General Astronomy (29:61)
Fall 2012
Lecture 37 Notes, December 03, 2012

1 The Jovian Planets

The book lumps all four of the outer planets, Jupiter, Saturn, Uranus, and Neptune into the category of the "Jovian Planet". A somewhat more discriminating classification would restrict membership in the Jovian planets to Jupiter and Saturn. Uranus and Neptune are sometimes called *ice giants*. We'll see later (or see for yourself now in Figure 10.12) that there are important differences in the structures of Jupiter and Saturn on one hand, and Uranus and Neptune on the other.

2 Jupiter and Saturn

Let's start with Jupiter and Saturn. $\longrightarrow$ Look at pictures in online notes and diagrams. Jupiter and Saturn are huge compared to any and all of the terrestrial planets. These differences reflect the different places in the solar system that they formed.

Jupiter and Saturn are predominantly formed of hydrogen and helium, the "star stuff" that makes up the Sun and all the other stars. $\longrightarrow$ Look at pictures in online notes and diagrams for arguments as to how we know this.

3 The Interior Structures of Jupiter and Saturn

Since Jupiter and Saturn are made of hydrogen and helium, this is a gas for the atmosphere that we see, and even deeper below the cloud layers where we can't see. Let's follow some mathematical arguments and see where this leads us.

The following is a variation of the derivation in Section 10.2.1 of the book.

When we discussed the atmosphere of the Earth, we discussed the property of *hydrostatic equilibrium*. As you go up higher in the atmosphere (larger value of altitude z), the change in the pressure is given by

$$\frac{dp}{dz} = -\rho g \tag{1}$$

Now we want to know how the pressure changes as we go outwards from the center of the planet to the surface and beyond, given by the radial coordinate r. If we are considering the change in pressure in spherical coordinates, can we just use the same formula as above and just swap in r for z?

It turns out in this case you can, although in general you can't be so quick in changing variables from Cartesian to spherical.

We then have an expression for the change in pressure as a function of distance r from the center of the planet,

$$\frac{dp}{dr} = -\rho g \tag{2}$$

It is easy enough to write down an expression for g. If the distribution of mass is spherically symmetric (a good approximation, even for rapidly rotating and flattened planets like Jupiter and Saturn), then

$$g = \frac{GM}{r^2} \tag{3}$$

Where $M = M(r)$ is the mass *interior* to a spherical surface of radius r. This equation tells us that $g = g(r)$, it is a function of r. M is given by the distribution of density (ρ) inside the planet.

Let's bring back our equation.

$$\frac{dp}{dr} = -\rho g \tag{4}$$

Now we really **do** seem to be stuck. To solve this equation for $p(r)$, we need to know how the density (ρ) varies with r. However, surely $\rho(r)$ depends on $p(r)$. How do we get out of this?

It turns out there are ways of approaching this problem.

However, for the moment, we will follow the example of the book and make a real gross approximation. It will give us some insight, and perhaps give us ballpark estimates for the value of the pressure in the interior of Jupiter and Saturn.

Let's assume that the density of Jupiter in the interior is uniform, and equal to the mean density $\bar{\rho}$. The total mass is given by

$$\frac{4}{3}\pi R^3 \bar{\rho} = M \tag{5}$$

$$\bar{\rho} = \frac{3M}{4\pi R^3} \tag{6}$$

In the interior of the planet,

$$g(r) = \frac{GM(r)}{r^2} \tag{7}$$

$$M(r) = \frac{4}{3}\pi r^3 \bar{\rho} \text{ so} \tag{8}$$

$$g(r) = \frac{4\pi G \bar{\rho} r^3}{3r^2} \text{ and} \tag{9}$$

$$\frac{dp}{dr} = -\frac{4\pi}{3} G \bar{\rho}^2 r \tag{10}$$

This equation tells us that in this approximation, the pressure drops rapidly as we approach the edge of the planet. $\longrightarrow$ drawing on blackboard.