Relativistic boosted potential and its nonlocality

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We employ a relativistic formalism which based on an exact realization of the symmetry of the Poincaré group [1]. There are some identifications of relativistic potential corresponding to the original nonrelativistic potential [2]. Using the Coester-Pieper-Serduke [3] method one solves a nonlinear equation [4] to obtain the relativistic potential.

$$\begin{aligned}&4mv(\vec{p},\vec{p}\,')\\&= v^r(\vec{p},\vec{p}\,')\left(2\sqrt{p^2+m^2}+2\sqrt{p'^2+m^2}\right)+\int d\vec{p}\,''v^r(\vec{p},\vec{p}\,'')v^r(\vec{p}\,'',\vec{p}\,'),\end{aligned} \tag{1}$$

where $v(\vec{p},\vec{p}\,')$ and $v^r(\vec{p},\vec{p}\,')$ are nonrelativistic and relativistic potential, respectively. This equation of v^r is directly solved by the iteration method [4] in the partial wave.
As well as solving t-matrix without partial wave decomposition [5] one can solve Eq.(1) introducing angle between $\vec{p}$ and $\vec{p}\,'$ ($x'=\cos\theta=\hat{p}\cdot\hat{p'}$) to get the relativistic potential v^r.

$$\begin{aligned}&4mv(p,p\,',x') = v^r(p,p\,',x')\left(2\sqrt{p^2+m^2}+2\sqrt{p'^2+m^2}\right)\\&+\int_0^\infty dp\,''\int_{-1}^1 dx''\int_0^{2\pi} d\varphi'' v^r(p,p\,'',y)v^r(p\,'',p\,',x''),\end{aligned} \tag{2}$$

where $x'=\hat{p}\cdot\hat{p'}, x''=\hat{p}''\cdot\hat{p}'$, and $y=\hat{p''}\cdot\hat{p}=xx''+\sqrt{1-x^2}\sqrt{1-x''^2}\cos\varphi''$.
Similarily we can solve the boosted potential v^r_q as

$$\begin{aligned}&4mv(p,p\,',x') = v^r_q(p,p\,',x')\left(2\sqrt{p^2+m^2+q^2/4}+2\sqrt{p'^2+m^2+q^2/4}\right)\\&+\int_0^\infty dp\,''\int_{-1}^1 dx''\int_0^{2\pi} d\varphi'' v^r_q(p,p\,'',y)v^r_q(p\,'',p\,',x''),\end{aligned} \tag{3}$$

where q is the total mometum of the pair.
Now, we have a question how large the nonlocality of the boosted potential is. We introduce the function $N(\vec{\rho})$ of the nonlocality in the NN interaction [6]

$$N(\vec{\rho};v)\equiv\frac{1}{(2\pi)^3}\int d\vec{K}e^{i\vec{K}\vec{\rho}}\frac{v(K,K,1)}{v(0,0,1)} \tag{4}$$

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where $\vec{K} \equiv (\vec{p} + \vec{p}\,')/2$, $\vec{\rho} = \vec{r}\,' - \vec{r}$, and $\vec{r}$ and $\vec{r}\,'$ are the relative coordinate of initial and final states, respectively.
In the case of local potential, the nonlocality is trivially given as [6]

$$N(\vec{\rho}; \text{local potential}) = \delta^3(\vec{\rho}) \tag{5}$$

As a measure for the width one can use

$$\langle \rho^2 \rangle^{1/2} \equiv \left(\int d\vec{\rho} \rho^2 N(\rho; v) \right)^{1/2} \tag{6}$$

In Fig 1. the width $\langle \rho^2 \rangle^{1/2}$ is demonstrated to the boost momentum q. The nonlocality will weaken by increasing boost momentum q.

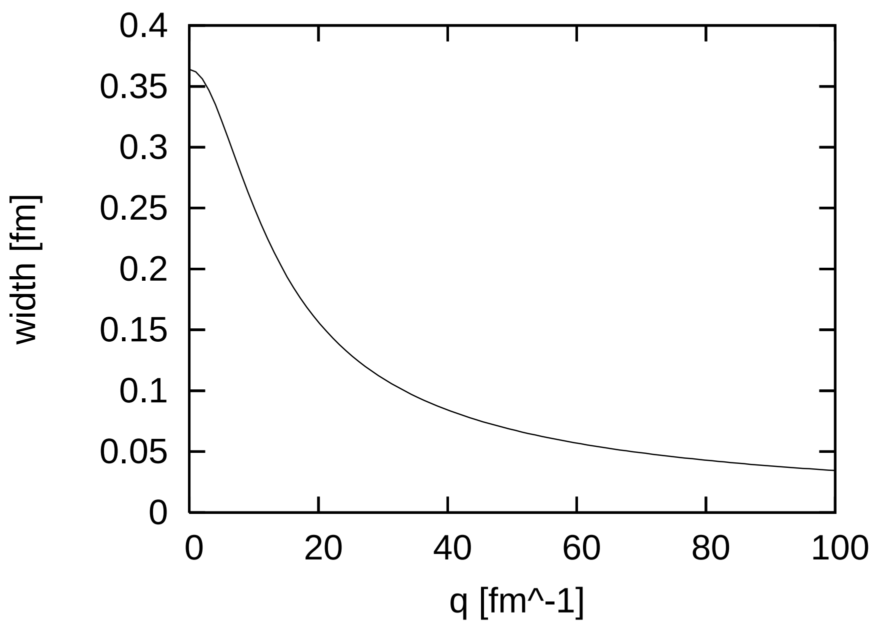

Figure 1: Width of the nonlocality. Malfliet-Tjon version V potential [7] is chosen as the original local nonrelativistic one.

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