General Astronomy (29:61)
Fall 2012
Lecture 29 Notes, November 9, 2012

1 The Physics of the Earth's Atmosphere

The Perfect Gas Law says

$$P = \frac{\rho k_B T}{\mu m_p} \tag{1}$$

In the case of N2, $\mu = 28$, and for O2, $\mu = 32$. If we form an average weighted by their abundances in the atmosphere, we have $\mu = 29$ (see book, p216).

Let's work out the case for the surface of the Earth. The density of air at sea level is 1.3 kg/m^3. Let's pick 290K as a typical temperature. We then have

$$P = \frac{\rho k_B T}{\mu m_p} \tag{2}$$

$$P = \frac{1.3(1.381 \times 10^{-23})290}{29(1.67 \times 10^{-27})} \tag{3}$$

$$P = \frac{5.22 \times 10^{-21}}{4.84 \times 10^{-26}} \tag{4}$$

$$P = 1.078 \times 10^5 \text{ N/m}^2 \tag{5}$$

$$\tag{6}$$

A unit of pressure in the SI system is also called a *Pascal*. By convention, a pressure of 1.013×10^5 N/m^2 is referred to as one atmosphere, and is taken as the average pressure at sea level.

The atmospheric pressure at the same altitude above sea level is remarkably constant across the Earth. Variations of 1 percent are associated with major weather systems. A strong hurricane would have a pressure in the eye of about 92 % of an atmosphere.

1.1 The vertical extent of the Earth's Atmosphere

How high does the Earth's atmosphere extend? This is closely related to the question, how high up is outer space? We can figure this out with basic physics considerations.

Let's consider a cylinder of air that goes straight up through the atmosphere. $\longrightarrow$ diagram on blackboard. The pressure in the cylinder changes as a function of

z, the altitude above the surface of the Earth. The pressure goes down as z goes up. The reason is as follows.

Consider a cross section of the cylinder at an altitude z, of area A. The force down on this area is equal to the weight of the overlying area. Air is a fluid, like a liquid, and has mass. This downward force is $F = Mg$, where M is the mass of all the air at higher altitudes.

If there were no force opposing this weight, the overlying column of air would fall. The opposing force is provided by airpressure, $F = PA$. (Remember that pressure times area equals force). In equilibrium these forces balance, so the net force is zero, and nothing accelerates. This equilibrium is called *hydrostatic equilibrium*.

This is nice to know, but it still doesn't allow us to solve for $P(z)$, since we don't know how the mass is distributed above the height z. But let's think this out.

Let the pressure at height z_1 be $P(z_1)$. Once again, $P(z_1)A = M(z_1)g$, where $M(z_1)$ is the mass of air above z_1. Now let's consider the situation at a level z_2 slightly higher, $z_2 = z_1 + dz$. Now,

$$(P(z_2) - P(z_1))A = dPA = (M(z_2) - M(z_1))g = -dMg \tag{7}$$

The negative sign in the last expression means there is less mass above z_2 than above z_1. Now,

$$dM = \rho A dz \tag{8}$$

We have picked the density to be a constant over the tiny interval dz. Now let's follow these ideas through.

$$dPA = -\rho A g dz \tag{9}$$

$$dP = -\rho g dz \tag{10}$$

$$\frac{dP}{dz} = -\rho g \tag{11}$$

In the language of Calculus, the derivative of the pressure with respect to altitude equals the density times g (times -1). We can solve this equation for the pressure as a function of altitude if we know the density as an altitude.

Remember from above that

$$P = \frac{\rho k_B T}{\mu m_p} \tag{12}$$

This says that there is a direct relationship between the pressure and the density. Let's make an *approximation* in which we say that the temperature T is roughly constant as a function of height. This is called the *isothermal approximation*. In this case, pressure equals density times a constant, and we can solve our equation easily. Let's follow through the steps.

$$\frac{dP}{dz} = -\rho g \tag{13}$$

$$\frac{dP}{dz} = -\left(\frac{k_B T}{\mu m_p}\right) \rho g \left(\frac{\mu m_p}{k_B T}\right) \tag{14}$$

$$\frac{dP}{dz} = -P\left(\frac{\mu m_p g}{k_B T}\right) \tag{15}$$

$$\frac{dP}{dz} = -\left(\frac{\mu m_p g}{k_B T}\right) P \tag{16}$$

$$\frac{dP}{dz} = -\alpha P \text{ with } \alpha \equiv \left(\frac{\mu m_p g}{k_B T}\right) \tag{17}$$

What's the solution to this equation?

$$P(z) = P_0 e^{-\alpha z} \tag{18}$$

It actually is a little more intuitively appealing if we define

$$\alpha = \frac{1}{H}, \text{ with } H \equiv \frac{k_B T}{\mu m_p g} \tag{19}$$

The height H is called the isothermal scale height. It tells us the altitude interval over which the atmospheric pressure falls to 37 % of its value at sea level.

So we have as our expression for the atmospheric pressure as a function of altitude,

$$P(z) = P_0 e^{-z/H} \tag{20}$$

This is the *bottom line* equation that you should know. You can substitute in values for the Earth's atmosphere, and find that $H = 8.41$ km, a little different from the value in the book.

We can plot up this function, and see what the atmospheric pressure is as a function of altitude $\longrightarrow$ see online notes.

The two vertical lines indicate altitudes of 9.75 km (32000 ft) and 11.6 km (38,000 ft), which are standard cruising altitudes for jet airliners. These plots illustrate the amazing fact that when we fly on commercial airliners, the outside air pressure is way smaller than it is at sea level. We are a good ways towards outer space! A related realization is that the atmosphere of the Earth is really a very thin film of dense gas above the Earth. To appreciate this, think about the radius of the Earth.

These equations can be used for planetary atmospheres throughout the solar system, and even for the atmospheres of stars.

1.2 Layers of the Earth's Atmosphere

There are a number of layers of the Earth's atmosphere. The distinctions between these are quite meaningful. See the discussion on pp 216 and 217 of the book.

- Troposphere: this is the level of the atmosphere in which we live, and to which the above calculation is relevant. It is a region in which heat transport from the surface of the Earth to the cold upper atmosphere is carried by convection.
- Stratosphere: this is the layer immediately above the troposphere. Convection is not typical there, so the atmosphere is stably stratified. We also speak of it as "convectively stable". Jet airliners fly in the upper troposphere or close to the troposphere-stratosphere boundary.
- Higher layers such as the mesosphere, thermosphere, and exosphere. A **very important distinction** comes at altitudes greater than 100 kilometers. At such altitudes, we are above so much of the atmosphere that there is not effective absorption of the ultraviolet light from the Sun. This UV radiation ionizes the nitrogen and oxygen atoms, so the gas is partially ionized. This produces the **ionosphere**. $\longrightarrow$ see diagram in online notes showing the density of ionized atoms as a function of altitude. The ionosphere is our nearest, naturally-occurring example of a form of matter called a *plasma*, which is the most common form of matter in the universe.

1.3 The Temperature Profile in the Earth's Atmosphere

Figure 9.5 of the textbook shows the measured temperature profile in the Earth's atmosphere. $\longrightarrow$ See also plot in online notes.

We can see some interesting features in this plot. First of all is the fall in the temperature with increasing height in the troposphere. In the stratosphere this trend reverses, and the temperature increases to a maximum at around 50 km. This stratospheric increase has an interesting explanation. It is due to ozone molecules absorbing solar ultraviolet light. Some of this energy goes into heating the stratospheric gas. This stratospheric temperature increase is a consequence of the same process that is protecting us from harmful UV light.

1.4 Auroras

Another important phenomenon related to the upper atmosphere is auroras. Auroras are glowing lights that are seen in the northern sky. $\longrightarrow$ see pictures in online notes and diagrams. They are rare here in Iowa, but occur almost nightly in the north (like central Alaska). They occur at the lower levels of the ionosphere, say 100 - 140 km above the surface of the Earth. They are caused when energetic electrons from far out in space are beamed down into the Earth's atmosphere. These electrons ionize and excite atoms in the atmosphere (oxygen and nitrogen), causing the air to glow. They are a hint that there are interesting things happening out in space beyond the Earth's atmosphere.