ionization, and important parameter is the *ionization potential*, which is the amount of energy necessary to ionize an atom from the ground state. For example, for the case of hydrogen, this is 13.6 electrons volts.

4 Kirchoff's Laws of Radiation

A compact set of statements about what kind of spectral appear under what circumstances are given by *Kirchoff's Laws*. These are given on p122 of the textbook. See online diagrams and plots.

4.1 The Planck Function

There is a more modern way to express Kirchoff's 1st Law. A *blackbody* produces a spectrum of radiation which is described by a function called the Planck function. This is given in Equation 5.86, and plotted in Figure 5.14 and 5.15.