General Astronomy (29:61)
Fall 2012
Lecture 18 Notes, October 12, 2012

1 Telescopes and Astronomical Detection of Light

Telescopes, in rough terms, carry out two functions.

1. The collect large amounts of radiant energy from an astronomical object and concentrate it for detection and measurement.

2. They form an image of the object.

Look at the discussion in the textbook about a pinhole camera, and how an image is formed in that case.

In class I showed a set of pictures of major telescopes, both refractors and reflectors, and also showed a picture of the radio telescope at North Liberty, Iowa

2 Image Formation

Let's begin with discussion of image formation by a lens. A lens has the property that parallel rays of light from a distant source of radiation are focused to a point. This point, called the *focus*, is a distance F behind the lens. We call F the *focal length*.

To do this, the lens must bend a ray of light towards the central axis of the lens. If the parallel ray strikes the lens a distance y from the central axis, the lens must bend the light ray through an angle θ,

$$\tan\theta = \frac{y}{F} \tag{1}$$

2.1 Image Formation with Extended Object

Let's assume an object subtends an angle 2ϕ, where one end is an angle ϕ from the central axis, and the other end at $-\phi$.

If we follow the ray through the center of the lens (the undeflected ray), it comes to a position $-d$ from the axis of the lens at the focal plane. d is given by

$$\tan\phi = \frac{d}{F} \tag{2}$$

The ray from the other end of the extended object, at $-\phi$, goes to $+d$ on the focal plane.

This is the position of the rays that go through the center of the lens, but where do the other rays end up?

Let's consider the ray coming from $+\phi$, that strikes the lens a distance y from the axis of the lens. That ray is bent by an angle θ (see above) so the total angle with respect to the axis of the lens is $\phi + \theta$. We can write the position where this ray strikes the focal plane as follows:

$$\tan(\theta + \phi) = \frac{y + x}{F} \tag{3}$$

where x is the position below (or above, if $x < 0$) the axis of the lens where this rays strikes the focal plane. We can use a law from trigonometry for sums of angles,

$$\tan(\theta + \phi) = \frac{\tan\phi + \tan\theta}{1 - \tan\theta\tan\phi} \tag{4}$$

So, as an equation for the unknown distance x in terms of θ, ϕ, F, we have

$$\frac{\tan\phi + \tan\theta}{1 - \tan\theta\tan\phi} = \frac{y + x}{F} \tag{5}$$

Which looks like a complicated mess. It simplifies a lot if we can make the approximations that $\phi \ll 1$, and $\phi \ll 1$. What this means in practice is that ϕ, θ are angles much less than one radian. An angle of 10 or 15 degrees would satisfy this. An angle of 45 degrees wouldn't.

In this case $\tan\phi \ll 1$, so the *product of the two is really small.* This means the denominator in the above equation is essentially 1, so our equation becomes

$$\tan\phi + \tan\theta = \frac{y}{F} + \frac{x}{F} \tag{6}$$

Since $\tan\theta$ is defined as $\frac{y}{F}$, this means we have the following equation for the unknown x

$$\tan\phi = \frac{x}{F} \tag{7}$$

Compare this equation with Equation (2). This shows that $x = d$.

"Big Deal!" you may be saying. What does this show? It shows that a ray, coming from a direction $+\phi$ and striking the lens a distance y from the axis of the lens, ends up at the same point on the focal plane as a ray going through the center of the lens. You could generalize this argument and convince yourself that all rays coming from a direction ϕ, regardless of where they strike the lens, end up at the same point on the focal plane. Thus, the lens forms an *image* of the object.

3 The Image Scale in the Focal Plane

How big is the image that is formed in the focal plane? This is important for plans in setting up observations. Let' s go back to the formula above,

$$\tan\phi = \frac{d}{F} \tag{8}$$

Let's assume $\phi \ll 1$, so $\tan\phi \simeq \phi$. We then have

$$\phi = \frac{d}{F} \tag{9}$$

where ϕ is in radians, and d and F can be in any units, as long as they are the same units. Let's convert ϕ from radians to arcseconds, which is a more convenient unit for astronomical objects. This is $\phi = \frac{\phi('')}{206{,}265}$. Let's also specify d and F in millimeters. We then have

$$\frac{\phi('')}{206,265} = \frac{d(mm)}{F(mm)} \tag{10}$$

$$\frac{F(mm)}{206,265} = \frac{d(mm)}{\phi('')} \tag{11}$$

It is astronomical convention to define the reciprocal of the above relation as the *image scale* or *plate scale* s,

$$s = \frac{\phi('')}{d(mm)} = \frac{206,265}{F(mm)} \tag{12}$$

Next time, we'll use this equation to come up with some interesting conclusions about astronomical observations.