General Astronomy (29:61)
Fall 2012
Lecture 26 Notes, November 2, 2012

1 Two Speeds that Determine Retention of an Atmosphere

We can use some of the physics we learned earlier in the semester. In fact, we can use the same arguments we used in understanding why the Sun formed a solar wind.

We have seen that the mean squared speed of particles in a gas is directly proportional to its temperature.

$$\frac{1}{2}m < v^2 >= \frac{3}{2}k_B T \tag{1}$$

$$\sqrt{< v^2 >} = \sqrt{\frac{3k_B T}{m}} \tag{2}$$

These equations say that the higher the temperature, the faster the particles are moving around. Remember that the "root-mean-squared speed" is slightly higher than the most probable speed. And remember that there is "tail of the distribution", consisting of particles with speeds much higher than the most probable speed. $\longrightarrow$ diagram on blackboard

Another speed we discussed was the orbital speed of a circular orbit of a small mass m around a large mass M.

$$V_c = \sqrt{\frac{GM}{r}} \tag{3}$$

Another related speed which we did not discuss is the escape speed V_{esc}. An object with this speed at a distance r from M will be on a parabolic (not elliptic) orbit.

$$V_c = \sqrt{\frac{2GM}{r}} \tag{4}$$

1.1 A Comparison of the Speeds on the Planetary Surface

$\longrightarrow$ diagram in online notes, Maxwell-Boltzmann distribution with escape speed indicated.

Particles with speeds greater than the escape speed leave the planet and are lost to interplanetary space. You might think that is the end of them, and it doesn't matter if a planet loses a tiny fraction of the molecules in its atmosphere.

However, nature really wants a Maxwell-Boltzmann distribution, so collisions "repopulate" the tail of the distribution ($v > V_c$) and generate a Maxwell-Boltzmann distribution again. This is then lost to space because the speeds are greater than the escape speed.

The atmosphere of a planet thus has a slow leak. The higher the escape speed relative to the most probable thermal speed, the slower the leak.

Calculations show (these are done in the junior level astrophysics course in our department) that for a planet to retain (hold on to) an atmosphere for a time comparable to the age of the solar system, the escape speed has to exceed the rms thermal speed by about a factor of 5-6. This relation helps us understand the existence (or non-existence) of atmospheres around different solar system objects.

1.2 Criterion for Retention of a Planetary Atmosphere

The above arguments indicate a simple, rule of thumb way of determining whether a planet can retain an atmosphere over the several billion year history of the planet.

1. Calculate the escape speed from the surface of a planet, then divide by 6. This represents the estimate of how much faster the escape speed must be than the root-mean-square speed.

2. Pick a gas of interest, say molecular oxygen or CO2.

3. Write down the surface temperature of the planet, either calculated via the equilibrium temperature above, or (better) the observed surface temperature. Calculate the rms (root-mean-square) speed of these molecules.

4. Compare the speeds calculated in (1) and (3) above. If $\frac{V_{esc}}{6} > \sqrt{< v^2 >}$, then there is a good chance that the planet or moon has retained the atmosphere. If $\frac{V_{esc}}{6} < \sqrt{< v^2 >}$, then the atmosphere has probably escaped to space over the 4.5 billion year history of the solar system.

We can combine this kind of calculation for all the planets and many kinds of gas on a single graph. This was done in the famous 1966 edition of the textbook *Exploration of the Universe* by George A. Abell. Like much (but not all) of the material in that book, this argument is still valid. $\longrightarrow$ diagram in online notes, planetary escape speeds and molecular speeds.

1.3 Conclusions from the Atmospheric Retention Diagram

Let's look at this diagram and see what it tells us.

1. Let's start with the upper 2 lines corresponding to the rms speeds for hydrogen and helium. The points representing all the Jovian planets are well above these lines, indicating that these planets could retain hydrogen and helium atmospheres for a long time, and indeed they do.

2. Earth and Venus look "iffy" in this respect; they are right on the lines. Since the Earth and Venus do not have significant hydrogen or helium in their atmospheres, this suggests that our rule of thumb might be a little generous for holding onto an atmosphere.

3. Now let's look at the lines corresponding to the common atmospheric gases other than hydrogen and helium. This means water vapor, methane, nitrogen, oxygen (diatomic), and carbon dioxide. Notice that all three of these lines are fairly close on this diagram, meaning we can think of them as a group. We see that the Earth and Venus are well above this set of three curves, indicating that they can hold on to all of the gases, and indeed, they have carbon dioxide and nitrogen and (for the Earth) oxygen and water vapor.

4. Mars and Titan seem to be above the three lines, but not by a lot. We could contend that they have atmospheres, but less dense than the other planets. Furthermore, this diagram would seem to suggest that Mars and Titan could hang on to carbon dioxide more easily than the other gases.

5. Finally, Mercury and the Moon are the "born losers"; they don't have strong enough gravity to retain any of these gases. That is even more the case for the asteroid Ceres (we will see it in our field trip the week after next).

The appealing feature of this diagram is that, on the basis of a couple of fundamental and simple physics principles, we can sort out the main features of planetary atmospheres in the solar system. That is which ones have atmospheres, can what the chemical compositions of the atmospheres are.

2 The Concept of an Exosphere

Read the text on the bottom of p200 and the top of p201 of the textbook dealing with the concept of an *exosphere*. It's important. Here is why.

In the discussion above, I argued that a molecule with a speed greater than the escape speed from the planet would escape. At the surface of the Earth, that's not true. The reason for this is that in the Earth's atmosphere at sea level, molecules make it a very tiny distance before they collide with another molecule. The average distance a molecule (or atom) goes before undergoing a collision and ricocheting off in another direction is called the *mean free path*. The general formula for the mean free path is

$$l_{mfp} = \frac{1}{\sigma n} \tag{5}$$

where σ is the cross section for a collision, and n is the number density (number/unit volume) of the targets. (Make sure you understand that the right hand side has units of length).

As the book states, that mean free path of molecules at sea level is about 40 nm, not exactly all the way to outer space.

However, as we go way up in the atmosphere, the situation changes. As the equation states, as the density drops, the mean free path increases. If you go high enough up in the atmosphere, the mean free path becomes comparable to the thickness of the atmosphere, and a molecule does have a good change of making it to space and escaping. This region of the atmosphere is called the *exosphere*. The book shows that in the Earth's atmosphere, this occurs at an altitude of about 500km.

The arguments given in the prior sections are valid; just keep in mind that the temperature to be used in the formulas is the temperature in the exosphere, not the planetary surface. The height of the exosphere will clearly vary from planet to planet.

3 A Cautionary Remark

It is worth emphasizing that the physics of atmosphere presented here (and in the book) is pretty simple. We haven't considered how the temperature depends on height, the effect of sunlight on the atmosphere, or a lot of other things. Given this, we should not be surprised if we find some discrepancies when we start studying these atmospheres in detail, as we have been able to do with spacecraft and modern instrumentation.

4 Oddities in the Atmospheric Retention Diagram

Somebody who looks closely at this diagram might note a couple of weird features.

First of all, it looks like Pluto is right up there comparable Jovian planets in its ability to hold onto a hydrogen or helium atmosphere, yet many students in this course know that Pluto has little or no atmosphere. Why is there such a large discrepancy in this case?

Second, Mars and Titan look above the three lines of common gases by about the same amount. This would seem to indicate that they should have similarly dense atmospheres. However, in reality Mars has a very tenuous atmosphere, while Titan (as we will learn) has a very dense one. What is causing this difference?

It turns out that one of these can be explained trivially, while the other remains unexplained and may be telling us something intriguing about the solar system.

With regard to Pluto, this diagram, and the book it came from, is showing its age. In 1966, we had very little knowledge about the properties of Pluto, most of all its mass. We didn't really know the mass of Pluto until 1980, when its satellite Charon was discovered. As a result, the estimate for the mass of Pluto in *Exploration of the Universe* was way off. If you use modern, accurate values for the mass and radius of Pluto, you get an escape speed of 1.21 km/sec. If you divide this by 5 to plot it on the diagram, you get a number about 0.24 km/sec, which puts it down comparable to the Moon and Mercury, so it is now understandable.

The case of Mars is more intriguing. Although we do not know if this is the answer, the resolution might be due to the fact that Mars is much closer to the Sun than Titan. This means that the "thrust" of the solar wind is much stronger (by about a factor of 100), and some scientists think that a more powerful solar wind early in the history of the solar system "sandblasted" an earlier, denser Martian atmosphere. We may find out the answer to this with later generations of Martian lander spacecraft.

5 Internal Heat Sources

In discussing factors which determine the surface temperature of a planet, we discussed sunlight. However, internal sources of heat were mentioned from the class. These may indeed be important in satellites of the outer planets, such as Ganymede, Europa, and Enceladus. The internal heat source may come from tidal flexing of the satellites.