General Astronomy (29:61)
Fall 2012
Lecture 25 Notes, October 30, 2012

1 Blackbody Radiation of a Bald Planet, the Stefan-Boltzmann Law

This sounds like a weird section title, but let's move on. Let's assume that a planet has a uniform temperature. You might say this isn't true for the Earth, but as we so above, what strike us as big temperature changes aren't really all that big. The *total power* radiated to space is

$$L_p = \sigma T^4 4\pi R_E^2 \tag{1}$$

This is Equation 8.7. Astronomers like this equation, but if you want to be precise about it, the right hand side should be multiplied by a constant ϵ which is a measure of how good a blackbody a planet is. For planets this number is usually close to 1, but it isn't exactly 1.

2 Radiative Equilibrium and the Equilibrium Blackbody Temperature of a Planet

Equation (4) gives power coming from the Sun to heat up the Earth. Equation (7) is the power radiated by the Earth as it cools down. In the condition of *equilibrium*, these two balance,

$$W_p = \left(\frac{L_\odot}{4\pi r^2}\right)\pi R_E^2(1-A) = L_p = \sigma T^4 4\pi R_E^2 \tag{2}$$

We can cancel terms to obtain the equilibrium temperature of a planet

$$\left(\frac{L_\odot}{4\pi r^2}\right)\pi R_E^2(1-A) = \sigma T^4 4\pi R_E^2 \tag{3}$$

$$T^4 = \left(\frac{L_\odot}{4\pi r^2}\right)\frac{1-A}{4\sigma} \tag{4}$$

$$T = \left[\left(\frac{L_\odot}{4\pi r^2}\right)\frac{1-A}{4\sigma}\right]^{1/4} \tag{5}$$

Let's work it out for the Earth. This is fun!

$$T = \left[\left(\frac{L_\odot}{4\pi r^2} \right) \frac{1 - A}{4\sigma} \right]^{1/4} \tag{6}$$

$$T = \left[1365 \frac{1 - 0.40}{4(5.67 \times 10^{-8}} \right]^{1/4} \tag{7}$$

$$T = 245K \tag{8}$$

The real value (global average temperature of Earth) is 290 K (check Table 8.3 of book).

There are two ways of considering the agreement, or lack of agreement, between these two numbers. The first is that the agreement really isn't too bad for a pencil-and-paper calculation. However, the difference, 45K, is the difference between 62 degrees Fahrenheit and -19 degrees Fahrenheit. That is the difference between the Earth as it is, and one locked in pole-to-pole glaciation.

$\longrightarrow$ Look at Table 8.3; the agreement isn't very good, and for Venus it is freakishly bad. What is wrong?

3 Surface Temperatures of Planets

To focus our attention, let's think about the Earth, where our equation above says the equilibrium temperature of the Earth should be 245K, but where the mean surface temperature really is 290K.

Question for the learned assembly: What do you think explains the difference?

Hint 1: Recall that the calculation above assumed that *all wavelengths* present in the Planck function are radiated freely to space.

Hint 2: Now look at Figure 6.17 of the book.

Any ideas?

4 The Retention of Planetary Atmospheres

Why do some planets have atmospheres, while others don't? In explaining this, astronomers assume that when the solar system formed, all planets and moons had a gaseous atmosphere. The present state of affairs (some have them and others don't) is a consequence of *retention of atmospheres.* Some planets and moons were able to hold on to their atmospheres, and others didn't.

The plot thickens when you note that the planets that have atmospheres differ in the chemical composition of them. Jupiter and Saturn are primarily hydrogen, like the Sun (you'll learn that later in the semester). The Earth has an atmosphere that is primarily nitrogen and oxygen, with water vapor and also carbon dioxide as a "trace gas".

Mars and Venus have vastly different atmospheres. The atmosphere of Mars is far thinner than that of Earth, while that of Venus is far thicker. Yet in both cases they are predominantly carbon dioxide. How do we explain this vast variety?

5 A Tale of Two Speeds

We can use some of the physics we learned earlier in the semester. In fact, we can use the same arguments we used in understanding why the Sun formed a solar wind.

We have seen that the mean squared speed of particles in a gas is directly proportional to its temperature.

$$\frac{1}{2}m < v^2 >= \frac{3}{2}k_BT \tag{9}$$

$$\sqrt{< v^2 >} = \sqrt{\frac{3k_BT}{m}} \tag{10}$$

These equations say that the higher the temperature, the faster the particles are moving around. Remember that the "root-mean-squared speed" is slightly higher than the most probable speed. And remember that there is "tail of the distribution", consisting of particles with speeds much higher than the most probable speed. $\longrightarrow$ diagram on blackboard

Another speed we discussed was the orbital speed of a circular orbit of a small mass m around a large mass M.

$$V_c = \sqrt{\frac{GM}{r}} \tag{11}$$

Another related speed which we did not discuss is the escape speed V_{esc}. An object with this speed at a distance r from M will be on a parabolic (not elliptic) orbit.

$$V_c = \sqrt{\frac{2GM}{r}} \tag{12}$$

5.1 A Comparison of the Speeds on the Planetary Surface

$\longrightarrow$ diagram on blackboard, Maxwell-Boltzmann distribution with escape speed indicated.

Particles with speeds greater than the escape speed leave the planet and are lost to interplanetary space. You might think that is the end of them, and it doesn't matter if a planet loses a tiny fraction of the molecules in its atmosphere.

However, nature really wants a Maxwell-Boltzmann distribution, so collisions "repopulate" the tail of the distribution ($v > V_c$) and generate a Maxwell-Boltzmann distribution again. This is then lost to space because the speeds are greater than the escape speed.

The atmosphere of a planet thus has a slow leak. The higher the escape speed relative to the most probable thermal speed, the slower the leak.

Calculations show (these are done in the junior level astrophysics course in our department) that for a planet to retain (hold on to) an atmosphere for a time comparable to the age of the solar system, the escape speed has to exceed the rms thermal speed by about a factor of 5-6. This relation helps us understand the existence (or non-existence) of atmospheres around different solar system objects.