General Astronomy (29:61)
Fall 2013
Lecture 5 Notes , September 6, 2013

1 Astronomical Fundamentals of Time

Astronomical phenomena help define our idea of time, and the measurement of time is tied inextricably with astronomy. Our fundamental terms for units of time, the day, and month, and the year, all relate to astronomical cycles.

1.1 The Day

You might think the definition of the day is ridiculously simple. However, in reality you have to be careful on how you define it. The book defines the day as *"the interval between successive transits of a celestial object"*.

The most common (and seemingly only) choice for this celestial object is the Sun. This leads to the definition of the **Solar Day**.

However, you could also choose a star, such as Vega, and measure the day to be the time between successive transits. This is the definition of the **Sidereal Day**. Aren't these the same?

No they aren't. To see why, and estimate the difference, look at Figure 1.11. The crucial point is that the Sidereal Day is the rotation period of the Earth in an *inertial reference frame* defined by the distant stars. The Solar Day is the rotation period in a rotating reference frame in which one axis is defined by the direction from the Earth to the Sun.

At the end of one sidereal day, the Earth has to still turn through a little angle to be back to the direction of the Sun.

1.2 Numbers, numbers, numbers

Let's work out the difference between the sideral day and the solar day. One way of doing this is with the derivation in Equation 1.1 - 1.5. Here's an easier (although not as exact) way of seeing the same thing. Look at Figure 1.11.

After 1 sidereal day, the Earth has advanced in its orbit by an angle given by

$$\Delta\theta = \omega_E P_{sid} \tag{1}$$

Where ω_E is the *angular speed* (radians/sec) of the Earth's orbital motion around the Sun ($\omega_E = \frac{2\pi}{P_E}$), and P_{sid} is the length of the sidereal day.

The time Δt it takes for the Earth to rotate through this angle is (approximately)

$$\Delta\theta \simeq \omega_{sid}\Delta t \text{ so} \tag{2}$$
$$\omega_{sid}\Delta t \simeq \omega_E P_{sid} \tag{3}$$
$$\Delta t \simeq \frac{\omega_E}{\omega_{sid}} P_{sid} \tag{4}$$
$$\Delta t \simeq \frac{P_{sid}}{P_E} P_{sid} \tag{5}$$

To the extent that P_{sid} and P_{sol} are the same, this gives the same result as Equation (1.5) of the book, 1/365 of a day, or about 4 minutes.

The reason I used the "nearly equal" sign $\simeq$ rather than the mathematical identity symbol $=$ is that my calculation assumed that the Earth stayed put in its orbit while it rotates through the angle $\Delta\theta$.

The bottom line of this is the very important result that the sidereal day is 4 minutes shorter than the solar day. This fact provides another way of understanding seasonal variations in the appearance of the night sky.

You should look at the alternative (and more rigorous) derivation of this result on p18 and 19 of the book.

1.3 Choice of the Solar Day

We choose to organize our lives using the solar day because human beings are up and around during daylight. However, a major complication arises. *The solar day is not a constant; it varies through the year.*

2 The Mean Solar Day and the Equation of Time

If you measured the length of the solar day with a precise clock, you would find that it varies through the year. You would also find that the Sun does not transit at noon every day. By contrast, the length of a sidereal day is vastly more constant. What is going on?

There are two effects responsible for the changing length of the day. To start with, the solar day is given by the sidereal day (the true, inertial rotation period of the Earth), plus "something extra". That something extra is the time it takes the Earth to rotate through the angle that the Sun has moved to the east during a day (look again at Figure 1.11). This angle is roughly a degree.

If the Earth were in a circular orbit, and the obliquity of the ecliptic were zero, then this "something extra" would be constant. In this case, the solar day would be as constant as the sidereal day. However, those conditions are not true. Let's see what the consequences are.

1. *Obliquity of the Ecliptic.* The Earth rotates parallel to its equator. Celestial objects move across the sky on diurnal circles parallel to the celestial equator. For this reason, it is eastward component of the Sun's daily motion relative to the equatorial coordinate system that matters.

   The fact that the ecliptic is inclined at 23.5° with respect to the celestial equator means that this eastward daily motion changed through the year. It is a maximum at the time of the solstices, and a minimum at the time of the equinoxes (see Figure in slides and pictures).

2. *Nonuniform Motion of the Sun Along the Ecliptic* On the other hand, even if the obliquity of the ecliptic were zero, the solar day would vary through the year because the daily angular speed of the Sun along the ecliptic changes.

   The reason for this is *Keplers's 2nd Law of Planetary Motion.* We will discuss this in detail in the next chapter. Let's pick the main features now.

   The motion of the Earth about the Sun is not a circle, but an ellipse. The Sun is not at the center of the ellipse, but offset at one of the focuses (see Figure in lecture notes). Kepler's 2nd Law says that the Earth moves at a small angular speed when it is far from the Sun, and at a high angular speed when it is close. *See diagram in slides.* This means we see the Sun shifting against the background stars rapidly when we are closest to the Sun (*perihelion*), and slowly when furthest from the Sun (*aphelion*).

In practice, both of these effects are present, and cause the length of the solar day to vary in a complicated way through the year.

2.1 Mean Solar Day

It would be horribly impractical to run daily affairs according to solar time, or specifically, *apparent solar time*, which is time based on the true position of the Sun in the sky.

For this reason, civil time is based on *Mean Solar Time*, which is that every day is taken to be equal to the average of the solar day during the year. Every day is defined to have the same length, so the length of a minute and hour every day is the same.

While this makes sense, it leads to an interesting astronomical phenomenon called *The Equation of Time*. Remember that the Sun transits at noon *Local Apparent Solar Time*. Because the length of the apparent solar day varies during the year, this can be before or after noon, mean solar time. The difference between the two is called *The Equation of Time*, and is equal to Apparent - Mean Solar Time.

A very nice plot is given in Figure 1.12. This figure shows how much of the The Equation of Time is due to the obliquity of the ecliptic, and how much due to the eccentricity of the Earth's orbit. You can see that the Sun will transit up to 17 minutes early or late relative to apparent solar time (civil time).

2.2 The Analemma

The Equation of Time takes on different values at different times of year. The Sun is at different declinations at different times of year, so the amount by which the apparent solar time leads or follows civil time changes with the declination of the Sun. Plotting up the value of the Equation of Time versus declination leads to an intriguing mathematical figure called the *Analemma* (See Figure 1.12). The Analemma can actually be photographed over the course of a year (see attached picture).

3 The Year

Read section 1.6 about the year. A lot of the complication about calendars arises because the year (technically, the —em tropical year) is not an integer number of mean solar days, but instead = 365.24219 mean solar days.

4 Orbits in the Solar System...the start

We now move on to Chapter 2. This chapter is mainly focused on historical information. While all that is nice, we aren't going to spend time in class on it. Read it yourself.

However, this chapter does have some fundamentals about *orbits*. Mainly, we will develop a vocabulary that we can use when we start discussing the physics of orbits in Chapter 3.

What are the concepts of interest to us?

Let's start with the definition of an **orbit**. An orbit is a path taken by a celestial object through space. Mathematically, we express it as the spatial coordinates of the celestial object *as a function of time.*

4.1 Orbits of the Major Planets

Our principal interest is in the orbits of the *major planets* about the center of the solar system

Isn't the center of the solar system the center of the Sun? Close, but not quite. We'll get to that later.

We will find out that the same physical laws that govern the orbits of the major planets also govern minor solar system objects like asteroids, comets, minor planets, and spacecraft.

4.2 Some real basic properties of orbits

There were some truths about orbits that were figured out very early, before the development of the science of physics as we understand it. These were probably known well to the ancient Greeks. If you observe the sky closely, and think about what you see, you would be able to figure it out. Now, we know they are consequences of fundamental laws of physics.

1. Orbits are plane figures. The celestial object stays in a plane as it moves (not like a knuckleball in baseball).

2. The center of the solar system (think center of the Sun!) is also in this plane.

3. The orbits are *periodic.* The celestial object comes back to its original place, and the motion repeats after a certain amount of time called the period. Physicists are very interested in periodic phenomena.

4.3 Figuring out the orbits of planets when we are on one.

One of the reasons it took a long time to deduce the true laws of planetary orbits (Kepler's Laws) is that we are observing the other planets from our own. The Mayans had an outstanding knowledge of the motion of the planets in the sky (the so-called synodic motion), but never deduced the planetary motions in an *inertial reference frame.*

5 Diagrams for Planetary Orbits

It is convenient to distinguish between planets whose orbits lie inside the orbit of the Earth, or *inferior planets* and those who orbits lie outside that of Earth, called *superior planets*.

5.1 Inferior Planets

Look at figure 2.9 of the textbook. The main "events" associated with an inferior planet are:

- inferior conjunction
- superior conjunction
- elongation
- Think how you would determine the distance (in astronomical units) of an inferior planet.

5.2 Superior Planets

Look at figure 2.8 of the textbook. The main "events" associated with a superior planet are:

- opposition
- conjunction
- quadrature
- elongation
- Be sure you understand how do determine the distance (in astronomical units) of a superior planet.