Exchange current contributions in null-plane quantum models of elastic electron deuteron scattering

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We investigate exchange current contributions to elastic electron-deuteron scattering using exactly Poincaré invariant quantum mechanics with a null-plane kinematic symmetry. Our model exchange current is motivated by one-pion-exchange physics. Exact current conservation and current covariance are satisfied by using this current to compute a linearly independent set of current matrix elements. The remaining current matrix elements are generated from the independent matrix elements using the covariance and current conservation constraints. The presence of the exchange current increases the sensitivity to the choice of independent current matrix elements. Two choices of independent matrix elements that have distinct motivations lead to a good description of all of the elastic scattering observables for momentum transfers below 8 $(\mathrm{GeV})^2$.

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I. INTRODUCTION

The scattering of electrons from nuclei in the impulse approximation is based on the assumptions that the cross section can be accurately calculated in the one-photon-exchange approximation and the nuclear current operator is a sum of single-nucleon current operators. Unfortunately, the dynamical constraints imposed by current conservation and current covariance cannot be satisfied with one-body current operators. In addition to the dynamical contributions to the current required by covariance and current conservation there are dynamical contributions to the nuclear current operator due to the exchange of charged particles and eliminated degrees of freedom. All of these effects require that the nuclear current operator has non-vanishing many-body contributions. Because the constraints on the current operator are dynamical, the structure of the current operator is representation dependent.

In a theory satisfying cluster separability the current operator has a well-defined cluster decomposition. Single-nucleon current operators and cluster properties fix the one-body contribution to the current operator. The additional many-body contributions to the current operator are not as well understood. The result is that the interpretation of electron-scattering experiments involves theoretical ambiguities that must be resolved in order to use electron scattering as a precision tool for understanding nuclear structure and dynamics.

Elastic electron-deuteron scattering is the simplest reaction where these questions arise [1]. In this paper we investigate two-body current operators in a formulation of the dynamics based on Poincaré-invariant quantum mechanics with a null-plane kinematic symmetry[2] [3][4]. We investigate the one-pion exchange contribution to the current. The long-range part of realistic nucleon-nucleon interactions includes a one-pion-exchange contribution. There is an additional short-range part that is modeled to fit measured nucleon-nucleon scattering cross sections. Our model exchange current is constructed using the same long-range pion-exchange interaction that is used in model two-body interactions. In this work we demonstrate that the addition of this two-body current operator to the impulse current operator provides an excellent description of all of the elastic electron-deuteron observables over a wide range of momentum transfers.

There are several theoretical approaches for calculating elastic electron-deuteron scattering observables. In all cases the problem is to construct matrix elements of a conserved covariant current operator

$$\langle P+Q,\mu',d|I^{\nu}(0)|P,\mu,d\rangle \tag{1.1}$$

for a range of space-like momentum transfers, Q, where P and $P+Q$ are the initial and final deuteron four momenta. The input to equation (1.1) is the deuteron eigenstates, a dynamical representation of the Lorentz group to transform the deuteron eigenstates to different frames related by the four-momentum transfer, and a current operator for the two-nucleon system. For the deuteron the current is a sum of one-body and two-body operators. The one-body current operators are fixed by covariance, current conservation, and nucleon form factors. Elastic electron-deuteron scattering has been studied by many authors [5][6] [7][8] [9][10] [11] [12][13][14] [15][16][17] using a variety of different methods and assumptions. These have been reviewed elsewhere [15] [18].

While true impulse approximations, that replace the current in (1.1) by a sum of single-nucleon currents, necessarily violate current conservation and current covariance, there are "generalized impulse approximations" in each formalism that use the sum of single-nucleon current matrix elements as input to generate approximations that enforce current conservation and current covariance. These "generalized impulse approximations" typically provide a qualitative description of all three elastic electron-deuteron scattering observables, however a quantitative agreement requires additional corrections. Such an agreement has been achieved in some formalisms [9][8]. The presence of a one-pion exchange "pair" current [19] or a related "contact current" [20] is a common element in corrections that explain a good part of the difference between the "generalized impulse approximations" and experiment.

In this paper we construct an exactly Poincaré invariant dynamical model of the two-nucleon system and a model of a covariant one-pion-exchange current operator. The dynamical representation of the Poincaré group is constructed using precision nucleon-nucleon interactions [21] [22] that are fit to scattering data. Current conservation, current covariance and discrete symmetries are used to identify linearly independent current matrix elements. The one-body contributions to the linearly *independent* current matrix elements are fixed by cluster properties. Our model exchange current is added to these independent current matrix elements. The remaining current matrix elements are generated using the dynamical constraints of current conservation and current covariance.

The Poincaré invariant dynamical models in this paper are formulated in a representation with a null-plane kinematic symmetry [2] [3][4]. Unlike models based on null-plane quantum field theory [23][24], where few-degree of freedom approximations break rotational covariance, the class of quantum mechanical models that we consider are *exactly* rotationally invariant. The null-plane treatment of electron scattering has a number of notable advantages:

1. For electron scattering the momentum transfer, Q, is spacelike, so the orientation of the null plane can be chosen so the +-component, $Q^+ := Q^0 + Q^3$, of the momentum transfer is zero. When $Q^+ = 0$ all current matrix elements can be constructed from a maximal independent set of matrix elements of the + component, $I^+(0) := I^0(0) + I^3(0)$, of the current:

$$\langle \tilde{\mathbf{P}}', \mu', d | I^+(0) | \tilde{\mathbf{P}}, \mu, d \rangle \qquad \tilde{\mathbf{P}} := (P^+, P_1, P_2). \tag{1.2}$$

2. There is a three-parameter subgroup of Lorentz boosts that leaves the null-plane invariant. Since these boosts form a subgroup, boosting does not lead to Wigner rotations. It follows that matrix elements of $I^+(0)$ with $Q^+ = 0$ and suitably normalized initial and final states are independent of frames related by null-plane boosts:

$$\langle \tilde{\mathbf{P}}''', \mu', d | I^+(0) | \tilde{\mathbf{P}}'', \mu, d \rangle = \langle \tilde{\mathbf{P}}', \mu', d | I^+(0) | \tilde{\mathbf{P}}, \mu, d \rangle. \tag{1.3}$$

This means that matrix elements of $I^+(0)$ are equal to Breit-frame matrix elements with the same null-plane spin magnetic quantum numbers.

3. A consequence of property two is that the *on-shell one-body* contributions to matrix elements of $I^+(0)$ *exactly* factor out[4] of the corresponding nuclear current matrix elements.

$$\langle \Psi, \tilde{\mathbf{P}} + \tilde{\mathbf{Q}} | I_i^+(0) | \Psi, \tilde{\mathbf{P}} \rangle =$$

$$\langle \tilde{\mathbf{p}}_{i0} + \tilde{\mathbf{Q}} | I_i^+(0) | \tilde{\mathbf{p}}_{i0} \rangle \int d\tilde{\mathbf{p}}_1 \cdots d\tilde{\mathbf{p}}_n \langle \Psi, \tilde{\mathbf{P}} + \tilde{\mathbf{Q}} | \tilde{\mathbf{p}}_1 \cdots \tilde{\mathbf{p}}_i + \tilde{\mathbf{Q}} \cdots \tilde{\mathbf{p}}_n \rangle \langle \tilde{\mathbf{p}}_1 \cdots \tilde{\mathbf{p}}_i \cdots \tilde{\mathbf{p}}_n | \Psi, \tilde{\mathbf{P}} \rangle. \tag{1.4}$$

This means that the matrix elements of the one-body contributions to $I^+(0)$ can be expressed as sums of products of Breit-frame nucleon matrix elements multiplied by functions of Q^2 that only depend on the initial and final wave functions.

This formalism has the advantage that it is possible to satisfy exact rotational covariance and the impulse contribution to the independent current matrix elements involves only experimentally observable on-shell single-nucleon matrix elements.

II. NULL-PLANE KINEMATICS

In this section we introduce our notation and discuss the null-plane kinematic subgroup. The null plane is the three-dimensional hyperplane of points tangent to the light cone satisfying the condition

$$\{x | x^+ := t + \mathbf{x} \cdot \hat{\mathbf{e}}_3 = 0\}. \tag{2.1}$$

The null-plane components of the four vector x^μ are

$$x^{\pm} := t \pm \mathbf{x}\cdot\hat{\mathbf{e}}_3 \qquad \mathbf{x}_\perp = (\mathbf{x}\cdot\hat{\mathbf{e}}_1, \mathbf{x}\cdot\hat{\mathbf{e}}_2). \tag{2.2}$$

Four vectors x^μ can be represented by 2×2 Hermitian matrices in the null-plane components of x^μ:

$$X = \begin{pmatrix} x^+ & x_\perp^* \\ x_\perp & x^- \end{pmatrix} = x^\mu \sigma_\mu \qquad x_\perp = x_1 + ix_2 \qquad x^\mu = \frac{1}{2}\mathrm{Tr}(\sigma_\mu X) \tag{2.3}$$

where $\sigma_\mu = (I, \boldsymbol{\sigma})$ and $\boldsymbol{\sigma}$ are the two $\times$ two Pauli matrices. Since the determinant of X is minus the square of the proper time, $-x^2$, if Λ is a complex 2×2 matrix with unit determinant then the transformation

$$X \to X' = \Lambda X \Lambda^\dagger \tag{2.4}$$

defines a real Lorentz transformation.

$$\Lambda^\mu{}_\nu = \frac{1}{2}\mathrm{Tr}(\sigma_\mu \Lambda \sigma_\nu \Lambda^\dagger). \tag{2.5}$$

Points on the null-plane can be represented by triangular matrices of the form

$$X = \begin{pmatrix} 0 & x_\perp^* \\ x_\perp & x^- \end{pmatrix}. \tag{2.6}$$

Poincaré transformations

$$X \to X' = \Lambda X \Lambda^\dagger + A \tag{2.7}$$

that leave the null plane invariant (i.e. preserve the form of (2.6)) have the form

$$\Lambda = \begin{pmatrix} \alpha & 0 \\ \beta & 1/\alpha \end{pmatrix} \qquad A = \begin{pmatrix} 0 & a_\perp^* \\ a_\perp & a^- \end{pmatrix}, \tag{2.8}$$

where $\alpha \neq 0$, β, and $a_\perp$ are complex and a^- is real. These transformations form a seven-parameter subgroup of the Poincaré group that leaves the null plane invariant. This subgroup is called the *kinematic subgroup* of the null plane. This subgroup includes the three-parameter subgroup of translations on the null plane, a three-parameter subgroup of null plane preserving boosts as well as rotations about the 3-axis.

The null plane-preserving boost that transforms a rest 4-momentum $(m, \mathbf{0})$ to a value p are matrix valued functions of $\tilde{\mathbf{v}} = \tilde{\mathbf{p}}/m$ given by

$$\Lambda_f(\tilde{\mathbf{p}}/m) := \Lambda_f(\tilde{\mathbf{v}}) := \begin{pmatrix} \sqrt{v^+} & 0 \\ v_\perp/\sqrt{v^+} & 1/\sqrt{v^+} \end{pmatrix}. \tag{2.9}$$

Since the reality of α is preserved under matrix multiplication, the null-plane boosts form a *subgroup* of the Lorentz group.

The action of a null-plane boost on an arbitrary four momentum vector is determined by the transformation properties of the $+$ and $\perp$ components of the four momentum

$$p^+ \to p^{+\prime} = v^+ p^+ \qquad \mathbf{p}_\perp \to \mathbf{p}'_\perp + \mathbf{v}_\perp p^+. \tag{2.10}$$

Since p^- does not appear in (2.10) the three components $\tilde{\mathbf{p}} := (p^+, \mathbf{p}_\perp)$ are called a “null-plane vector”. The $-$ component can be calculated using the mass-shell condition

$$p^- = \frac{m^2 + \mathbf{p}_\perp^2}{p^+}. \tag{2.11}$$

The null-plane spin of a particle of mass m is defined [25][4] so that (1) it agrees with ordinary canonical spin in the particle’s rest frame and (2) is invariant with respect to null-plane boosts, (2.9).

The null-plane representation of the single-nucleon Hilbert space is the space of square integrable functions of the null-plane vector components of the particle’s four momentum and the 3-component of its null-plane spin:

$$\psi(\tilde{\mathbf{p}}, \mu) = \langle \tilde{\mathbf{p}}, \mu | \psi \rangle \qquad \int_0^\infty dp^+ \int_{\mathbb{R}^2} d\mathbf{p}_\perp \sum_{\mu=-j}^{j} |\psi(\tilde{\mathbf{p}}, \mu)|^2 < \infty. \tag{2.12}$$

The Poincaré group acts irreducibly on single-nucleon states

$$\langle \tilde{\mathbf{p}}, \mu | U_1(\Lambda, A) | \psi \rangle =$$

$$\int_0^\infty dp^{+\prime} \int_{\mathbb{R}^2} d\mathbf{p}'_\perp \sum_{\mu'=-j}^{j} \mathcal{D}^{m,j}_{\tilde{\mathbf{p}},\mu;\tilde{\mathbf{p}}',\mu'}[\Lambda, A] \langle \tilde{\mathbf{p}}', \mu' | \psi \rangle, \tag{2.13}$$

where the Poincaré group Wigner-$\mathcal{D}$ function in the null-plane irreducible basis [4] is

$$\mathcal{D}^{m,j}_{\tilde{\mathbf{p}},\mu;\tilde{\mathbf{p}}',\mu'}[\Lambda, A] := \langle \tilde{\mathbf{p}}, \mu | U(\Lambda, a) | \tilde{\mathbf{p}}', \mu' \rangle =$$

$$\delta(\tilde{\mathbf{p}} - \tilde{\mathbf{\Lambda}}(p')) \sqrt{\frac{p^+}{p^{+\prime}}} D^j_{\mu\mu'}[\Lambda_f^{-1}(\tilde{\mathbf{p}}/m)\Lambda\Lambda_f(\tilde{\mathbf{p}}'/m)] e^{ip\cdot a} \tag{2.14}$$

where $D^j_{\mu\mu'}(R)$ is the ordinary $SU(2)$ Wigner D-function.

Our model Hilbert space for the two-nucleon system is the tensor product of two single-nucleon Hilbert spaces. The tensor product of two single-nucleon representations of the Poincaré group defines the kinematic representation of the Poincaré group on the two-nucleon Hilbert space.

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_1 \qquad U_0(\Lambda, A) := U_1(\Lambda, A) \otimes U_1(\Lambda, A). \tag{2.15}$$

The tensor product of two one-body irreducible representations of the Poincaré group is reducible. It can be decomposed into an orthogonal linear superposition of irreducible representations using the Poincaré group Clebsch-Gordan coefficients in the null-plane basis:

$$\Psi(\tilde{\mathbf{p}}_1, \mu_1, \tilde{\mathbf{p}}_2, \mu_2) = \sum \int \langle m_1, j_1, \tilde{\mathbf{p}}_1, \mu_1; m_2, j_2, \tilde{\mathbf{p}}_2, \mu_2 | k, j(l,s) \tilde{\mathbf{P}}, \mu \rangle d\tilde{\mathbf{P}} k^2 dk \Psi(m, j, \tilde{\mathbf{P}}, \mu, l, s) \tag{2.16}$$

where the Poincaré group Clebsch-Gordan coefficient [4] in the null-plane basis is

$$\langle m_1, j_1, \tilde{\mathbf{p}}_1, \mu_1; m_2, j_2, \tilde{\mathbf{p}}_2, \mu_2 | k, j(l,s) \tilde{\mathbf{P}}, \mu \rangle =$$

$$\delta(\tilde{\mathbf{P}} - \tilde{\mathbf{p}}_1 - \tilde{\mathbf{p}}_2) \frac{\delta(k - k(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2))}{k^2} | \sqrt{\frac{P^+ \omega_1(\mathbf{k}) \omega_2(\mathbf{k})}{(\omega_1(\mathbf{k}) + \omega_2(\mathbf{k})) p_1^+ p_2^+}} \times$$

$$\sum Y_{lm}(\hat{\mathbf{k}}(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2)) D^{1/2}_{\mu_1,\mu_1'}[\Lambda_f^{-1}(\mathbf{k}/m_1)\Lambda_c(\mathbf{k}/m_1)] D^{1/2}_{\mu_2,\mu_2'}[\Lambda_f^{-1}(-\mathbf{k}/m_2)\Lambda_c(-\mathbf{k}/m_2)] \times$$

$$\langle \frac{1}{2}, \mu_1', \frac{1}{2}, \mu_2' | s, \mu_s \rangle \langle l, m, s, \mu_s | j, \mu \rangle \tag{2.17}$$

where $(\omega(\mathbf{k}), \mathbf{k}) = k(p_1, p_2) = \Lambda_f^{-1} p_1$. In (2.17) the two-body invariant mass M_0, which has continuous spectrum, is replaced by $\mathbf{k}^2$ which is related to M_0 by

$$M_0 = 2\sqrt{\mathbf{k}^2 + m^2}. \tag{2.18}$$

The transformations $\Lambda_f(\mathbf{k}/m)$ and $\Lambda_c(\mathbf{k}/m)$ are null-plane, (2.9), and canonical boosts,

$$\Lambda_c(\mathbf{k}/m) = e^{\frac{1}{2}\boldsymbol{\rho}\cdot\boldsymbol{\sigma}}, \tag{2.19}$$

where $\boldsymbol{\rho}$ is the rapidity

$$\boldsymbol{\rho} = \hat{\mathbf{k}} |\boldsymbol{\rho}| \qquad \cosh(|\boldsymbol{\rho}|) = \sqrt{\mathbf{k}^2 + m^2}/m \qquad \sinh(|\boldsymbol{\rho}|) = \mathbf{k}/m. \tag{2.20}$$

The combination

$$\Lambda_f^{-1}(\pm\mathbf{k}/m)\Lambda_c(\pm\mathbf{k}/m) \tag{2.21}$$

is a rotation, called a Melosh rotation [26][4] that transforms the null-plane spins so they rotate under a single representation of $SU(2)$ which allows them to be coupled using ordinary $SU(2)$ Clebsch-Gordan coefficients. The absence of Wigner rotations in (2.17) is a consequence of the null-plane boosts forming a subgroup.

In this basis two-nucleon wave functions have the form

$$\psi(k,j(l,s)\tilde{\mathbf{P}},\mu) = \langle k,j(l,s)\tilde{\mathbf{P}},\mu|\psi\rangle \qquad \int_0^\infty dP^+ \int_{\mathbb{R}^2} d\mathbf{P}_\perp k^2 dk \sum_{\mu=-j}^{j} \sum_{s=0}^{1} \sum_{l=|j-s|}^{|j+s|} |\psi((k,j(l,s)\tilde{\mathbf{P}},\mu)|^2 < \infty \tag{2.22}$$

and $U_0(\Lambda, A)$ acts irreducibly on these states:

$$\langle k,j(l,s)\tilde{\mathbf{P}},\mu|U_0(\Lambda,A)|\psi\rangle =$$

$$\int_0^\infty dP^{+\prime} \int_{\mathbb{R}^2} d\mathbf{P}'_\perp \sum_{\mu'=-j}^{j} \mathcal{D}^{M_0(k),j}_{\tilde{\mathbf{P}},\mu;\tilde{\mathbf{P}}',\mu'}[\Lambda,A]\langle k,j(l,s)\tilde{\mathbf{P}}',\mu'|\psi\rangle. \tag{2.23}$$

This basis is used in the formulation of our dynamical model in the next section.

III. DYNAMICS

Dynamical models of the two nucleon system in Poincaré invariant quantum mechanics are defined by a dynamical unitary representation of the Poincaré group acting on two-nucleon Hilbert space. The mass Casimir operator for this representation can be defined by adding a realistic nucleon-nucleon interaction v_{nn} [27][4] to the square of the non-interacting two-nucleon mass operator as follows:

$$M^2 = M_0^2 + 4mv_{nn} \tag{3.1}$$

where for a null-plane dynamics v_{nn} has the form

$$\langle k',j',(l',s')\tilde{\mathbf{P}}',\mu'|v_{nn}|k,j,(l,s)\tilde{\mathbf{P}},\mu\rangle = \delta(\tilde{\mathbf{P}}'-\tilde{\mathbf{P}})\delta_{j'j}\delta_{\mu'\mu}\langle k',l',s'\|v^j\|k,l,s\rangle \tag{3.2}$$

in the non-interacting irreducible basis (2.17). The interaction v_{nn} is restricted so M^2 is a positive operator.

Simultaneous eigenstates of M^2, j^2 ,$\tilde{\mathbf{P}}$, j_z in the non-interacting irreducible basis have the form

$$\langle k',j',(l',s')\tilde{\mathbf{P}}',\mu'|\lambda,j,\tilde{\mathbf{P}},\mu\rangle = \delta(\tilde{\mathbf{P}}'-\tilde{\mathbf{P}})\delta_{j'j}\delta_{\mu'\mu}\phi_{\lambda,j}(k,l,s) \tag{3.3}$$

where the wave function $\phi_{\lambda,j}(k,l,s)$ is a solution of the eigenvalue equation

$$(4k^2+4m^2-\lambda^2)\phi_{\lambda,j}(k,l,s) = \sum_{s=0}^{1}\sum_{l=|j-s|}^{l=|j+s|}\int_0^\infty 4m\langle k,l,s\|v^j_{nn}\|k',l',s'\rangle k'^2 dk' \phi_{\lambda,j}(k',l',s'). \tag{3.4}$$

The eigenfunctions, $\phi_{\lambda,j}(k',l',s')$, of this mass operator are also solutions to the non-relativistic Schrödinger equation with interaction v_{nn}, which can be seen by dividing both sides of (3.4) by $4m$. The deuteron is an even parity bound state with $j = s = 1$ and parity limits the l sum in (3.4) to $l \in \{0,2\}$.

If $\{M^2, j, \tilde{\mathbf{P}}, \mu\}$ have the same interpretation as $\{M_0^2, j, \tilde{\mathbf{P}}, \mu\}$, then it follows that the eigenstates (3.4) transform irreducibly with respect to a *dynamical* representation of the Poincaré group defined by replacing the eigenvalues of M_0 by the eigenvalues λ of M:

$$\langle k',j',(l',s')\tilde{\mathbf{P}}',\mu'|U(\Lambda,A)|\lambda,j,\tilde{\mathbf{P}},\mu\rangle =$$

$$\int \sum_{\mu''=-j}^{j} \langle k',j',(l',s')\tilde{\mathbf{P}}',\mu'|\lambda,j,\tilde{\mathbf{P}}'',\mu''\rangle d\tilde{\mathbf{P}}''\langle\lambda,j,\tilde{\mathbf{P}}'',\mu''|U(\Lambda,A)|\lambda,j,\tilde{\mathbf{P}},\mu\rangle =$$

$$\phi_{\lambda,j}(k',l',s')\mathcal{D}^{\lambda;j'}_{\tilde{\mathbf{P}}',\mu';\tilde{\mathbf{P}},\mu}[\Lambda,A]. \tag{3.5}$$

Since the eigenstates (3.3) are complete, $U(\Lambda, A)$, defined by (3.5), is a dynamical unitary representation of the Poincaré group on the two-nucleon Hilbert space. *When* (Λ, A) is an element of the kinematic subgroup of the null plane, the Poincaré Wigner $\mathcal{D}$-function in the null-plane basis, $\mathcal{D}^{\lambda,j}_{\tilde{\mathbf{P}}',\mu';\tilde{\mathbf{P}},\mu}[\Lambda, A]$, is independent of the mass eigenvalue, λ, and is thus identical to the Poincaré Wigner $\mathcal{D}$-function for the non-interacting irreducible representation. This shows that the null-plane kinematic subgroup defined by (2.8) is the kinematic subgroup of the dynamical representation (3.5).

The rotational symmetry is *exact*, but because $\mathcal{D}^{\lambda,j}_{\tilde{\mathbf{P}}',\mu';\tilde{\mathbf{P}},\mu}[\Lambda, A]$, depends explicitly on the mass eigenvalue λ when $\Lambda = R$ is a rotation, the rotations are dynamical transformations.

The eigenfunctions of the interacting mass operator, M, are identical functions of k, l, s to the eigenfunctions of the non-relativistic Schrödinger equation with interaction v_{nn}. The identity

$$\Omega_\pm(H_{nr}, H_{0nr}) = \Omega_\pm(H_r, H_{0r}), \tag{3.6}$$

where $\Omega_\pm(H, H_0)$ are the scattering wave operators for the non-relativistic and relativistic system respectively, follows from the elementary calculation:

$$\Omega_\pm(H_r, H_{0r}) = \lim_{t\to\pm\infty} e^{i(P_0^+ + P^-)t/2} e^{-i(P_0^+ + P_0^-)t/2} = \lim_{t'\to\pm\infty} e^{iP^- t'} e^{-iP_0^- t'} =$$

$$\lim_{t'\to\pm\infty} e^{i(M^2 + \mathbf{P}_{0\perp}^2)t'/P_0^+} e^{-i(M_0^2 + \mathbf{P}_{0\perp}^2)t'/P_0^+} = \lim_{t''\to\pm\infty} e^{iM^2 t''} e^{-iM_0^2 t''} =$$

$$\lim_{t''\to\pm\infty} e^{i(\mathbf{k}^2/m + v_{nn})4mt'' + i4m^2 t''} e^{-i(\mathbf{k}^2/m)4mt'' - i4m^2 t''} =$$

$$\lim_{t'''\to\pm\infty} e^{i(\mathbf{P}^2/4m + \mathbf{k}^2/m + v_{nn})t'''} e^{-i(\mathbf{P}^2/4m + \mathbf{k}^2/m)t'''} =$$

$$\Omega_\pm(H_{nr}, H_{0nr}) \tag{3.7}$$

where the limits are strong or absolute abelian [28] limits and the times in (3.7) were reparameterized as follows

$$t''' = 4mt'' = 4mt'/P_0^+ = 2mt/P_0^+. \tag{3.8}$$

This proof uses kinematic symmetries that follow from (3.2) and the spectral condition, $P_0^+ > 0$. It follows from eq. (3.7) that relativistic and non-relativistic scattering matrices are *identical* functions of $\mathbf{k}$:

$$S(H_{nr}, H_{0nr}) = \Omega_+^\dagger(H_{nr}, H_{0nr})\Omega_-(H_{nr}, H_{0nr}) = \Omega_+^\dagger(H_r, H_{0r})\Omega_-(H_r, H_{0r}) = S(H_r, H_{0r}). \tag{3.9}$$

Since realistic nn interactions are constructed by correctly transforming experimental differential cross section data to the center of momentum frame and then fitting the solution of the scattering problem to the transformed data, these interactions can be used in (3.1) *without modification*. There is a small binding energy correction of about 1 part in 2000, which we ignore.

We use this method to construct the deuteron eigenstates and representation of the Poincaré group that we use to evaluate the deuteron current matrix elements (1.1).

IV. OBSERVABLES

The differential cross section for elastic-electron deuteron scattering in the one-photon exchange approximation,

$$d\sigma = \frac{(2\pi)^4 e}{\sqrt{(p_e \cdot p_d)^2 - m_e^2 m_d^2}} |\sqrt{\omega_e(\mathbf{p}'_e)}\langle p'_e, \mu'_e | I_{e\mu}(0) | p_e, \mu_e\rangle \sqrt{\omega_e(\mathbf{p}_e)} \frac{(2\pi)^3}{q^2} \times$$

$$\sqrt{\omega_d(\mathbf{p}'_d)}\langle p'_d, \mu'_d | I_s^\mu(0) | p_d, \mu_d\rangle \sqrt{\omega_d(\mathbf{p}_d)}|^2 \delta^4(p'_d - p_d + p'_e - p_e) \frac{d\mathbf{p}'_e}{\omega_e(\mathbf{p}'_e)} \frac{d\mathbf{p}'_d}{\omega_d(\mathbf{p}'_d)}, \tag{4.1}$$

is a quadratic function of the deuteron current matrix elements. Because only three of the deuteron current matrix elements are linearly independent, there are three independent elastic scattering spin observables for a given momentum transfer.

The observables are the structure functions $A(Q^2)$ and $B(Q^2)$ and the tensor polarization $T_{20}(Q^2,\theta)$ at $\theta_{lab}=70^o$.

The quantities $A(Q^2)$, $B(Q^2)$ can be extracted from the unpolarized laboratory frame differential cross section:

$$\frac{d\sigma}{d\Omega}(Q^2,\theta) = \frac{\alpha^2\cos^2(\theta/2)}{4E_i^2\sin^4(\theta/2)}\frac{E_f}{E_i}[A(Q^2)+B(Q^2)\tan^2(\theta/2)] \tag{4.2}$$

while $T_{20}(Q^2,\theta)$ is extracted from the difference in the cross sections for target deuterons having canonical spin polarizations $\mu_d=1$ and $\mu_d=0$ at a fixed laboratory scattering angle:

$$T_{20}(Q^2,\theta) = \sqrt{2}\frac{\frac{d\sigma}{d\Omega}_1(Q^2,\theta) - \frac{d\sigma}{d\Omega}_0(Q^2,\theta)}{\frac{d\sigma}{d\Omega}(Q^2,\theta)}. \tag{4.3}$$

V. CURRENTS

The current operator $I^\mu(x)$ transforms as a four-vector density under the dynamical representation (3.5) of the Poincaré group,

$$U(\Lambda,A)I^\mu(x)U^\dagger(\Lambda,A) = (\Lambda^{-1})^\mu{}_\nu I^\nu(\Lambda x+a). \tag{5.1}$$

The current operator must also satisfy current conservation

$$g_{\mu\nu}[P^\mu, I^\nu(0)]_- = 0, \tag{5.2}$$

in addition to symmetries with respect to space reflections and time reversal. Here $g_{\mu\nu}$ is the Minkowski metric with signature $(-+++)$. Current covariance and current conservation are dynamical constraints on the current operator.

Cluster properties imply that the current operator for the two-nucleon system is the sum of two single-nucleon current operators plus a two-body operator that vanishes when the nucleons are asymptotically separated. A non-vanishing two-body current is needed to satisfy (5.1) and (5.2) when the nucleons interact. The two-body current is constrained, but not fixed by the requirements of current covariance and current conservation.

Current matrix elements are matrix elements of $I^\mu(x)$ in irreducible eigenstates of $U(\Lambda,A)$. Combining current covariance (5.1) with the transformation properties (3.5) of the irreducible eigenstates of $U(\Lambda,A)$ gives a set of linear equations that relate the different matrix elements:

$$\langle d,j,\tilde{\mathbf{P}}',\mu'|I^\mu(x)|d,j,\tilde{\mathbf{P}},\mu\rangle =$$

$$(\Lambda^{-1})^\mu{}_{\mu'}\int\sum \mathcal{D}^{M_d,1*}_{\tilde{\mathbf{P}}''',\mu''';\tilde{\mathbf{P}}',\nu'}[\Lambda,A]d\tilde{\mathbf{P}}'''\langle\tilde{\mathbf{P}}''',\nu''',d|I^{\mu'}(\Lambda x+a)|\tilde{\mathbf{P}}'',\nu,d\rangle d\tilde{\mathbf{P}}''\mathcal{D}^{M_d,1}_{\tilde{\mathbf{P}}'',\nu'';\tilde{\mathbf{P}},\nu}[\Lambda,A]. \tag{5.3}$$

Current conservation leads to the additional linear constraints on the current matrix elements:

$$g_{\alpha\mu}(P^{'\alpha}-P^\alpha)\langle\tilde{\mathbf{P}}',\nu',d|I^\mu(0)|\tilde{\mathbf{P}},\nu,d\rangle = 0. \tag{5.4}$$

For the deuteron the constraints (5.3), (5.4), parity and time reversal imply that all of the current matrix elements can be constructed from three linearly independent matrix elements for each value of Q^2. If the coordinate axes are chosen so the + component of the momentum transfer is zero, which can always be done for electron scattering, then these relations imply that all of the current matrix elements can be calculated from matrix elements of $I^+(0)$ [7] .

In Poincaré invariant quantum mechanics " generalized impulse approximations" are defined by using the one-body current operators determined by cluster properties to compute linearly independent current matrix elements; the remaining current matrix elements are generated by the dynamical constraints discussed above. Different choices of independent current matrix elements lead to different "generalized impulse approximations"; however *any* choice leads to matrix elements of an exactly conserved covariant current operator. The non-uniqueness of Poincaré invariant "generalized impulse approximations" suggests that it is unreasonable to expect that the two-body contributions to a chosen set of independent current matrix elements will be exactly zero.

If the deuteron eigenstates $|\tilde{\mathbf{P}},\mu,d\rangle$ are given a delta-function normalization, then the matrix elements

$$I^+_{\mu,\nu} := \langle\tilde{\mathbf{P}}',\mu,d|I^+(0)|\tilde{\mathbf{P}},\nu,d\rangle \tag{5.5}$$

with $Q^+ = P^{+\prime} - P^+ = 0$ are invariant with respect to null-plane boosts. There are 9 spin combination; all but four are related by kinematic symmetries [7]. The remaining four matrix elements of $I^+(0)$, which can be taken as $I^+_{1,1} I^+_{1,0} I^+_{0,0}$ and $I^+_{1,-1}$, are related by rotational covariance. Only three linear combinations of these matrix elements can be taken as independent.

The three independent linear combinations of the current matrix elements used in ref. [7] were:

$$I^+_{1,1} + I^+_{0,0}, I^+_{1,0}, I^+_{1,-1}. \tag{5.6}$$

We refer to this choice of independent current matrix elements as choice I. These independent matrix elements can be distinguished by the number of spin flips between the initial and final states.

The difference $I^+_{1,1} - I^+_{0,0}$ is related to the matrix elements (5.6) by dynamical transformations (i.e they involve the deuteron mass, M_d):

$$I^+_{1,1} - I^+_{0,0} = -\frac{1}{1+\eta}[\eta(I^+_{1,1} + I^+_{0,0}) - 2\sqrt{2\eta} I^+_{1,0} + I^+_{1,-1}] \tag{5.7}$$

where

$$\eta := Q^2/4M_d^2. \tag{5.8}$$

The difference between the direct computation of the difference $I^+_{1,1} - I^+_{0,0}$ and the difference determined by the constraint (5.7) of current covariance

$$\Delta_- := I^+_{1,1} - I^+_{0,0} + \frac{1}{1+\eta}[\eta(I^+_{1,1} + I^+_{0,0}) - 2\sqrt{2\eta} I^+_{1,0} + I^+_{1,-1}] \tag{5.9}$$

gives a measure of the size of the additional required dynamical contribution to the current operator that is generated by the covariance constraint. If this difference is small compared to the size of independent matrix elements, then there will not be too much sensitivity to the choice of independent matrix elements; on the other hand, if this difference is large, there will be an increased sensitivity to the choice of independent current matrix elements.

Both impulse or impulse plus an exchange current can be used to compute the independent current matrix elements. Our calculations, shown in fig. 14, indicate that Δ_- is larger when the pion-exchange current contributions are added to the independent matrix elements. This means that the presence of the exchange current enhances the sensitivity of the results to the choice of independent matrix elements. Because of this increased sensitivity, we investigate the impact of using different choices of independent linear combinations of current matrix elements to compute the elastic electron-deuteron scattering observables.

Any matrix elements of a conserved covariant current in a set of deuteron eigenstates can be expressed in terms of a rank 3 covariant current tensor. Since the deuteron spin operator is constructed by boosting the Pauli-Lubanski 4-vector divided by the mass to the rest frame [3][4], it is possible to label the deuteron spins by four vectors by multiplying the current matrix element by the inverse of the boost used to define spin of the irreducible eigenstate. Since the Pauli-Lubanski vector is orthogonal to the four momentum, this gives a four vector that is orthogonal to the four momentum. If this boost is applied to the initial and final deuteron states in a current matrix element then the result is a rank-three tensor density, $T^\mu_{\rho\sigma}(P', P)$ satisfying

$$P^{\rho\prime} T^\mu_{\rho\sigma}(P', P) = T^\mu_{\rho\sigma}(P', P) P^\sigma = (P'_\mu - P_\mu) T^\mu_{\rho\sigma}(P', P) = 0. \tag{5.10}$$

Formally this tensor density is related to the current matrix elements by

$$T^\mu_{\rho\sigma}(P', P) := \Lambda_f(\tilde{\mathbf{P}}'/M_d)^i{}_\rho O^*_{\nu' i} \sqrt{P^{+\prime}} \langle \tilde{\mathbf{P}}', \nu' | I^\mu(0) | \tilde{\mathbf{P}}, \nu \rangle \sqrt{P^+} O_{\nu j} \Lambda_f(\tilde{\mathbf{P}}/M_d)^j{}_\sigma \tag{5.11}$$

where $O_{\nu j}$ is the unitary matrix

$$O_{\nu j} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 \end{pmatrix} \tag{5.12}$$

that converts Cartesian components of vectors to spherical components and the i, j sums go from 1 to 3. The factors $\sqrt{P^+}$ give $|\tilde{\mathbf{P}}, \nu\rangle \sqrt{P^+}$ a covariant normalization.

The general form of this Lorentz covariant tensor density $T^{\mu}_{\rho\sigma}(P',P)$ can be used to define invariant form factors $F_1(Q^2), F_2(Q^2)$ and ,$F_3(Q^2)$

$$T^{\mu}_{\rho\sigma}(P',P) = \frac{1}{2}(P'+P)^{\mu}E(P')_{\rho\alpha}[g^{\alpha\beta}F_1(Q^2) + \frac{Q^{\alpha}Q^{\beta}}{M_d^2}F_2(Q^2)]E(P)_{\beta\sigma}+$$

$$\frac{1}{2}E(P')_{\rho\alpha}[Q^{\alpha}g^{\mu\beta} - g^{\mu\alpha}Q^{\beta}]F_3(Q^2)E(P)_{\beta\sigma} \tag{5.13}$$

where

$$E(P)_{\alpha\beta} = g_{\alpha\beta} - \frac{P_{\alpha}P_{\beta}}{P^2} \tag{5.14}$$

is the covariant projector on the subspace orthogonal to P. The projectors are not needed when this tensor is contracted with vectors orthogonal to the final or initial four-momentum.

The current tensor constructed using formula (5.11) will only transform as a tensor if the underlying current matrix elements satisfy current covariance, while the expression (5.13) is always a tensor. In applications where the underlying current operator is only covariant with respect to the null-plane kinematic subgroup we can relate the form factor to a maximal set of independent current matrix elements by contracting the current tensor with three sets of "polarization 4-vectors". These are 4-momentum dependent vectors $v^{\nu}(P)$ that are orthogonal P^{μ}. The contraction has the form

$$v'_{af}(P')^{\rho}T^{\mu}_{\rho\sigma}(P',P)v_{bi}(P)^{\sigma} = \frac{1}{2}(P'+P)^{\mu}[v'_{af}(P')\cdot v_{bi}(P)F_1(Q^2) + \frac{(v'_{af}(P')\cdot Q)(Q\cdot v_{bi}(P)}{Q^2}F_2(Q^2)]+$$

$$\frac{1}{2}[(v'_{af}(P')\cdot Q)v_{bi}(P)^{\mu} - v_{af}(P')^{\mu}(Q\cdot v_{bi}(P)))F_3(Q^2). \tag{5.15}$$

In this case the three invariant scalar products

$$v'_{af}(P')\cdot v_{bi}(P) \qquad v'_{af}(P')\cdot Q \qquad Q\cdot v_{bi}(P) \tag{5.16}$$

determine the form factors in terms of the current matrix elements. When the underlying current matrix elements are only kinematically covariant then it is necessary to pick a "frame" to evaluate the contractions. The result will give consistent results in all frames related to the original frame by a null-plane kinematic transformation. The three independent sets of contractions can be used to calculate the three form factors which fix the remaining matrix elements by the assumed covariance. This provides an equivalent means for constructing independent linear combinations of current matrix elements.

The form factors $F_1(Q^2)$, $F_2(Q^2), F_3(Q^2)$ defined by (5.11) and (5.13) are related to the form factors $G_1(Q^2)$, $G_2(Q^2), G_3(Q^2)$ [1] which are defined in terms of canonical-spin Breit-frame matrix elements with momentum transfer in the 3 direction:

$$F_1(Q^2) - 4\eta^2(1+\eta)F_2(Q^2) =_c \langle\frac{\mathbf{Q}}{2},0,d|I^0(0)|-\frac{\mathbf{Q}}{2},0,d\rangle_c = G_0(0) + \sqrt{2}G_2(Q^2) \tag{5.17}$$

$$F_1(Q^2) =_c \langle\frac{\mathbf{Q}}{2},1,d|I^0(0)|-\frac{\mathbf{Q}}{2},1,d\rangle_c = G_0(0) - \frac{1}{\sqrt{2}}G_2(Q^2) \tag{5.18}$$

$$F_3(Q^2) = \sqrt{\frac{2}{\eta}}\,_c\langle\frac{\mathbf{Q}}{2},-1,d|I^1(0)|-\frac{\mathbf{Q}}{2},0,d\rangle_c = G_1(Q^2). \tag{5.19}$$

The deuteron elastic scattering observables can be expressed in terms of the deuteron form factors $G_i(Q^2)$, (5.17,5.18,5.19):

$$A(Q^2) = G_0^2(Q^2) + \frac{2}{3}\eta G_1^2(Q^2) + G_2^2(Q^2) \tag{5.20}$$

$$B(Q^2) = \frac{4}{3}\eta(1+\eta)G_1^2(Q^2) \tag{5.21}$$

$$T_{20}(Q^2,\theta) = -\frac{G_2^2(Q^2) + \sqrt{8}G_0(Q^2)G_2(Q^2) + \frac{1}{3}\eta G_1^2(Q^2)[1+2(1+\eta)\tan^2(\theta/2)]}{\sqrt{2}[A(Q^2) + B(Q^2)\tan^2(\theta/2)]}. \tag{5.22}$$

The enhanced sensitivity to the choice of linearly independent matrix elements when the exchange current is included, as shown in fig. 14, suggests that in addition to the choice (5.6) of linearly independent matrix elements used in [7], other choices should be considered.

Frankrurt, Frederico, and Strikman [29] defined independent current matrix elements using polarization vectors constructed from the last three columns $v_{1c}(P_b), v_{2c}(P_b), v_{3c}(P_b)$, of the canonical boost in the Breit frame

$$(v_{1c}(P_b), v_{2c}(P_b)v_{3c}(P_b)) = \begin{pmatrix} -\sqrt{\eta} & 0 & 0 \\ \sqrt{1+\eta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{5.23}$$

These are automatically orthogonal to the momentum. Three independent current matrix elements are extracted from the contractions $v_{1c}(P'_b)^\rho T^+_{\rho\sigma}(P'_b,P_b)v_{1c}(P_b)^\sigma$, $v_{2c}(P'_b)^\rho T^+_{\rho\sigma}(P'_b,P_b)v_{2c}(P_b)^\sigma$ and $v_{3c}(P'_b)^\rho T^+_{\rho\sigma}(P'_b,P_b)v_{1c}(P_b)^\sigma$. These are consistent in all frames related to the Breit frame by null-plane kinematic transformations. Their preference for this choice is to avoid using the $3'3$ matrix element, which is maximally suppressed in the infinite momentum frame. We call this choice of independent current matrix elements choice II.

We are indebted to Fritz Coester [30] for suggesting a third choice of independent current matrix elements that directly emphasizes the kinematic symmetries of the null plane. This choice uses columns of the null-plane boost as polarization vectors.

$$(v_{1f}(P), v_{2f}(P)v_{3f}(P)) = \begin{pmatrix} P^x/M_d & P^y/M_d & P^0/M_d - M_d/P^+ \\ 1 & 0 & P^x/M_d \\ 0 & 1 & P^y/M_d \\ -P^x/M_d & -P^y/M_d & P^z/M_d + M_d/P^+ \end{pmatrix} \tag{5.24}$$

The independent combinations are $v_{1f}(P')^\rho T^+_{\rho\sigma}(P',P)v_{1f}(P)^\sigma$, $v_{2f}(P')^\rho T^+_{\rho\sigma}(P',P)v_{2f}(P)^\sigma$, and $v_{3c}(P')^\rho T^+_{\rho\sigma}(P',P)v_{1c}(P)^\sigma$. When the first two terms are evaluated in the Breit frame the scalar products (5.16) that relate the contractions to the form factors are mass independent, which is preserved under kinematic transformations. This means that the natural boosts in the impulse approximation are independent of the constituent masses. To construct a third independent linear combination that exploits this mass independence would require using matrix elements of $I^2(0)$. Rather than using $I^2(0)$, this choice also uses the $3'1$ Breit frame canonical spin polarization vectors that are also used in [29]. In this case the mass dependence is trivial in the Breit frame with respect to rotations. We call this choice of independent matrix elements choice III.

These three choices lead to three different expressions for deuteron form factors in terms of independent linear combinations of matrix elements of $I^+(0)$:
Choice I:

$$(1+\eta)G_0(Q^2) = (\frac{1}{2} - \frac{\eta}{3})(I_{11} + I_{00}) + \frac{5\sqrt{2\eta}}{3}I_{10} + (\frac{2\eta}{3} - \frac{1}{6})I_{1-1} \tag{5.25}$$

$$(1+\eta)G_1(Q^2) = I_{11} + I_{00} - I_{1-1} - (1-\eta)\sqrt{\frac{2}{\eta}}I_{10} \tag{5.26}$$

$$(1+\eta)G_2(Q^2) = -\frac{\sqrt{2}\eta}{3}(I_{11} + I_{00}) + \frac{4\sqrt{\eta}}{3}I_{10} - \frac{\sqrt{2}}{3}(2+\eta)I_{1-1} \tag{5.27}$$

Choice II ($G_0(Q^2)$ of choice I replaced by):

$$(1+\eta)G_0(Q^2) = (\frac{2\eta}{3} + 1)I^+_{1,1} - \frac{\eta}{3}I^+_{00} + \frac{2\sqrt{2\eta}}{3}I^+_{1,0} + (\frac{2\eta+1}{3})I^+_{1,-1}] \tag{5.28}$$

Choice III ($G_0(Q^2)$ and $G_2(Q^2)$ of choice I replaced by):

$$G_0(Q^2) = (1 + \frac{2\eta}{3})I^+_{11} + \frac{1}{3}I^+_{1,-1} + -\frac{2\eta}{3}G_1(Q^2) \tag{5.29}$$

$$G_2(Q^2) = \frac{2\sqrt{2}}{3}(\eta I^+_{1,1} - I^+_{1,-1} - \eta G_1(Q^2)) \tag{5.30}$$

Since $B(Q^2)$ only depends on $G_1(Q^2)$ which uses the same linear combination of current matrix elements for all three choices, the computation of $B(Q^2)$, is unchanged. These choices lead to a $B(Q^2)$ that fits the experimental data.

The different linear combinations of null-plane current matrix elements used in choices I , II and III have a non-trivial effect on the scattering observables $A(Q^2)$ and $T_{20}(Q^2, \theta)$. Selecting a choice is a model assumption.

VI. IMPULSE CURRENTS

The impulse current is the sum of single nucleon current operators for the proton and neutron. For spin 1/2 systems that are eigenstates of parity there are two independent matrix elements. They are related to the Dirac nucleon form factors $F_1(Q^2)$ and $F_2(Q^2)$ by

$$\langle \tilde{\mathbf{p}}', \nu' | I^\mu(0) | \tilde{\mathbf{p}}, \nu\rangle = \sqrt{\frac{m}{p^{+\prime}}}\bar{u}(p')\Gamma^\mu u(p)\sqrt{\frac{m}{p^+}} \tag{6.1}$$

where

$$\Gamma^\mu = \gamma^\mu F_1(Q^2) + \frac{1}{2}[\gamma^\mu, \frac{1}{2m}\gamma\cdot Q]F_2(Q^2) \tag{6.2}$$

where $u(p)$ is a null plane Dirac spinor if the spin in the current matrix element is a null plane spin.

For spacelike momentum transfers with $Q^+ = 0$ the form factors can be expressed in terms of the independent null-plane matrix elements

$$\langle \frac{1}{2}|I^+(0)|\frac{1}{2}\rangle = \langle \tilde{\mathbf{p}}', \frac{1}{2}|I^+(0)|\tilde{\mathbf{p}}, \frac{1}{2}\rangle = F_1(\mathbf{Q}^2) \tag{6.3}$$

$$\langle \frac{1}{2}|I^+(0)| - \frac{1}{2}\rangle = \langle \tilde{\mathbf{p}}', \frac{1}{2}|I^+(0)|\tilde{\mathbf{p}}, -\frac{1}{2}\rangle = -\sqrt{\tau}F_2(\mathbf{Q}^2). \tag{6.4}$$

where $\tau := \frac{\mathbf{Q}^2}{4m^2}$.

The Dirac form factors $F_1(Q^2)$ and $F_2(Q^2)$ are related to the Sachs form factors by [31]

$$F_1(\mathbf{Q}^2) = \frac{G_e(\mathbf{Q}^2) + \tau G_m(\mathbf{Q}^2)}{1+\tau}, \tag{6.5}$$

$$F_2(\mathbf{Q}^2) = \frac{G_m(\mathbf{Q}^2) - G_e(\mathbf{Q}^2)}{1+\tau}. \tag{6.6}$$

We consider recent parameterizations due to Bijker and Iachello [32]; Bradford, Bodek, Budd, and Arrington [33]; Budd, Bodek, and Arrington [34]; Kelly [35] and Lomon [36][37] [38]. These parameterizations all determine the proton electric form factors using the polarization experiments [39]. The input to our calculations is the isoscalar form factors, which are the sum of the proton and neutron form factors.

VII. DYNAMICAL EXCHANGE CURRENTS

The realistic interactions [21][22] used to construct the mass operator (3.1) include a one-pion exchange contribution plus a short-range contribution that is designed so the two-body cross sections fit experiment.

"Required" two-body currents are needed to satisfy the constraints of current covariance and current conservation. These constraints can be satisfied by directly computing a set of linearly independent matrix elements and then using the constraints to determine the remaining current matrix elements. Even if the independent current matrix elements are computed using the one-body impulse current, the matrix elements generated by the constraints will have two-body contributions. The interaction dependence arises because the covariance and current conservation constraints involve the mass eigenvalues of the initial and final states.

In addition to the two-body currents directly generated by covariance and current conservation, realistic interactions contain terms that involve the exchange of charged mesons, leading to the exchange of the nucleon charges, and the possibility of two-body currents associated with these exchanges. These currents appear as additional contributions to the linearly independent matrix elements of $I^+(0)$ that are needed to compute electron-scattering observables in null-plane quantum mechanics. The form of the dynamical exchange current contribution to $I^+(0)$ is model dependent, but we assume that the most important contribution is motivated by one-pion exchange physics.

The interactions used in the dynamical equation (3.2) are constructed from realistic rotationally invariant interactions that are functions of relative momentum variable and spins. These interactions can be used to construct the invariant mass and spin operators for scattering equivalent dynamical models with any kinematic symmetry by multiplying by the appropriate delta function. In (3.2) they are made into interactions with a null-plane kinematic symmetry by embedding them in the full two-nucleon Hilbert space by multiplying the rotationally invariant interaction by a null-plane momentum-conserving delta function.

Any "derivation" of model interactions in Poincaré invariant quantum mechanics should result in a rotationally invariant kernel in terms of relative momentum variables and spins. "Derivations" of exchange current should have a consistent treatment of the interactions. Meson-nucleon field theories provide a framework that explicitly includes the degrees of freedom whose elimination leads to exchange currents in dynamical models involving nucleon degrees of freedom. These theories can be used to motivate the structure of model exchange currents. The precise "derivation" of any current involves model assumptions.

To construct a model exchange current we begin by constructing off-shell transition amplitudes for nucleon-nucleon scattering and nucleon-nucleon scattering in the presence of an external field electromagnetic field using one-pion exchange Feynman diagrams. We define a rotationally invariant model one-pion exchange interaction, represented as a kernel in the relative momenta and spins (3.2), by identifying the interaction kernel with this off-shell transition amplitude in the rest frame of the two-nucleon system.

We use a similar method to construct our model one-pion exchange current from the off-shell transition amplitude for nucleon-nucleon scattering in the presence of an external field. The origin of this current is the part of the pion-exchange interaction that couples to the photon with the Dirac $\Lambda_-(p)$ projection operator. The details of how we relate this operator to the rotationally invariant kernel that represents the one-pion exchange interaction and how we realize the null-plane kinematic symmetry is outlined in Appendix A. In our model current the rotationally invariant kernel that represents the one-pion exchange interaction is replaced by the corresponding one-poin-exchange contribution to our realistic model interaction.

We use the relation between the consistently derived one-pion exchange interaction and the pion exchange current to motivate the construction of a corresponding exchange current from the one-pion exchange part of a realistic interaction. Details are given in Appendix A and Appendix B.

The resulting matrix element of the exchange current has the following form

$$\langle \tilde{\mathbf{P}}', \nu', d | I^{\mu}_{ex}(0) | \tilde{\mathbf{P}}, \nu, d \rangle :=$$

$$\int \langle \tilde{\mathbf{P}}', \nu', d | \tilde{\mathbf{p}}_1', \nu_1', \tilde{\mathbf{p}}_2', \nu_2' \rangle (-\frac{1}{2m}) \sqrt{\frac{m}{p_1^+}} \bar{u}_f(p_1') \Gamma^{\mu} \gamma_5 (\frac{P\cdot\gamma}{M_d}) u_f(p_1'') \sqrt{\frac{m}{p_1^{+\prime\prime}}} d\tilde{\mathbf{p}}_1' d\tilde{\mathbf{p}}_1'' d\tilde{\mathbf{p}}_2' \times$$

$$\langle \tilde{\mathbf{p}}_1'', \nu_1'', \tilde{\mathbf{p}}_2', \nu_2' | U(\Lambda_f(P/M_d)) \tilde{v}_1 | \tilde{\mathbf{P}}_0, \nu, d \rangle +$$

$$\int \langle \tilde{\mathbf{P}}', \nu', d | \tilde{\mathbf{p}}_1', \nu_1', \tilde{\mathbf{p}}_2', \nu_2' \rangle (-\frac{1}{2m}) \sqrt{\frac{m}{p_2^+}} \bar{u}_f(p_2') \Gamma^{\mu} \gamma_5 (\frac{P\cdot\gamma}{M_d}) u_f(p_2'') \sqrt{\frac{m}{p_2^{+\prime\prime}}} d\tilde{\mathbf{p}}_1' d\tilde{\mathbf{p}}_2'' d\tilde{\mathbf{p}}_2' \times$$

$$\langle \tilde{\mathbf{p}}_1', \nu_1', \tilde{\mathbf{p}}_2'', \nu_2'' | U(\Lambda_f(P/M_d)) \tilde{v}_2 | \tilde{\mathbf{P}}_0, \nu, d \rangle +$$

$$\int \langle \tilde{\mathbf{P}}_0', \nu', d | \tilde{v}_1^{\dagger} U^{\dagger}(\Lambda_f(P'/M_d)) | \tilde{\mathbf{p}}_1', \nu_1', \tilde{\mathbf{p}}_2', \nu_2' \rangle (-\frac{1}{2m}) \sqrt{\frac{m}{p_1^{+\prime}}} \bar{u}_f(p_1') (\frac{P'\cdot\gamma}{M_d}) \gamma_5 \Gamma^{\mu} u_f(p_1'') \sqrt{\frac{m}{p_1^{+\prime\prime}}} \times$$

$$d\tilde{\mathbf{p}}_1' d\tilde{\mathbf{p}}_1'' d\tilde{\mathbf{p}}_2 \langle \tilde{\mathbf{p}}_1'', \nu_1'', \tilde{\mathbf{p}}_2, \nu_2' | \tilde{\mathbf{P}}, \nu, d \rangle +$$

$$\int \langle \tilde{\mathbf{P}}_0' \nu', d | \tilde{v}_2^{\dagger} U^{\dagger}(\Lambda_f(P'/M_d)) | \tilde{\mathbf{p}}_1', \nu_1', \tilde{\mathbf{p}}_2', \nu_2' \rangle (-\frac{1}{2m}) \sqrt{\frac{m}{p_2^{+\prime}}} \bar{u}_f(p_2') (\frac{P'\cdot\gamma}{M_d}) \gamma_5 \Gamma^{\mu} u_f(p_2'') \sqrt{\frac{m}{p_2^{+\prime\prime}}} d\tilde{\mathbf{p}}_1' d\tilde{\mathbf{p}}_2'' d\tilde{\mathbf{p}}_2' \times$$

$$\langle \tilde{\mathbf{p}}_1', \nu_1', \tilde{\mathbf{p}}_2'', \nu_2'' | \tilde{\mathbf{P}}, \nu, d \rangle \tag{7.1}$$

where

$$\langle \tilde{\mathbf{P}}', \nu', d | \tilde{\mathbf{p}}_1', \nu_1', \tilde{\mathbf{p}}_2', \nu_2' \rangle \tag{7.2}$$

is the deuteron eigenstate in the tensor product basis, $\tilde{\mathbf{P}}_0 = (M_d, 0, 0)$ is the rest frame value of the deuteron null-plane momentum, $U(\Lambda_f(P/M_d))$ represents a kinematic null-plane boost to the Breit frame. The field theory motivated argument in Appendix A leads to modified pion-exchange interactions $\tilde{v}_i$ of the form

$$\langle \tilde{\mathbf{P}}', \mathbf{k}', \nu_1', \nu_2' | v_1 | \tilde{\mathbf{P}}, \mathbf{k}, \nu_1, \nu_2 \ \rangle =$$

$$\delta(\tilde{\mathbf{P}}' - \tilde{\mathbf{P}}) \frac{g_\pi^2}{(2\pi)^3} \sqrt{\frac{m}{\omega(\mathbf{k}')}} \bar{u}(\mathbf{k}') \beta u(\mathbf{k}) \sqrt{\frac{m}{\omega(\mathbf{k})}} \frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{m^2 + (k'-k)^2 - i0^+} \sqrt{\frac{m}{\omega(\mathbf{k}')}} \bar{u}(-\mathbf{k}') \gamma_5 u(-\mathbf{k}) \sqrt{\frac{m}{\omega(\mathbf{k})}} \tag{7.3}$$

and

$$\langle \tilde{\mathbf{P}}', \mathbf{k}', \nu_1', \nu_2' | v_2 | \tilde{\mathbf{P}}, \mathbf{k}, \nu_1, \nu_2 \rangle =$$

$$\delta(\tilde{\mathbf{P}}' - \tilde{\mathbf{P}}) \frac{g_\pi^2}{(2\pi)^3} \sqrt{\frac{m}{\omega(\mathbf{k}')}} \bar{u}(\mathbf{k}') \gamma_5 u(\mathbf{k}) \sqrt{\frac{m}{\omega(\mathbf{k})}} \frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{m^2 + (k'-k)^2 - i0^+} \sqrt{\frac{m}{\omega(\mathbf{k}')}} \bar{u}(-\mathbf{k}') \beta u(-\mathbf{k}) \sqrt{\frac{m}{\omega(\mathbf{k})}} \tag{7.4}$$

They differ from the standard one pion exchange interaction by the replacement of one of the factor of $\bar{u}(\mathbf{k}')\gamma_5 u(\mathbf{k})$ by $\bar{u}(\mathbf{k}')\beta u(\mathbf{k})$.

In a dynamical model based on a phenomenological interaction we replace these interactions by the corresponding contribution to the model interaction:

$$\langle \tilde{\mathbf{P}}', \mathbf{k}', \nu_1', \nu_2' | \tilde{v}_1 | \tilde{\mathbf{P}}, \mathbf{k}, \nu_1, \nu_2 \ \rangle \rightarrow$$

$$\delta(\tilde{\mathbf{P}}' - \tilde{\mathbf{P}}) \frac{g_\pi^2}{(2\pi)^3} \frac{2m}{2m} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 v_\pi(\mathbf{k} - \mathbf{k}') \frac{(\mathbf{k}' - \mathbf{k}) \cdot \boldsymbol{\sigma}_2}{2m} \tag{7.5}$$

where $v_\pi(\mathbf{k} - \mathbf{k}')$ is the coefficient function of the one-pion-exchange contribution to the operator

$$\delta(\tilde{\mathbf{P}}' - \tilde{\mathbf{P}}) \frac{g_\pi^2}{(2\pi)^3} \frac{(\mathbf{k} - \mathbf{k}') \cdot \boldsymbol{\sigma}_1}{2m} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 v_\pi(\mathbf{k} - \mathbf{k}') \frac{(\mathbf{k}' - \mathbf{k}) \cdot \boldsymbol{\sigma}_2}{2m} \tag{7.6}$$

in the phenomenological interaction. For the Argonne V18 interaction $v_\pi(\mathbf{k} - \mathbf{k}')$ is extracted from the one-pion-exchange contribution to the tensor $\times(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$ and spin-spin $\times(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$ parts of the interaction using the method discussed in [40] and [41]. The extraction is discussed in Appendix B. The resulting interaction $v_\pi(\mathbf{k}-\mathbf{k}')$ differs from a $1/(m_\pi^2 + (\mathbf{k} - \mathbf{k}')^2)$ by the effects of the short distance cutoff in the AV18 interaction.

The important property of this exchange current is that it is constructed to transform covariantly with respect to the null-plane kinematic subgroup. This current is added to the impulse current when computing the independent matrix elements. These are then used to generate the remaining current matrix elements using the constraint of current covariance and current conservation.

Each term in the expression **j.1** for the exchange current matrix element can be represented as the product of $(-\frac{1}{2m})$ with the modified current $\sqrt{\frac{m}{p_1^+}} \bar{u}_f(p_1') \Gamma^\mu \gamma_5 (\frac{P \cdot \gamma}{M_d}) u_f(p_1'') \sqrt{\frac{m}{p_1^{+\prime\prime}}}$ evaluated between a deuteron eigenstate and a pseudo-state defined by applying the rotationally invariant modified interaction $\tilde{v}_i$ to the rest deuteron state and kinematically boosting the result to the Breit frame.

VIII. RESULTS

The input to our calculation of the elastic electron-deuteron scattering observables is (1) the choice of nucleon form factors [32] [37][38] [34][33][35] (2) the choice of nucleon-nucleon interaction [21][22] (3) the choice of independent linear combinations of current matrix elements used to compute the required two-body currents, [7],[29], [6][42].

The extraction of proton electric form factors based on polarization measurements compared with measurements based on the Rosenbluth separation were found to be inconsistent[39] ; these inconsistencies have been explained [43] by including two photon exchange corrections in the Rosenbluth separation. This has led to a modification of the phenomenological parameterizations of the proton electric form factor. All of the nucleon form factors that we use are consistent with the extractions based on the polarization measurements.

Neutron electric form factor data is only available for a limited range of momentum transfers. The high momentum transfer behavior of the different parameterizations is consequence of different theoretical assumptions. This leads to some variation of the parameterizations for momentum transfers above $Q^2 \sim 1(GeV)^2$. Different parameterizations of the neutron electric form factor are compared to a dipole form factor in fig 1. The form factors that we compare are recent parameterizations given by Lomon [37], Budd, Bodeck and Arrington (BBA) [34], Bradford Budd, Bodeck and Arrington (BBBA) [33], Kelly [35], and Bijker and Iachello (BI) [32]. All of these parameterizations agree for $Q^2 < 1$. The curves in figure 1 are given as ratios to dipole form factors, which emphasize differences in the parameterizations. The input to our calculations are the isoscalar linear combinations of the nucleon form factors, $F_{1N}(Q^2)$ and $F_{2N}(Q^2)$. These form factors are plotted in figures 2 and 3 for the same parameterizations that are compared in figure 1. These plots show no significant variation among the various nucleon form factors. Our calculations of elastic scattering observables are not very sensitive to these differences.

As shown in section VII, our model exchange current breaks up into a product of an effective "one-body" current and an interaction. The interaction is a modification of the one-pion exchange contribution to the tensor part of the phenomenological interaction. Most of our calculations are based on the Argonne V18 interaction. We extract the one-pion exchange contribution to the V18 potential by first discarding the short-range parts of the interaction that contributes to the spin-spin and tensor forces, then we use a method developed by Riska [40] and Schiavilla, Pandharipande, and Riska [41] to extract the one-pion exchange contribution to the tensor force from the remaining parts of the interactions. The procedure that we use is discussed in Appendix B. This differs from $\frac{1}{m_\pi^2+(\mathbf{k}-\mathbf{k}')^2}$ by the effects of the short distance configuration space cutoff that appears in the Argonne V18 interaction. The Fourier transform of the extracted interaction is compared to the pion-exchange potential without the cutoff in figure 4. The dotted curve includes the cutoff parameters that are used in in the Argonne V18 interaction. The most important differences are for momenta above 1-2 fm^{-1}.

Figures 5,6,7 show the three deuteron form factors, G_0, G_1 and G_2 with and without the exchange current included for the two-nucleon form factors (BI and BBBA) that have the largest high momentum transfer difference in the neutron electric form factor. The independent matrix elements (5.6) are calculated using the one-body parts of the current and with the pion exchange current added. The remaining current matrix elements are determined by the constraints of current conservation and current covariance. The figures show that the addition of the pion exchange current contributions lead to an enhancement of G_0 above the minimum at $Q^2 = 1GeV^2$, The minimum of G_1 shifts to the right, and G_2 is enhanced. Except for momentum transfers Q^2 between 6 and 7 these form factors are not very sensitive to the different assumptions made about the neutron electric form factor.

Data for the observables A are labeled Stanford Mark III [44], CEA [45], Orsay [46], SLAC E101 [47], Saclay ALS [48], DESY [49], Bonn [50], Mainz [51], JLab Hall C [52], JLab Hall A [53] and Monterey [54]. Data for B are labeled: SLAC NPSA NE4 [55], Martin [56], Bonn [50], Saclay ALS [57], Mainz [51], Stanford Mark III [44]. Data for T_{20} are labeled: Novosibirsk-85 [58] [59], Novosibirsk-90 [60], Bates-84 [61], Bates-91 [62] and JLab Hall C [63]

Calculations of A, B, and T_{20} using the independent matrix elements (5.6) (choice I) and the five parameterizations of the nucleon form factors used in figures 1-3 are shown in figures 8-10. While the generalized impulse calculations give a qualitative understanding of the data, it is clear from these calculations that the generalized impulse approximation is inadequate.

Figures 11,12,13 show the effects of including the phenomenological pion-exchange current defined in section VII. We see that A and B provide acceptable fits to the data when the pion exchange current is included. The results are insensitive to the assumptions used in parameterizing the high momentum transfer behavior of the neutron electric form factor. T_{20} is closer to the data, but it is still below the most recent Jlab hall C data [63] between Q^2 of .5 and $2GeV^2$.

The calculation displayed in figures 11,12,13 used the choice I of independent current matrix elements given by (5.6). The presence of the pion exchange current (7.1) increases the sensitivity to the choice of the independent current matrix elements. This is shown in figure 14. What is plotted is the difference $I^+_{11}(0) - I^+_{00}(0)$ with and without the pion exchange current using a direct calculation of the difference or by generating the difference using current conservation and current covariance. The difference between the dashed curve and solid curve shows that the required two-body contributions to the current in this difference is small when the independent current matrix elements are computed in the impulse approximation. Comparing the dotted and dash-dot curve indicates that much larger required two-body contributions to the current are needed when the exchange current contributions are included in all matrix elements. This suggests that there will be a non-trivial sensitivity to the choice in independent current matrix elements when the pair current contribution is added.

To test this we examined the two other choices, II and III, of independent matrix elements discussed in section IV. These methods relate form factors to independent current matrix elements by contracting different sets of polarization vectors into the current tensor. In both approaches there are preferred polarization vectors; in one case the vectors are chosen to minimize the dependence on matrix elements that are maximally suppressed in the infinite momentum frame ($P^+ \to 0$), while the other choice minimizes the mass dependence in the contractions used to define the independent current matrix elements. Both choices were discussed in section IV.

Figures 15,16,17, and 18 show the deuteron elastic scattering observables $A(Q^2)$ and $T_{20}(Q^2)$ for both choices, II and III, of polarization vectors. In both case G_1 is computed using the same linear combination of current matrix elements used for choice I. Since B only depends on G_1, B is identical for all three choices. All three choices of independent matrix elements give different predictions for A and T_{20}. For choices II and III there is a mild enhancement of A at higher momentum transfers compared to choice of independent matrix elements given by (5.6). There is also a larger effect on the tensor polarization that brings the curve to within the experimental error bars.

The result is that both choice II and III of independent current matrix elements give consistent results for the elastic scattering observables and they both provide a good description of the existing data over a wide range of momentum transfers. It is clear that there is a non-trivial sensitivity to the choice of independent current matrix elements when these results are compared to the corresponding results based on choice I.

Another potential source of sensitivity to the input is the choice of nucleon-nucleon interaction. Any phase equivalent change in the nucleon-nucleon interaction is automatically accompanied by a corresponding change in the current operator. For interactions with a long-range meson exchange tail one might expect that the same data could be understood by simply adjusting the cutoff parameter. For typical soft interactions that are useful in low energy problems, one expects that a more significant modification of the current would be necessary.

In figs. 19,20 and 21 we compare calculations of $A(Q^2)$, $B(Q^2)$, and $T_{20}(Q^2)$ using the CD Bonn wave functions with and without the exchange current. In these calculations the exchange current is still based on the Argonne V18 cutoffs. The calculations show the generalized impulse calculations and calculations where the exchange current is added to the impulse current. without adjusting the cutoff parameters. The calculations clearly show that there is more sensitivity to the choice of nucleon-nucleon interactions than to the choice of nucleon form factor.

Good consistency with all experimental observables is obtained using the V18 interaction with nucleon form factors [33] and the choice of independent current matrix elements suggested by Frankfrut, Frederico and Strickman or Coester, discussed in section IV. These calculations are shown in figs. 22,23 and 24. For these choices the model exchange current explains the difference between the generalized impulse approximation and the experimental data. While our exchange current, which used the one-pion exchange part of the Argonne V18 interaction, required the numerical computation of Fourier transforms, the resulting interaction contribution to the exchange current differed very little from a simple static momentum space pion exchange interaction, fig 4, which indicates the simplicity of our exchange current.

Our results indicate that this simple pion exchange current is sufficient to provide a good quantitative understanding of elastic electron deuteron elastic scattering for a wide range of momentum transfers. The sensitivities to both the choice of interaction and the choice of independent matrix elements are the largest uncertainties in the calculations, and these uncertainties are all larger than the experimental uncertainties. For momentum transfers where data is available there is very little sensitivity to the uncertainties in the neutron electric form factors.

Finally we compare the magnetic and quadrupole moments of the deuteron with and without the exchange current and using different choices of independent current matrix elements. The results are shown in table 1. Our calculations do not exhibit any sensitivity to the choice of nucleon form factor, which are sufficiently well constrained by experiment at low momentum transfers. The exchange current contributions affect the results for all of the moments. In both cases they are closer to experiment than the moments computed using the generalized impulse approximation. The magnetic moment is in good agreement (to within computational accuracy) with experiment, while the quadrupole moment differs from the experimental result by a few percent.

The conclusion of our research is that a simple exchange current motivated by one-pion exchange and the freedom to define a conserved covariant current operator by choosing a preferred set of independent current matrix elements is sufficient to provide a good fit to all three elastic scattering observables using Poincaré invariant quantum theory with a null-plane kinematic symmetry. The model exchange current is essentially a pair current designed with a null plane kinematic symmetry. The quality of the nucleon form factors has progressed to the point that our results are insensitive to the choice of nucleon form factor.

While is it straightforward to include low-order pion-exchange physics in a more general class of models, the strategy for making the best choice of independent current matrix elements in a general class of electron-nucleus reactions requires more investigation. The principles used to derive choices I, II and III of independent current matrix elements all can be generalized to treat initial and final states with different spins. Whether there is one consistent set of principles that works universally for all reactions is not yet known.

A second observation is that our model, with one of the two preferred choices of independent current matrix elements, provides a better description of all three observables than methods based on truncations of null-plane field theory or instant form relativistic quantum mechanics. Our model has features of both - unlike the instant form model our model has the full-null plane kinematic symmetry with all of the advantages discussed at the beginning of this paper. Unlike truncations of a null plane field theory, which emphasizes cluster properties at the expense of exact Poincaré invariance, our model is exactly Poincaré invariant. We expect that the Poincaré invariance constraint to be more important for momenta near or slightly above the deuteron mass scale, since the deuteron mass scale is involved in the implementation of the symmetry.

This research provides a useful first step in trying to devise a more systematic treatment of model exchange currents in Poincaré invariant quantum mechanics with a null plane kinematic symmetry. It leads to a simple current that provides a significant improvement in all three elastic scattering observables when compared with the corresponding impulse calculations, however additional research is still needed in order to determine if these methods can be successfully applied to larger class of reactions.

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APPENDIX A: CURRENT CONSTRUCTION

The steps motivating the structure of the model exchange current (7.1) are summarized in this appendix. We consider a pseudoscalar pion-nucleon vertex:

$$\mathcal{L}(x) = -ig_\pi : \bar{\Psi}(x)\gamma_5\Psi(x)\boldsymbol{\tau}\cdot\boldsymbol{\phi}(x) : \tag{A1}$$

where $g_\pi = 2m\frac{f_\pi}{m_\pi}$ is the pseudoscalar pion-nucleon coupling constant and m is the nucleon mass. The off-shell transition amplitude in the two-body rest frame is given by the rotationally invariant kernel:

$$\langle \mathbf{k}', \mu_1', \mu_2' | v_\pi | \mathbf{k}, \mu_1, \mu_2 \rangle :=$$

$$\frac{g_\pi^2}{(2\pi)^3}\sqrt{\frac{m}{\omega(\mathbf{k}')}}\bar{u}(\mathbf{k}')\gamma_5 u(\mathbf{k})\sqrt{\frac{m}{\omega(\mathbf{k})}}\frac{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{m_\pi^2+(k'-k)^2-i0^+}\sqrt{\frac{m}{\omega(\mathbf{k}')}}\bar{u}(-\mathbf{k}')\gamma_5 u(-\mathbf{k})\sqrt{\frac{m}{\omega(\mathbf{k})}} \tag{A2}$$

where $(k'-k)^2 := (\mathbf{k}-\mathbf{k}')^2 - (\omega_m(\mathbf{k})-\omega_m(\mathbf{k}'))^2$ and we have assumed a plane wave normalization $\langle \mathbf{k}'|\mathbf{k}\rangle = \delta(\mathbf{k}'-\mathbf{k})$.

When the external field is turned on the off-shell transition amplitude has the form of a current contracted into the 4-vector potential of the field. The expression for the current matrix element using the vertex (A1), with

$$A^\mu(q) := \frac{1}{(2\pi)^4}\int e^{-iq\cdot y}A_\mu(y)d^4y, \tag{A3}$$

includes four terms, one of which is

$$\langle \mathbf{p}_1', \mu_1', \mathbf{p}_2', \mu_2' | I^\mu(0)^\mu | \mathbf{p}_1, \mu_1, \mathbf{p}_2, \mu_2 \rangle =$$

$$\sqrt{\frac{m}{\omega(\mathbf{p}_1')}}\bar{u}(\mathbf{p}_1')\Gamma^\mu u(\mathbf{r})\sqrt{\frac{m}{\omega(\mathbf{r})}}\sqrt{\frac{m}{\omega(\mathbf{r})}}\bar{u}(\mathbf{r})\gamma_5 u(\mathbf{p}_1)\sqrt{\frac{m}{\omega(\mathbf{p}_1)}}\frac{1}{E_{12}-\omega(\mathbf{r})-\omega(\mathbf{p}_2')+i0^+}\times$$

$$\frac{g_\pi^2}{(2\pi)^3}\frac{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{m^2+(p_2-p_2')^2-i0^+}\sqrt{\frac{m}{\omega(\mathbf{p}_2')}}\bar{u}(\mathbf{p}_2')\gamma_5 u(\mathbf{p}_2)\sqrt{\frac{m}{\omega(\mathbf{p}_2)}}+$$

$$\sqrt{\frac{m}{\omega(\mathbf{p}_1')}}\bar{u}(\mathbf{p}_1')\Gamma^\mu v(-\mathbf{r})\sqrt{\frac{m}{\omega(\mathbf{r})}}\sqrt{\frac{m}{\omega(\mathbf{r})}}\bar{v}(-\mathbf{r})\gamma_5 u(\mathbf{p}_1)\sqrt{\frac{m}{\omega(\mathbf{p}_1)}}\frac{1}{E_{12}-\omega(\mathbf{p}_2')+\omega(\mathbf{r})-i0^+}\times$$

$$\frac{g_\pi^2}{(2\pi)^3}\frac{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{m^2+(p_2-p_2')^2-i0^+}\sqrt{\frac{m}{\omega(\mathbf{p}_2')}}\bar{u}(\mathbf{p}_2')\gamma_5 u(\mathbf{p}_2)\sqrt{\frac{m}{\omega(\mathbf{p}_2)}}+\cdots \tag{A4}$$

where

$$E_{12} = \omega(\mathbf{p}_1) + \omega(\mathbf{p}_2) \tag{A5}$$

is the initial energy,

$$\mathbf{r} = \mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_2'. \tag{A6}$$

and

$$\Gamma^\mu = \gamma^\mu F_1(Q^2) + \frac{1}{4m}[\gamma^\mu, \gamma \cdot Q]_- \tag{A7}$$

is the nucleon impulse current. The $+\cdots$ represents the contribution from the other three terms related by either Hermitian conjugation and/or exchanging the proton and neutron.

To construct our model exchange current we first evaluate this covariant expression in the rest frame of the initial two-body system in so the pion-exchange interaction appears as a rotationally invariant function of the relative momenta. This allows us to relate the interaction part of this kernel to the interaction kernel in (A2). Next we treat the initial energy of the two-body system, E_{12}, as a parameter that can be expressed in terms of the mass of the initial state (deuteron) and the kinematically conserved momenta. We use the assumed null-plane kinematic symmetry of the interaction to express the energy denominators in terms of masses and null-plane kinematic variables; $E_{12} \to \frac{1}{2}(P^+ + \frac{M^2+\mathbf{P}_\perp}{P^+})$, $E_{120} \to \frac{1}{2}(P^+ + \frac{M_0^2+\mathbf{P}_\perp}{P^+})$. These are all model assumptions. In the rest frame of the initial deuteron $P^- = M_d$ which gives

$$E_{12} - \omega(\mathbf{r}) - \omega(\mathbf{p}_2') \to E_{12} - E_{120} \to \frac{1}{2}\frac{M_d^2 - M_0^2(\mathbf{k}')}{P^+_{rest}} \to \frac{M_d^2 - M_0^2(\mathbf{k}')}{2M_d} \approx \frac{M_d^2 - M_0^2(\mathbf{k}')}{4m} \tag{A8}$$

and

$$E_{12} - \omega(\mathbf{p}_2') + \omega(\mathbf{r}) \to E_{12} \to \frac{1}{2}(P^+ + \frac{M^2 + \mathbf{P}_\perp}{P^+}) \to \frac{1}{2}(M_d + \frac{M_d^2}{M_d}) = M_d \approx 2m \tag{A9}$$

Using (A8) and (A9) in (A4) gives

$$\langle \mathbf{p}_1', \mu_1', -\mathbf{k}', \mu_2' | I^\mu(0)^\mu | \mathbf{k}, \mu_1, -\mathbf{k}, \mu_2 \rangle =$$

$$\sqrt{\frac{m}{\omega(\mathbf{p}_1')}}\bar{u}(\mathbf{p}_1')\Gamma^\mu u(\mathbf{k}')\sqrt{\frac{m}{\omega(\mathbf{k}')}}\frac{4m}{M_d^2 - M_0^2 - i0^+}\sqrt{\frac{m}{\omega(\mathbf{k}')}}\bar{u}(\mathbf{k}')\gamma_5 u(\mathbf{k})\sqrt{\frac{m}{\omega(\mathbf{k})}}\times$$

$$\frac{g_\pi^2}{(2\pi)^3}\frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{m_\pi^2 + (k' - k)^2 - i0^+}\sqrt{\frac{m}{\omega(\mathbf{k}')}}\bar{u}(-\mathbf{k}')\gamma_5 u(-\mathbf{k})\sqrt{\frac{m}{\omega(\mathbf{k})}} -$$

$$\sqrt{\frac{m}{\omega(\mathbf{p}_1')}}\bar{u}(\mathbf{p}_1')\Gamma^\mu v(-\mathbf{k}')\sqrt{\frac{m}{\omega(\mathbf{k}')}}\frac{1}{2m}\sqrt{\frac{m}{\omega(\mathbf{k}')}}\bar{v}(-\mathbf{k}')\gamma_5 u(\mathbf{k})\sqrt{\frac{m}{\omega(\mathbf{k})}}\times$$

$$\frac{g_\pi^2}{(2\pi)^3}\frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{m_\pi^2 + (k' - k)^2 - i0^+}\sqrt{\frac{m}{\omega(\mathbf{k}')}}\bar{u}(-\mathbf{k}')\gamma_5 u(-\mathbf{k})\sqrt{\frac{m}{\omega(\mathbf{k})}} + \cdots \tag{A10}$$

The terms in this expression are easy to interpret. The first two lines represent the product of a one-body current matrix element, the propagator, $\frac{4m}{M_d^2 - M_0^2 - i0^+}$, associated with equation (3.4) and the rotationally invariant kernel of the pion-exchange interaction (A2). This term is already included in the one-body contribution to the current matrix element.

The last two lines have a similar form except one of the $u(\mathbf{k})$ spinors in the current and the rotationally invariant interaction kernel are replaced by $v(-\mathbf{k})$ spinors. In addition, the propagator term is replaced by the factor $1/2m$. We find it convenient for computational purposes to split the v spinor terms that give the covariant Dirac projector $\Lambda_-(-\mathbf{k}) = -v(-\mathbf{k})\bar{v}(-\mathbf{k})$ and to use $\gamma_5\beta$ to convert the $v(-\mathbf{k})$ to $u(\mathbf{k})$. The price paid for this is the introduction of two factors of β, neither of which are separately covariant. This causes no problem if we stay in the rest frame of the initial system, but if we transform to the Breit frame, it is necessary to replace both terms by kinematically covariant expressions that reduce to β in the rest frame.

Using

$$v(-\mathbf{p})\bar{v}(-\mathbf{p}) = \gamma_5\beta u(\mathbf{p})\bar{u}(\mathbf{p})\beta\gamma_5 \tag{A11}$$

the last two lines of (A10) become

$$\sqrt{\frac{m}{\omega(\mathbf{p}_1')}}\bar{u}(\mathbf{p}_1')\Gamma^\mu\gamma_5\beta u(\mathbf{k}')\sqrt{\frac{m}{\omega(\mathbf{k}')}}\frac{1}{2m}\times$$

$$\frac{g_\pi^2}{(2\pi)^3}\sqrt{\frac{m}{\omega(\mathbf{k}')}}\bar{u}(\mathbf{k}')\beta u(\mathbf{k})\sqrt{\frac{m}{\omega(\mathbf{k})}}\frac{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{m^2+(k'-k)^2-i0^+}\sqrt{\frac{m}{\omega(\mathbf{k}')}}\bar{u}(-\mathbf{k}')\gamma_5 u(-\mathbf{k})\sqrt{\frac{m}{\omega(\mathbf{k})}}+\cdots \tag{A12}$$

This separates into the product of an effective one-body current,

$$\sqrt{\frac{m}{\omega(\mathbf{p}_1')}}\bar{u}(\mathbf{p}_1')\Gamma^\mu\gamma_5\beta u(\mathbf{k}')\sqrt{\frac{m}{\omega(\mathbf{k}')}} \tag{A13}$$

a factor $1/2m$, and a modified one-pion-exchange interaction.

$$\frac{g_\pi^2}{(2\pi)^3}\sqrt{\frac{m}{\omega(\mathbf{k}')}}\bar{u}(\mathbf{k}')\beta u(\mathbf{k})\sqrt{\frac{m}{\omega(\mathbf{k})}}\frac{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{m^2+(k'-k)^2-i0^+}\sqrt{\frac{m}{\omega(\mathbf{k}')}}\bar{u}(-\mathbf{k}')\gamma_5 u(-\mathbf{k})\sqrt{\frac{m}{\omega(\mathbf{k})}} \tag{A14}$$

The interaction term is identical to the rotationally invariant kernel of the one-pion-exchange interaction (A2) with the replacement: $\bar{u}(\mathbf{k}')\gamma_5 u(\mathbf{k}) \to \bar{u}(\mathbf{k}')\beta u(\mathbf{k})$. This replacement preserves the rotational invariance of this kernel in the rest frame, but it is not covariant with respect to null-plane boosts.

This splitting has the advantage that the computation of the exchange current matrix element has the same structure as the computation of an impulse current matrix element with the current replaced by (A13) and the deuteron wave function $|\psi\rangle$ replaced by $\frac{1}{2m}\tilde{v}|\psi\rangle$ where $\tilde{v}$ is the modified interaction (A14). The input to this calculation is defined in rest frame of the initial deuteron, but we need it in the Breit frame. This is elementary if both modified state and current transform covariantly with respect to the null-plane kinematic subgroup.

The effective one-body current (A13) can be extended to a kinematically covariant operator that agrees with (A13) in the deuteron rest frame by replacing the β in (A13) by

$$\beta \to -P\cdot\gamma/M_d \tag{A15}$$

This leads to a kinematically covariant modified current kernel

$$-\sqrt{\frac{m}{\omega(\mathbf{p}_1')}}\bar{u}(\mathbf{p}_1')\Gamma^\mu\gamma_5\frac{(P\cdot\gamma)}{M_d}u(\mathbf{k}')\sqrt{\frac{m}{\omega(\mathbf{k}')}}. \tag{A16}$$

Kinematic covariance is all that this needed for a consistent computation of the current matrix elements.

The modified interaction (A14) is rotationally invariant so when it is applied to the rest deuteron eigenstate the resulting pesudo-state has the same spin as the deuteron. This can be consistently defined in any other kinematically related frame using null-plane boosts. This, along with the replacement (A15) restores the kinematic covariance.

The last step is to replace the modified interaction (A14) by the corresponding modified one-pion-exchange part of a realistic model interaction. In a typical realistic interaction the pseudoscalar pion exchange interaction is obtained by replacing the spinor terms in the rotationally invariant kernel (A2) by

$$\sqrt{\frac{m}{\omega(\mathbf{k}')}}\bar{u}(\mathbf{k}')\gamma_5 u(\mathbf{k})\sqrt{\frac{m}{\omega(\mathbf{k})}} \to \frac{\boldsymbol{\sigma}\cdot(\mathbf{k}-\mathbf{k}')}{2m} \tag{A17}$$

This expression is obtained by retaining the leading term in a $\mathbf{k}/m$ expansion of the spinor term, which is normally justified because the one-pion exchange interaction also includes a high-momentum or short-distance cutoff. The net effect is that the resulting interaction, when included in the full nucleon-nucleon interaction, provides a good description of the two-nucleon bound state and scattering observables.

Expanding $\bar{u}(\mathbf{k}')\beta u(\mathbf{k})$ to the same order in $\mathbf{k}/m$ gives 1. This suggests that the modified interaction (A14) can be modeled by replacing the one-pion exchange contribution to the realistic interaction

$$\boldsymbol{\sigma}_1\cdot(\mathbf{k}-\mathbf{k}')v_\pi(\mathbf{k}-\mathbf{k}')\boldsymbol{\sigma}_2\cdot(\mathbf{k}'-\mathbf{k})\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \tag{A18}$$

by

$$2mv_\pi(\mathbf{k}-\mathbf{k}')\boldsymbol{\sigma}_2\cdot(\mathbf{k}'-\mathbf{k})\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \tag{A19}$$

The one-pion exchange interaction (A2) only contributes to the part of the tensor force in the nucleon-nucleon interaction that multiplies the isospin exchange operator $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$. The tensor interaction also has contributions from the vector exchanges which also contribute to the spin-spin interaction. Riska [40] and Schiavilla, Pandharipande, and Riska [41] introduced a method for isolating the pion-exchange contribution to the tensor force of a phenomenological interaction using linear combinations of the tensor and spin-spin interactions. The resulting interaction is

$$v_{ps}(\mathbf{k}-\mathbf{k}') = \frac{1}{3}(2v_t(\mathbf{k}-\mathbf{k}') - v_{ss}(\mathbf{k}-\mathbf{k}')) \tag{A20}$$

where v_t and v_{ss} are tensor and spin-spin contributions to the charge exchange part of the Argonne V18 interaction. We extract this from the pion-exchange contribution to the Argonne V18 potential. This is compared to $\frac{1}{m_\pi^2+(\mathbf{k}-\mathbf{k}')^2}$ in figure 4. The difference between these curves is due to the short-distance cutoff used in the AV18 interaction.

When this interaction is applied to the deuteron bound state vector the result is a spin 1 "pseudo wave function". The resulting "pseudo vector" can be defined in the Breit frame by requiring that it transforms covariantly with respect to the null-plane kinematic subgroup. This kinematic covariance ensures that the current kernel is kinematically covariant provided the pseudo-current is modified following (A15). The kinematic covariance of the current is needed for a consistent calculation matrix elements of $I^+(0)$.

We can now write the form of the Breit frame matrix elements of this one-pion exchange contribution to the exchange current:

$$\langle\tilde{\mathbf{P}}',\mu',d|I^+_{ex}(0)|\tilde{\mathbf{P}},\mu,d\rangle =$$

$$\sum\int\langle\tilde{\mathbf{P}}',\mu',d|\tilde{\mathbf{P}}''',k',j',l',s',\mu'''\rangle k'^2dk'd\tilde{\mathbf{P}}'''\times$$

$$\langle\tilde{\mathbf{P}}''',k',j',l',s',\mu'''|\tilde{\mathbf{p}}'_p,\mu'_p,\tilde{\mathbf{p}}'_n,\mu'_n\rangle d\tilde{\mathbf{p}}'_p d\tilde{\mathbf{p}}'_n\times$$

$$\langle\tilde{\mathbf{p}}'_p,\mu'_p,\tilde{\mathbf{p}}'_n,\mu'_n|I^+_{ex-eff}(0)|\tilde{\mathbf{p}}_p,\mu_p,\tilde{\mathbf{p}}_n,\mu_n\rangle d\tilde{\mathbf{p}}_p d\tilde{\mathbf{p}}_n\times$$

$$\langle\tilde{\mathbf{p}}_p,\mu_p,\tilde{\mathbf{p}}_n,\mu_n|\tilde{\mathbf{P}}'',k,j,l,s,\mu''\rangle k^2dkd\tilde{\mathbf{P}}''\langle\tilde{\mathbf{P}}'',k,j,l,s,\mu''|\tilde{\mathbf{P}},\mu,\chi\rangle \tag{A21}$$

where the terms in this expression are the deuteron wave function in the free-particle irreducible null-plane basis

$$\langle\tilde{\mathbf{P}}',\mu',d|\tilde{\mathbf{P}}''',k',j',l',s',\mu'''\rangle = \delta(\tilde{\mathbf{P}}'-\tilde{\mathbf{P}}''')\delta_{j'j'''}\delta_{\mu'\mu'''}\phi^*_{j'}(k',l',s') \qquad (j'=s'=1), \tag{A22}$$

the Poincaré group Clebsch-Gordan coefficients in the null-plane basis

$$\langle\tilde{\mathbf{P}}''',k',j',l',s',\mu'''|\tilde{\mathbf{p}}'_p,\mu'_p,\tilde{\mathbf{p}}'_n,\mu'_n\rangle =$$

$$\delta(\tilde{\mathbf{P}}'''-\tilde{\mathbf{p}}'_p-\tilde{\mathbf{p}}'_n)\frac{\delta(k'-k(\tilde{\mathbf{p}}'_p,\tilde{\mathbf{p}}'_n))}{k'^2}\sqrt{\frac{\partial(\tilde{\mathbf{P}},\mathbf{k})}{\partial(\tilde{\mathbf{p}}'_p,\tilde{\mathbf{p}}'_n)}}\langle j,\mu|l,m_l,s,m_s\rangle\times$$

$$\langle s,m_s|\frac{1}{2},\mu_p,\frac{1}{2},\mu_n\rangle Y^*_{lm_l}(\hat{\mathbf{k}}(\tilde{\mathbf{p}}'_p\tilde{\mathbf{p}}'_n))\times$$

$$D^{1/2}_{\mu_p\mu'_p}[\Lambda_c^{-1}(\mathbf{k}/m)\Lambda_f(\mathbf{k}/m)]D^{1/2}_{\mu_n\mu'_n}[\Lambda_c^{-1}(-\mathbf{k}/m)\Lambda_f(-\mathbf{k}/m)], \tag{A23}$$

the proton effective current

$$\langle\tilde{\mathbf{p}}'_p,\mu'_p,\tilde{\mathbf{p}}'_n,\mu'_n|I^+_{ex-eff}(0)|\tilde{\mathbf{p}}_p,\mu_p,\tilde{\mathbf{p}}_n,\mu_n\rangle = \tag{A24}$$

$$\delta(\tilde{\mathbf{p}}_n - \tilde{\mathbf{p}}_n)\sqrt{\frac{m}{p_p^{+\prime}}}\bar{u}_f(\tilde{\mathbf{p}}'_p, \mu'_p)\Gamma_p^\mu(q)\gamma^5(\eta^0\gamma^0 - \eta^1\gamma^1)u_f(\tilde{\mathbf{p}}_p, \mu_p)\sqrt{\frac{m}{p_p^+}}, \tag{A25}$$

where the null plane Dirac spinors are related to the canonical Dirac spinors by a Melosh rotation

$$u_f(\tilde{\mathbf{p}}_p, \mu_p)_\mu = u_c(\tilde{\mathbf{p}}_p, \mu_p)_{\mu'} D^{1/2}_{\mu'\mu}[\Lambda_c^{-1}(\mathbf{p}_p/m)\Lambda_f(-\mathbf{p}_p/m)] \tag{A26}$$

another Poincaré Clebsch-Gordan coefficient

$$\langle \tilde{\mathbf{p}}_p, \mu_p, \tilde{\mathbf{p}}_n, \mu_n | \tilde{\mathbf{P}}'', k, j, l, s, \mu'' \rangle = \tag{A27}$$

$$\delta(\tilde{\mathbf{P}}''' - \tilde{\mathbf{p}}'_p - \tilde{\mathbf{p}}'_n)\frac{\delta(k' - k(\tilde{\mathbf{p}}'_p, \tilde{\mathbf{p}}'_n))}{k'^2}\sqrt{\frac{\partial(\tilde{\mathbf{P}}, \mathbf{k})}{\partial(\tilde{\mathbf{p}}'_p, \tilde{\mathbf{p}}'_n)}}\times \tag{A28}$$

$$D^{1/2}_{\mu_p\mu'_p}[\Lambda_f^{-1}(\mathbf{k}/m)\Lambda_c(\mathbf{k}/m)]D^{1/2}_{\mu_n\mu'_n}[\Lambda_f^{-1}(-\mathbf{k}/m)\Lambda_c(-\mathbf{k}/m)]\times$$

$$\langle s, m_s | \frac{1}{2}, \mu'_p, \frac{1}{2}, \mu'_n \rangle \langle j, \mu | l, m_l, s, m_s \rangle Y_{lm_l}(\hat{\mathbf{k}}(\tilde{\mathbf{p}}'_p, \tilde{\mathbf{p}}'_n)), \tag{A29}$$

and the pseudo wave function

$$\langle \tilde{\mathbf{P}}'', k, j, l, s, \mu'' | \tilde{\mathbf{P}}, \mu, \chi \rangle =$$

$$\delta(\tilde{\mathbf{P}}'' - \tilde{\mathbf{P}}) \int \sum \langle j, \mu'' | l, m_l, s, m_s \rangle \langle s, m_s | \frac{1}{2}, \mu_p, \frac{1}{2}, \mu_n \rangle \frac{1}{(2\pi)^{3/2}} Y^*_{lm_l}(\hat{\mathbf{k}}) d\hat{\mathbf{k}} \times$$

$$(\mathbf{k} - \mathbf{k}') \cdot \boldsymbol{\sigma}_{\mu_n\mu'_n} v_{ps}(\mathbf{k}' - \mathbf{k}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 d\mathbf{k}' Y_{l'm'_l}(\hat{\mathbf{k}}') \langle s', m'_s, | \frac{1}{2}, \mu'_p, \frac{1}{2}, \mu'_n \rangle \langle j, \mu, |, s', m'_s, l', m'_l \rangle . \phi_{lsj}(k') \tag{A30}$$

where v_{ps} is given by (A17). The full exchange current is the sum of the above quantity and the three other terms related by taking Hermitian conjugates or exchanging the particle that couples to the photon. The details of how these quantities are computed is discussed in Appendix B.

APPENDIX B: CALCULATION OF PSEUDO WAVE FUNCTION

The calculation of the pseudo wave function $(A30)$ requires the one-pion exchange part of the tensor interaction. Riska [40] and Schiavilla, Pandharipande, and Riska [41]outlined a method to extract the pseudoscalar contribution to the tensor force as a linear combination of the radial coefficient functions in the Argonne V18 interaction. The method is based on the observation that both pseudoscalar and vector meson exchange contribute to both the tensor and spin-spin interaction in the static limit[40].

The Fourier transform of the interaction has the structure

$$v_{nn}(\boldsymbol{\kappa}) = \Omega_t v_t(\boldsymbol{\kappa}^2) + \Omega_{ss} v_{ss}(\boldsymbol{\kappa}^2) + \cdots \tag{B1}$$

where $\boldsymbol{\kappa} := \mathbf{k}' - \mathbf{k}$,

$$\Omega_t := (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\kappa}^2 - 3\boldsymbol{\sigma}_1 \cdot \boldsymbol{\kappa} \boldsymbol{\sigma}_2 \cdot \boldsymbol{\kappa})(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \tag{B2}$$

$$\Omega_s := \boldsymbol{\kappa}^2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \tag{B3}$$

and we have explicitly exhibited the tensor and spin-spin contributions to the interaction.

The coefficient functions in (B1) are Fourier transforms of the coefficient functions the corresponding terms in the Argonne V18 interaction,

$$v_{ss}(\boldsymbol{\kappa}^2) = \frac{4\pi}{\boldsymbol{\kappa}^2} \int_0^\infty dr r^2 v^{\sigma\tau}(r)(j_0(\kappa r) - 1), \tag{B4}$$

$$v_t(\boldsymbol{\kappa}^2) = \frac{4\pi}{\boldsymbol{\kappa}^2}\int_0^\infty dr r^2 v^{t\tau}(r) j_2(\kappa r), \tag{B5}$$

where the factor $1/\boldsymbol{\kappa}^2$ is due to the difference in the conventions used to define tensor operator in momentum and configuration space.

These coefficient functions have contributions from both pseudoscalar and vector meson exchange. To separate them, following [40], define the tensor and spinor operators and note that

$$(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\kappa} \boldsymbol{\sigma}_2 \cdot \boldsymbol{\kappa})((\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)) = -\frac{1}{3}(\Omega_t - \Omega_{ss}) \tag{B6}$$

while vector meson exchange gives the combination [40]

$$((\boldsymbol{\sigma}_1 \times \boldsymbol{\kappa}) \cdot (\boldsymbol{\sigma}_2 \times \boldsymbol{\kappa}))(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) = \frac{1}{3}[2\Omega_{ss} + \Omega_t]. \tag{B7}$$

This implies that

$$v_t \Omega_t + v_{ss}\Omega_{ss} = v_{ps}\frac{1}{3}(\Omega_{ss} - \Omega_t) + v_v \frac{1}{3}(2\Omega_s + \Omega_t) = \tag{B8}$$

$$\frac{1}{3}(v_v - v_{ps})\Omega_t + \frac{1}{3}(v_{ps} + 2v_v)\Omega_s. \tag{B9}$$

Using these expressions we can isolate the pseudoscalar and vector contribution to the interaction using the following linear combinations of the tensor and spin-spin interactions:

$$v_{ps} = v_{ss} - 2v_t \tag{B10}$$

$$v_v = v_{ss} + v_t. \tag{B11}$$

For the Argonne V18 interaction, which is used in our calculations, there remains a small difference between the pseudoscalar interaction calculated using (B10) and the contribution that comes directly from terms that can be directly identified with the one-pion exchange contribution. We only retain the one-pion exchange contribution to these terms, this still includes the short-distance cutoff used in the Argonne V18 r-space potential.

The interactions $v^{\sigma\tau}(r)$ and $v^{t\tau}(r)$ have the form

$$v^{\sigma\tau}(r) \equiv \frac{f^2}{9}\left\{\left(\frac{m_0}{m_\pm}\right)^2 m_0 Y(\mu_0, r) + 2m_\pm Y(\mu_\pm, r)\right\} + v^c(r) \tag{B12}$$

$$v^{t\tau}(r) \equiv \frac{f^2}{9}\left\{\left(\frac{m_0}{m_\pm}\right)^2 m_0 T(\mu_0, r) + 2m_\pm T(\mu_\pm, r)\right\} + v^t(r). \tag{B13}$$

Here $Y(\mu, r)$ and $T(\mu, r)$ are the usual Yukawa and tensor functions with the exponential cutoff of the Urbana and Argonne V_{14} models:

$$Y(\mu, r) = \frac{e^{-\mu r}}{\mu r}(1 - e^{-cr^2}), \tag{B14}$$

$$T(\mu, r) = (1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2})Y(\mu r)(1 - e^{-cr^2}), \tag{B15}$$

The terms $v^c(r)$ and $v^t(r)$ are short-range phenomenological interactions that we discard for the computation of the exchange current.

In order to carry out the angular part of the integral in (A30), we expand the pseudoscalar potential $v_{ps}(\mathbf{k} - \mathbf{k}')$ in terms of partial waves:

$$v_{ps}(|\mathbf{k} - \mathbf{k}'|) = \sum_{lm} v_l(\mathbf{k}^2, \mathbf{k}'^2) Y^*_{lm}(\hat{\mathbf{k}}) Y_{lm}(\hat{\mathbf{k}}'), \tag{B16}$$

where

$$v_l(\mathbf{k}^2,\mathbf{k}'^2) = 2\pi \int_{-1}^{1} P_l(u) v_{ps}(\sqrt{\mathbf{k}^2+\mathbf{k}'^2-2kk''u})du. \tag{B17}$$

We expand the vector $\mathbf{k}$ and $\mathbf{k}'$ in spherical harmonics

$$\mathbf{k} = k(\sqrt{\frac{2\pi}{3}}(Y_{1-1}(\hat{\mathbf{k}}) - Y_{11}(\hat{\mathbf{k}})), i\sqrt{\frac{2\pi}{3}}(Y_{1-1}(\hat{\mathbf{k}}) + Y_{11}(\hat{\mathbf{k}})), \sqrt{\frac{4\pi}{3}}Y_{10}(\hat{\mathbf{k}})). \tag{B18}$$

to obtain the following expression for the pseudo wave function:

$$\begin{aligned}
&\chi^{(c)}_{\tilde{P}_d,\mu_d}(\vec{k}'',\mu_p'',\mu_n') \qquad\qquad (B19)\\
\equiv\ & \frac{1}{(2\pi)^3}\int d\hat{\mathbf{k}} k^2 dk \sum_{l=0,2}\sum_{\mu_l=-l}^{l}\sum_{l'=0}^{\infty}\sum_{\mu_l'=-l'}^{l'} v_{l'}(\mathbf{k}^2,\mathbf{k}''^2) Y_{l'\mu_l'}(\hat{\mathbf{k}}'') Y^*_{l'\mu_l'}(\hat{\mathbf{k}}) Y_{l\mu_l}(\hat{\mathbf{k}})\\
\times\ & [k''(\sqrt{\frac{2\pi}{3}}(Y_{1-1}(\hat{\mathbf{k}''}) - Y_{11}(\hat{\mathbf{k}''})), i\sqrt{\frac{2\pi}{3}}(Y_{1-1}(\hat{\mathbf{k}''}) + Y_{11}(\hat{\mathbf{k}})), \sqrt{\frac{4\pi}{3}}Y_{10}(\hat{\mathbf{k}}))\\
-\ & k(\sqrt{\frac{2\pi}{3}}(Y_{1-1}(\hat{\mathbf{k}}) - Y_{11}(\hat{\mathbf{k}})), i\sqrt{\frac{2\pi}{3}}(Y_{1-1}(\hat{\mathbf{k}}) + Y_{11}(\hat{\mathbf{k}})), \sqrt{\frac{4\pi}{3}}Y_{10}(\hat{\mathbf{k}}))]\\
\times\ & \langle \mu_n'|\vec{\sigma}|\mu_n\rangle \langle s_p,\mu_p'',s_n,\mu_n|s,\mu_s\rangle\langle s,\mu_s,l,\mu_l|j,\mu\rangle Y^l_{\mu_l}(\hat{\mathbf{k}})u_l(k),
\end{aligned}$$

The angular integrals are evaluated using:

$$\int d\hat{\mathbf{k}} Y^{*l'}_{\mu_l'}(\hat{\mathbf{k}}) Y^l_{\mu_l}(\hat{\mathbf{k}}) = \delta_{l'l}\delta_{\mu_l'\mu_l}, \tag{B20}$$

$$\begin{aligned}
&\int d\hat{\mathbf{k}} Y^{*l'}_{\mu_l'}(\hat{\mathbf{k}}) Y^l_{\mu_l}(\hat{\mathbf{k}}) Y^1_{\mu_l''}(\hat{\mathbf{k}}) \qquad\qquad (B21)\\
=\ & \sqrt{\frac{(2\cdot 1+1)(2\cdot l+1)}{4\pi(2\cdot l'+1)}}\langle 1,\mu_l'',l,\mu_l|l',\mu_l'\rangle\langle 1,0,l,0|l',0\rangle.
\end{aligned}$$

After carrying out the integral on $\hat{\mathbf{k}}$: the expression for the pseudo wave function becomes

$$\begin{aligned}
&\chi^{(c)}_{\tilde{P}_d,\mu_d}(\vec{k}'',\mu_p'',\mu_n') \qquad\qquad (B22)\\
\equiv\ & \frac{1}{(2\pi)^3}\int k^2 dk \sum_{l=0,2}\sum_{\mu_l=-l}^{l}\sum_{l'}\sum_{\mu_l'=-l'}^{l'} v_{l'}(\mathbf{k}^2,\mathbf{k}''^2,) Y_{l'\mu_{l'}}(\hat{\mathbf{k}}'') u_l(k)\\
\times\ & \langle s_p,\mu_p'',s_n,\mu_n|s,\mu_s\rangle\langle s,\mu_s,l,\mu_l|j,\mu\rangle\\
\times\ & [\sqrt{\frac{2\pi}{3}}\langle\mu_n'|\sigma_x|\mu_n\rangle \quad [\quad k''(Y_{1-1}(\hat{\mathbf{k}''}) - Y_{11}(\hat{\mathbf{k}''}))\delta_{l'l}\delta_{\mu_{l'}\mu_l}\\
+\ & k\sqrt{\frac{(2\cdot 1+1)(2\cdot l+1)}{4\pi(2\cdot l'+1)}}\langle 1,0,l,0|l',0\rangle(-\langle 1,-1,l,\mu_l|l',\mu_{l'}\rangle + \langle 1,1,l,\mu_l|l',\mu_{l'}\rangle)\quad]\\
+\ & i\sqrt{\frac{2\pi}{3}}\langle\mu_n'|\sigma_y|\mu_n\rangle \quad [\quad k''(Y_{1-1}(\hat{\mathbf{k}''}) + Y_{11}(\hat{\mathbf{k}''}))\delta_{l'l}\delta_{\mu_{l'}\mu_l}\\
+\ & k\sqrt{\frac{(2\cdot 1+1)(2\cdot l+1)}{4\pi(2\cdot l'+1)}}\langle 1,0,l,0|l',0\rangle(-\langle 1,-1,l,\mu_l|l',\mu_{l'}\rangle - \langle 1,1,l,\mu_l|l',\mu_{l'}\rangle)\quad]\\
+\ & \sqrt{\frac{4\pi}{3}}\langle\mu_n'|\sigma_z|\mu_n\rangle \quad [\quad k''Y_{10}(\hat{\mathbf{k}''})\delta_{l'l}\delta_{\mu_{l'}\mu_l}\\
-\ & k\sqrt{\frac{(2\cdot 1+1)(2\cdot l+1)}{4\pi(2\cdot l'+1)}}\langle 1,0,l,0|l',0\rangle\langle 1,0,l,\mu_l|l',\mu_{l'}\rangle\quad]\quad]
\end{aligned}$$

The sum over l' includes only a finite number of values. It can only be 0 or 1 for $l = 0$ and 1, 2 or 3 when $l = 2$. This limits the sum on l' to the first four partial waves, $l' = 0, 1, 2, 3$. The final expression of the pseudo wave function can be written in the form:

$$\chi^{(c)}_{\tilde{P}_d,\mu_d}(\vec{k}'', \mu''_p, \mu'_n) = \sum_{ll'} I_{ll'}(k'') f_{ll'}(\hat{k}'', \mu''_p, \mu'_n), \tag{B23}$$

where $f_{ll'}(\hat{k}'', \mu''_p, \mu'_n)$ are angle-dependent coefficients and $I_{ll'}(k'')$ are the relevant scalar quantities which have the following form:

$$I_{ll'}(k'') \equiv \int k^2 dk v_l(k, k'') u_{l'}(k) \tag{B24}$$

or

$$I_{ll'}(k'') \int k'^3 dk v_l(k, k'') u_{l'}(k). \tag{B25}$$

The allowed combinations of l and l' pairs requires the following integrals $I_{ll'}(k'')$:

$$I_{00}(k'') \equiv \int v_0(k, k'') u_0(k) k^2 dk \tag{B26}$$

$$I_{10}(k'') \equiv \int v_1(k, k'') u_0(k) k^3 dk \tag{B27}$$

$$I_{12}(k'') \equiv \int v_1(k, k'') u_2(k) k^3 dk \tag{B28}$$

$$I_{22}(k'') \equiv \int v_2(k, k'') u_2(k) k^2 dk \tag{B29}$$

$$I_{32}(k'') \equiv \int v_3(k, k'') u_2(k) k^3 dk \tag{B30}$$

The total exchange current contribution:

$$\begin{aligned} &\langle \tilde{\mathbf{P}}'_d, \mu'_d, d | I^+_{ex}(0) | \tilde{\mathbf{P}}_d, \mu_d, d\rangle = \sum \int d\mathbf{k}' J_i \\ \times\ & [\Psi^*_{\tilde{\mathbf{P}}'_d,\mu'_d}(\mathbf{k}', \mu'_p, \mu_n)[\bar{u}_f(\tilde{\mathbf{p}}'_p, \mu'_p)\Gamma^\mu_p(q)\gamma^5(-P\cdot\gamma/M_d)u_f(\tilde{\mathbf{p}}_p, \mu_p)]\chi^{(f)}_{\tilde{\mathbf{P}}_d,\mu_d}(\mathbf{k}, \mu_p, \mu_n) \\ +\ & \chi^{(f)*}_{\tilde{\mathbf{P}}'_d,\mu'_d}(\mathbf{k}, \mu'_p, \mu_n)[\bar{u}_f(\tilde{\mathbf{p}}'_p, \mu'_p)(-P\cdot\gamma/M_d)\gamma^5\Gamma^\mu_p(q)u_f(\tilde{p}_p, \mu_p)]\Psi_{\tilde{\mathbf{P}}_d,\mu_d}(\mathbf{k}, \mu_p, \mu_n)] \\ +\ & [p \leftrightarrow n] \end{aligned} \tag{B31}$$

where J_i is the jacobian

$$J_i = \left|\frac{\partial(\tilde{\mathbf{p}}''_p, \tilde{\mathbf{p}}''_n)}{\partial(\tilde{\mathbf{P}}'', \mathbf{k}'')}\right|^{\frac{1}{2}} \left|\frac{\partial(\tilde{\mathbf{P}}', \mathbf{k}')}{\partial(\tilde{\mathbf{p}}'_p, \tilde{\mathbf{p}}'_n)}\right|^{\frac{1}{2}} \tag{B32}$$

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	IM	IM+Exchange	IM(WSS)	IM+MEC(WSS)
Q_d	0.2698	0.2752	0.270	0.275
μ_d	0.8535	0.8596	0.847	0.871

TABLE I: deuteron magnetic and quadrupole moments evaluated in the impulse approximation and including the exchange current contribution. The values are the same using all six different nucleon form factor parameterizations and three combinations of independent current matrix elements. Argonne $V18$ potential is used in the calculation. The values labeled with WSS are from [21]. The experimental values are $0.2860 \pm 0.0015 fm^2$ [64] and $0.857406 \pm 0.000001 \mu_N$ [65].

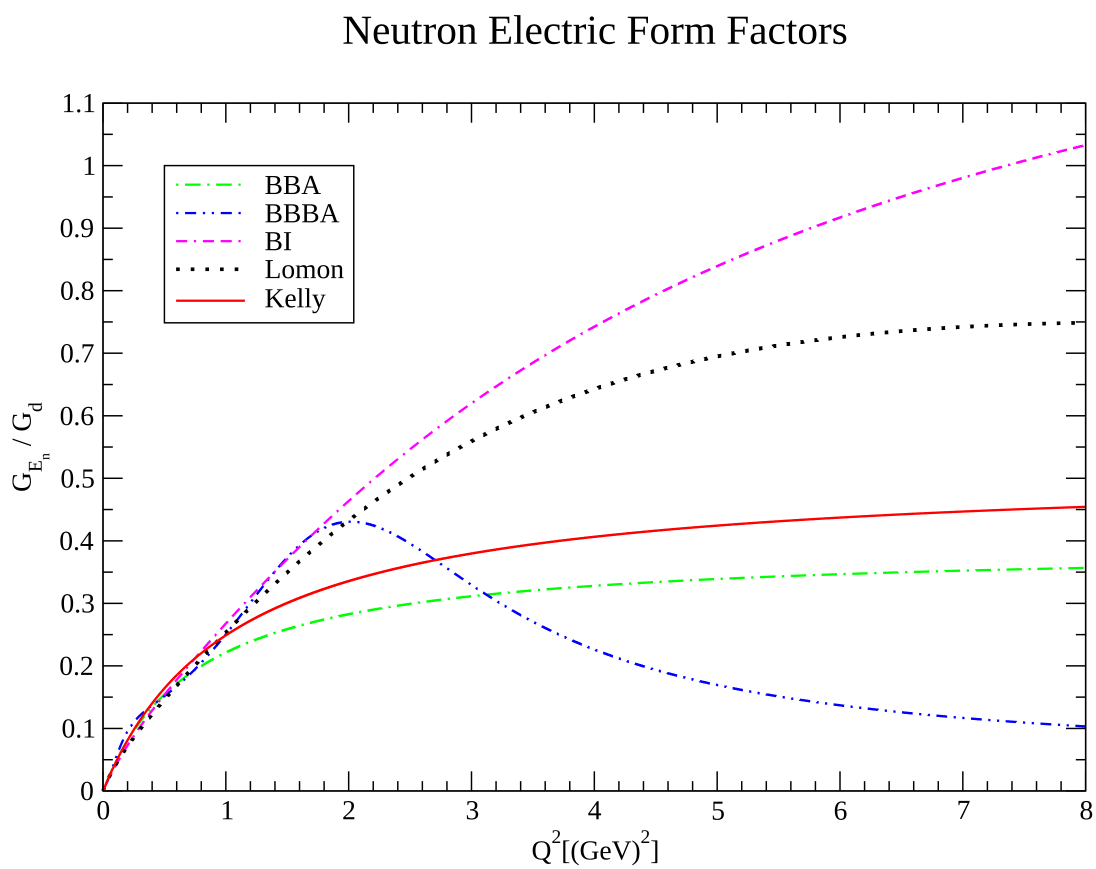

FIG. 1: Neutron electric form factor parameterizations

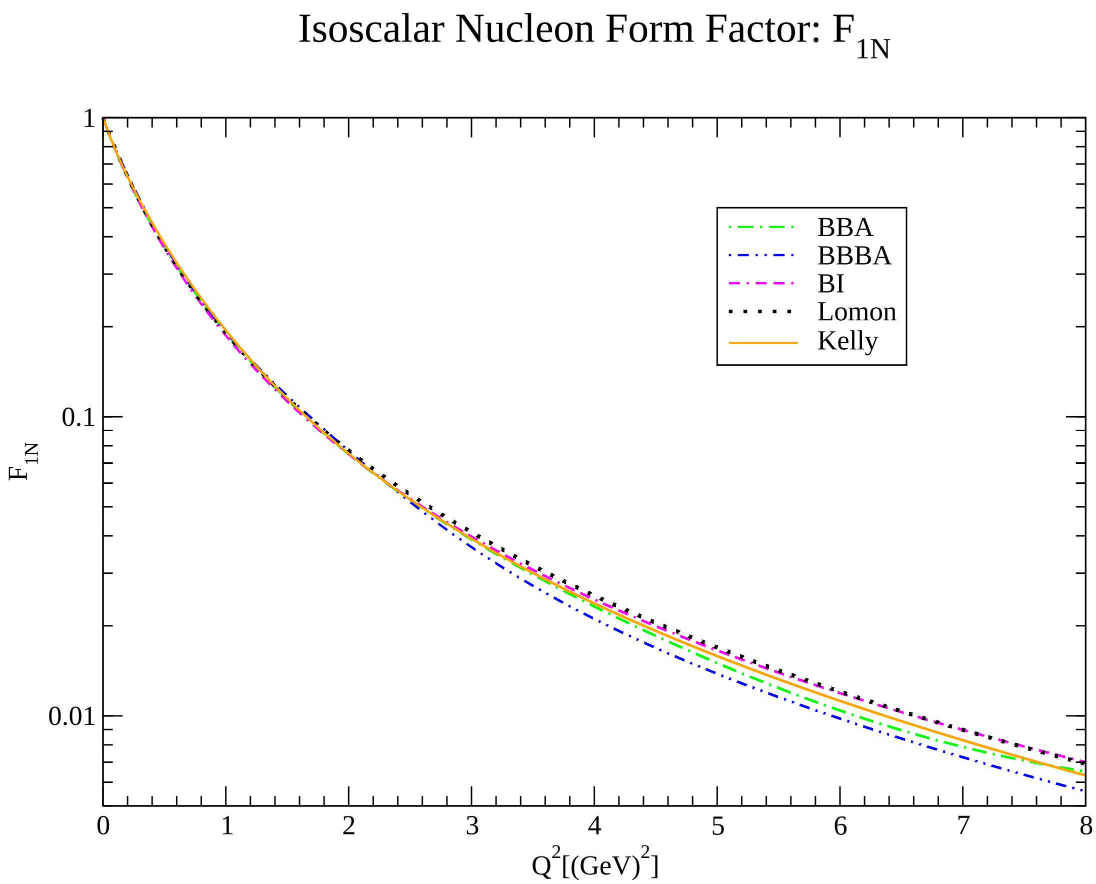

FIG. 2: Isoscalar nucleon form factor $F_{1N}(Q^2)$

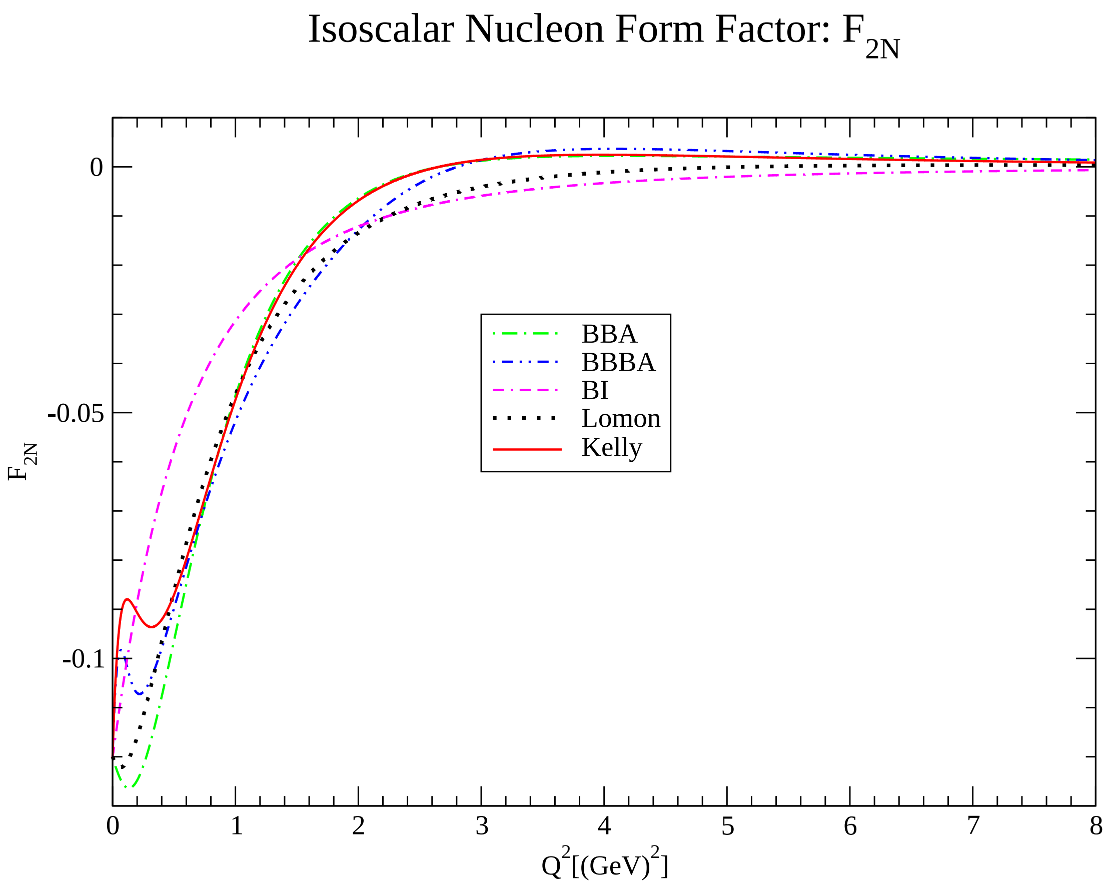

FIG. 3: Isoscalar nucleon form factor $F_{2N}(Q^2)$

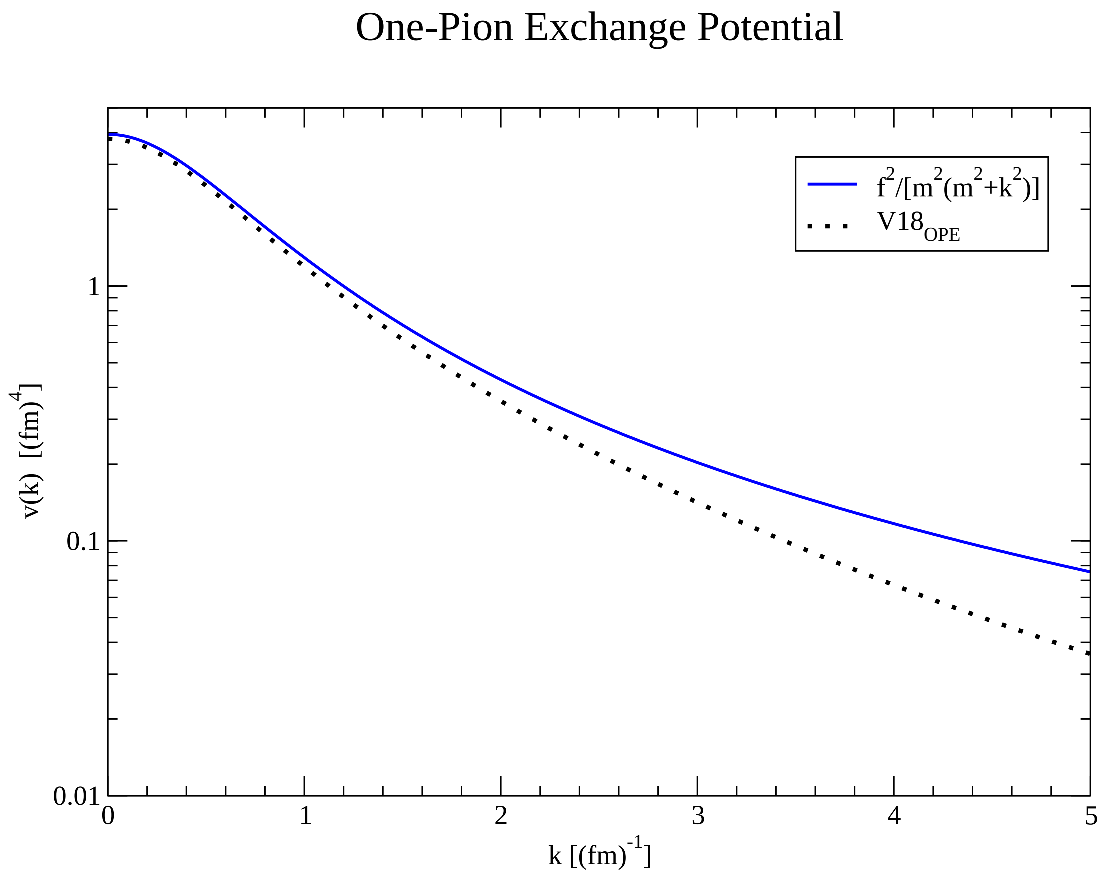

FIG. 4: One π-exchange interactions.

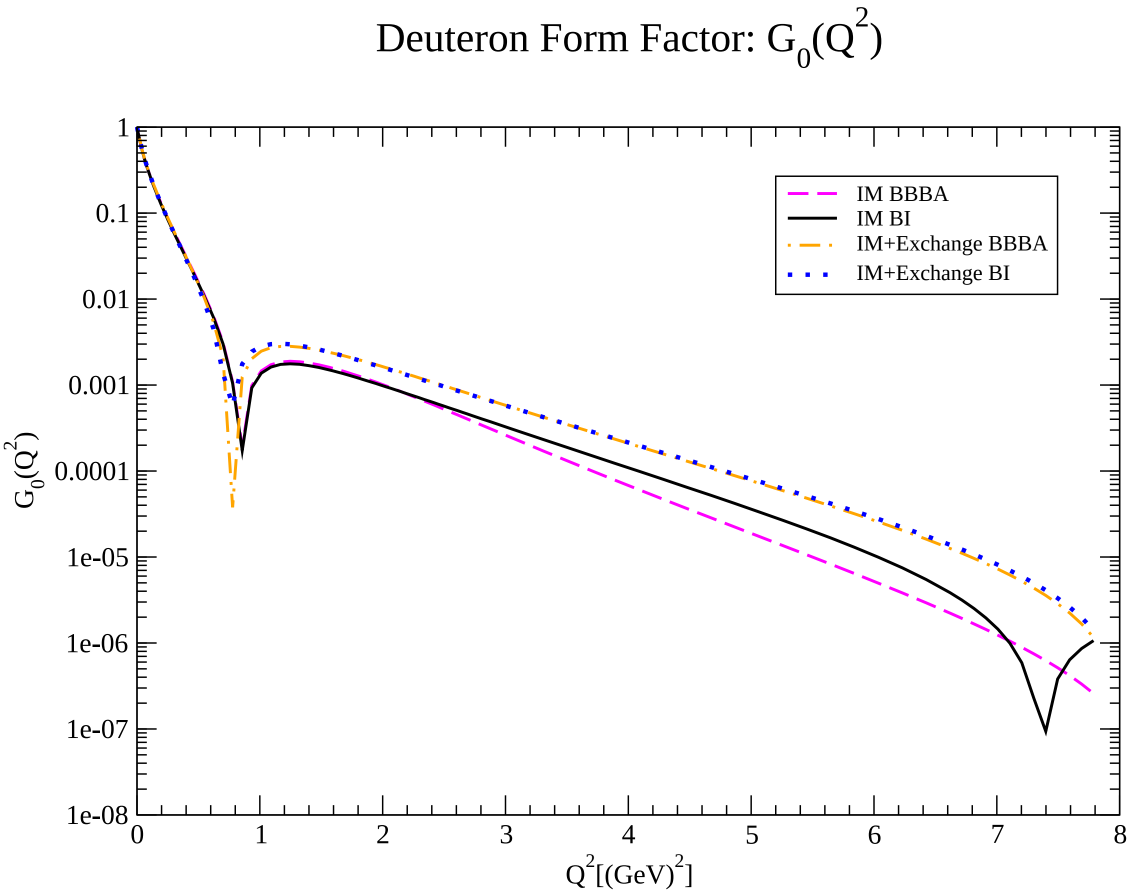

FIG. 5: Deuteron form factor $G_0(Q^2)$ with and without exchange current

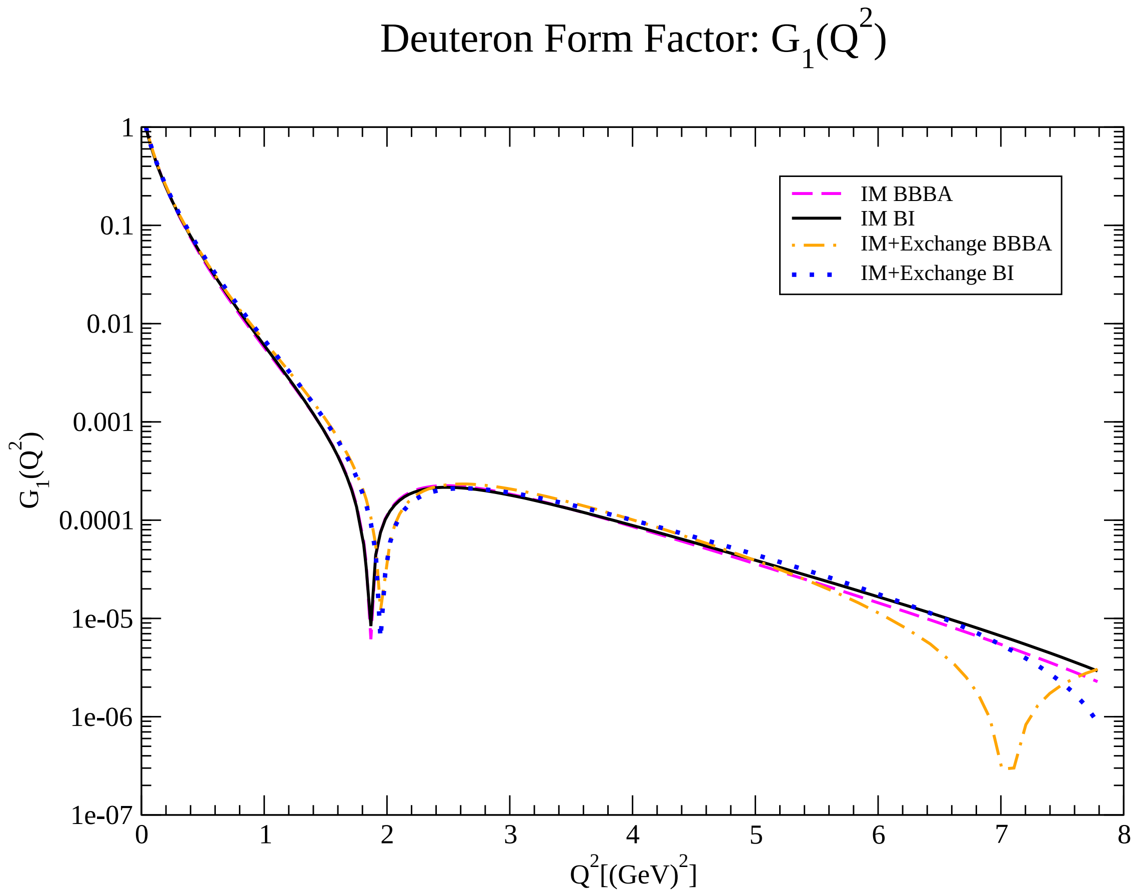

FIG. 6: Deuteron form factor $G_1(Q^2)$ with and without exchange current

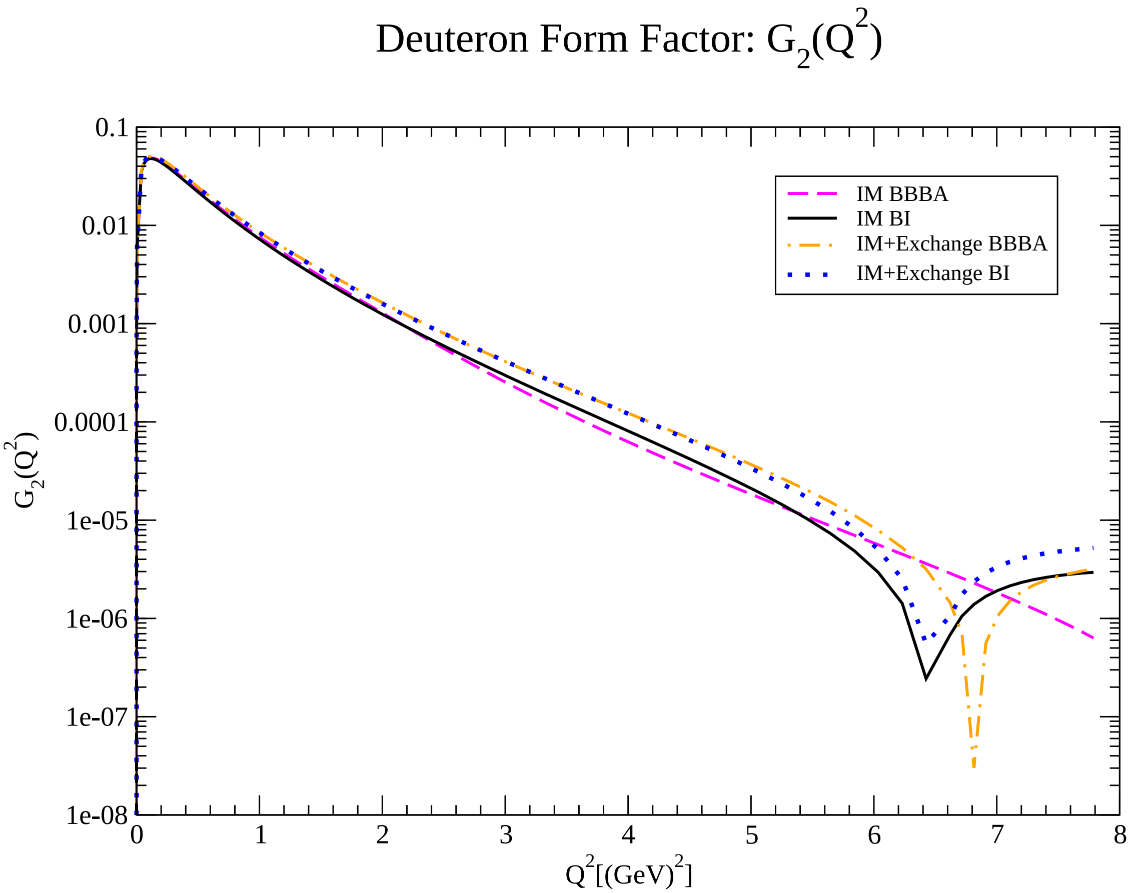

FIG. 7: Deuteron form factor $G_2(Q^2)$ with and without exchange current

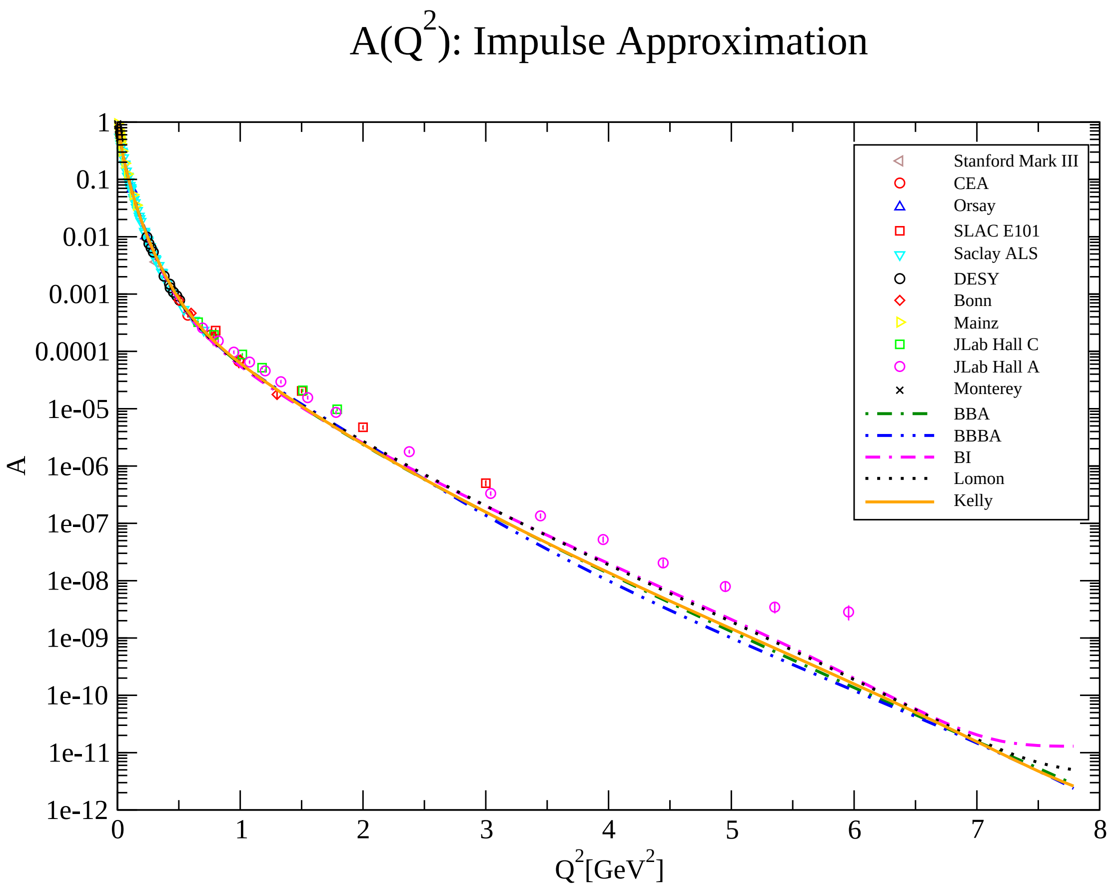

FIG. 8: $A(Q^2)$: Generalized impulse approximation

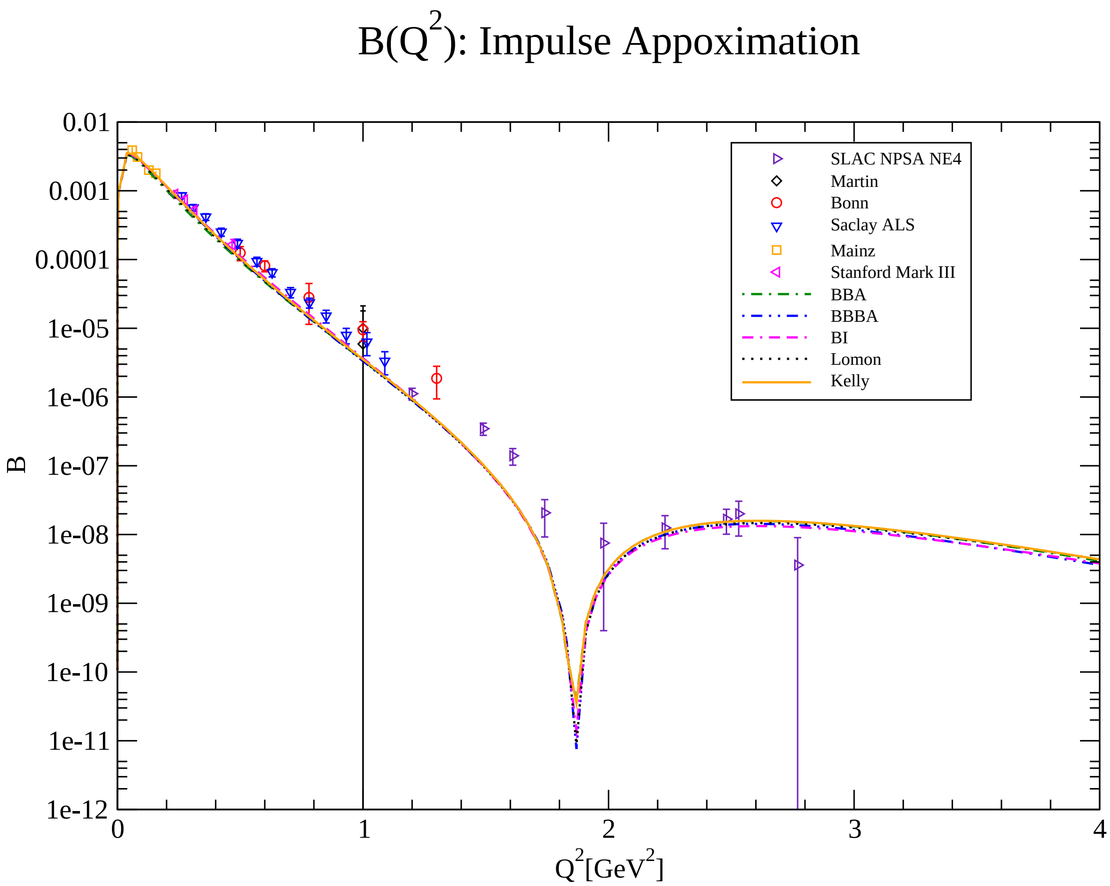

FIG. 9: $B(Q^2)$: Generalized impulse approximation

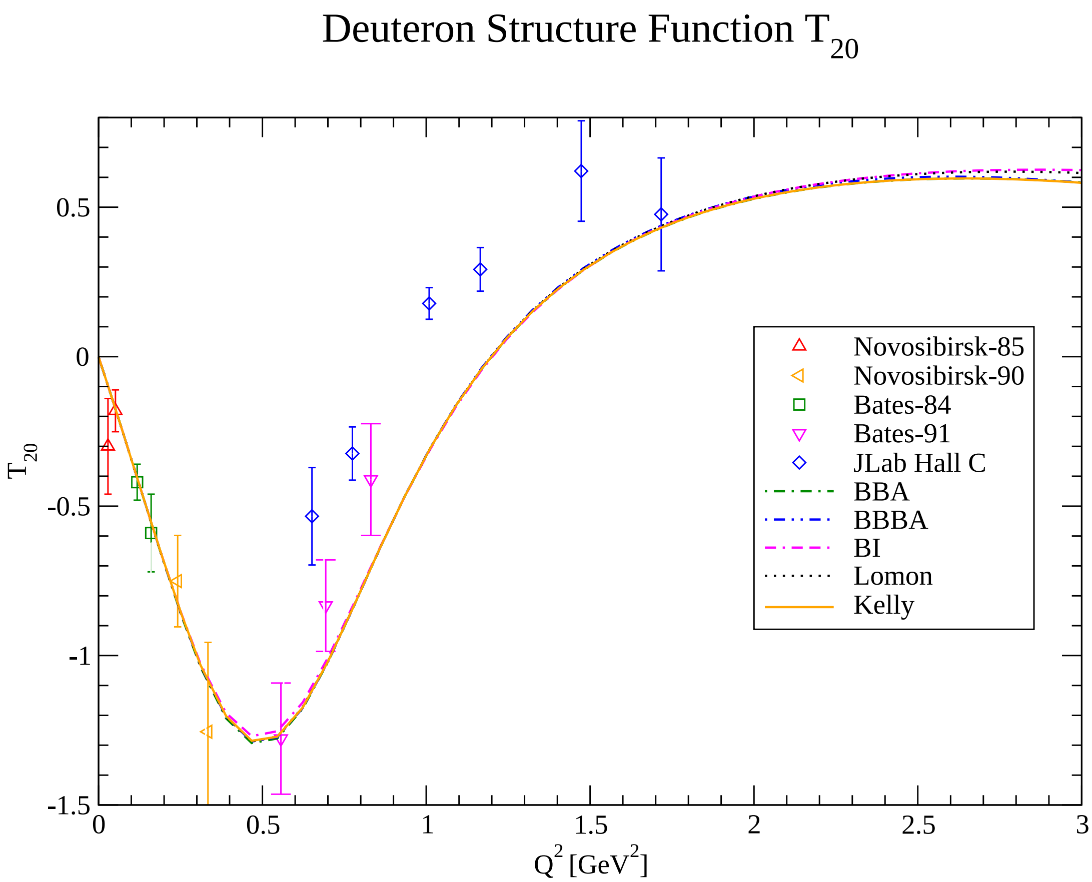

FIG. 10: $T_{20}(Q^2, 70^o)$: Generalized impulse approximation

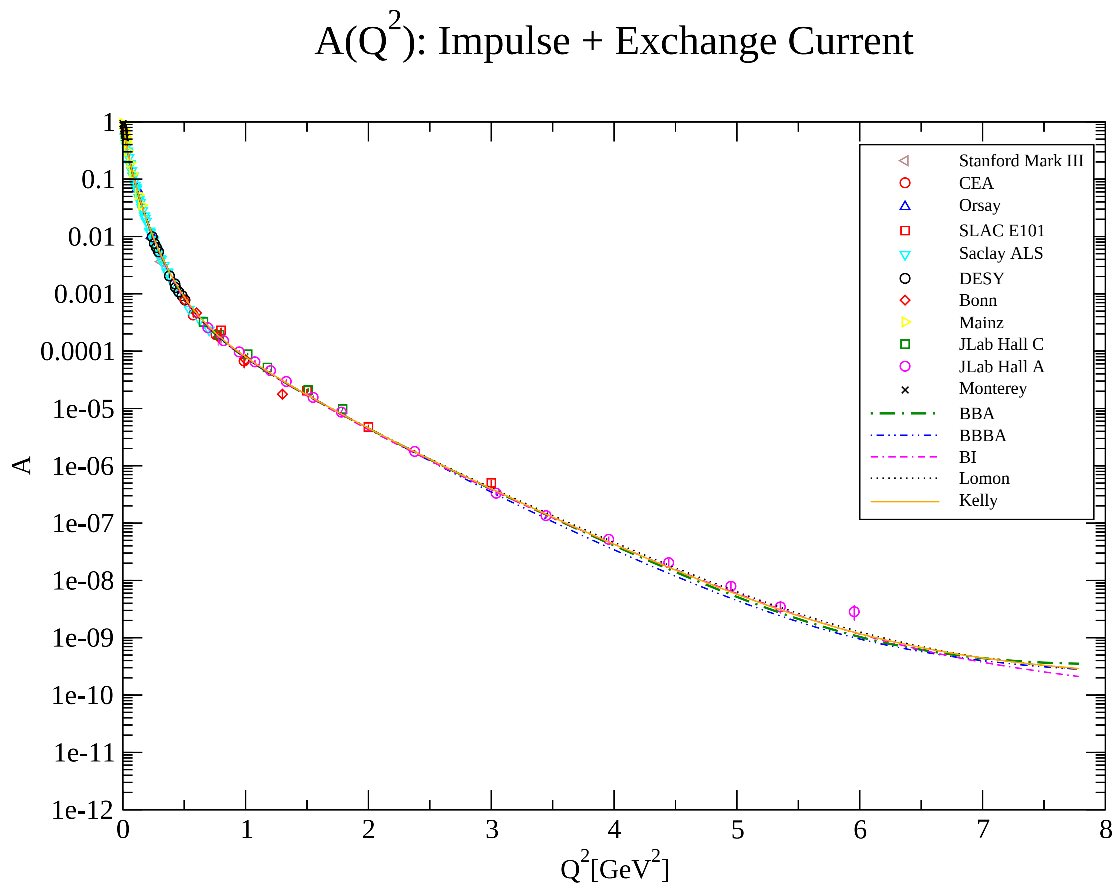

FIG. 11: $A(Q^2)$: Generalized impulse approximation plus exchange current I

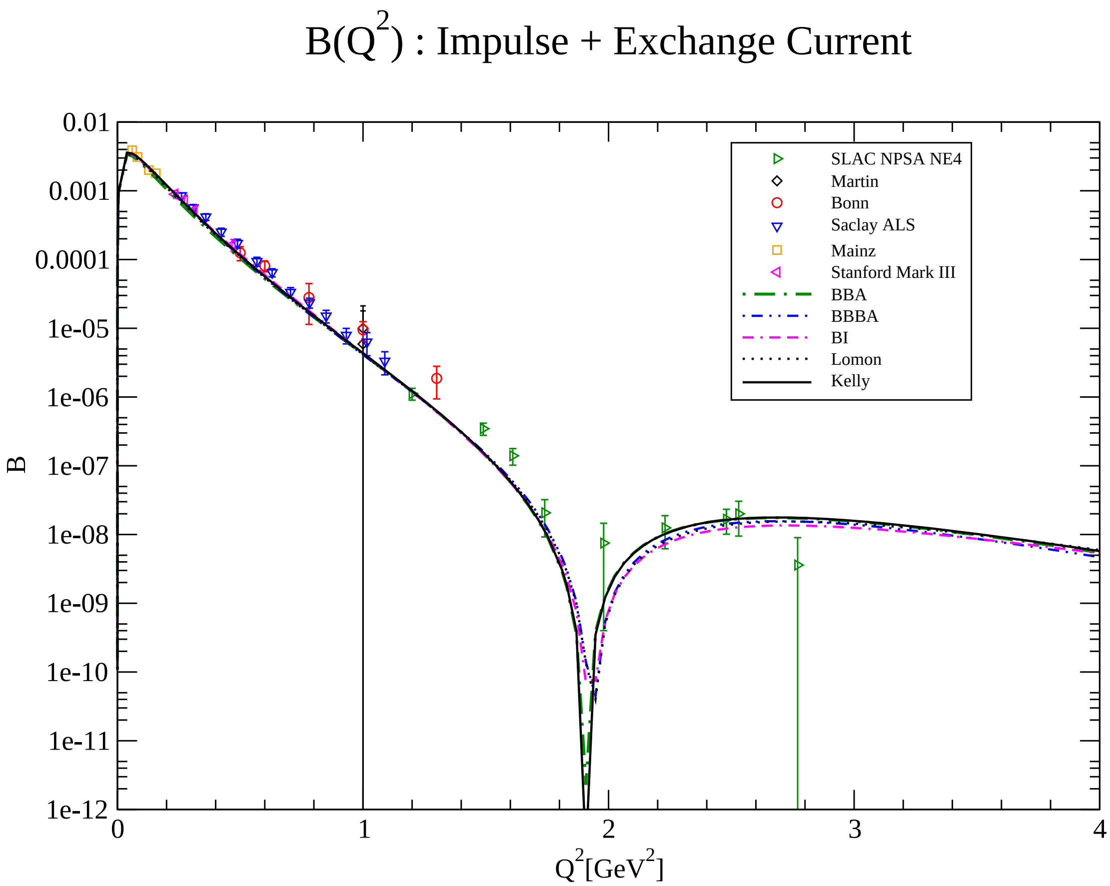

FIG. 12: $B(Q^2)$: Generalized impulse approximation plus exchange current I

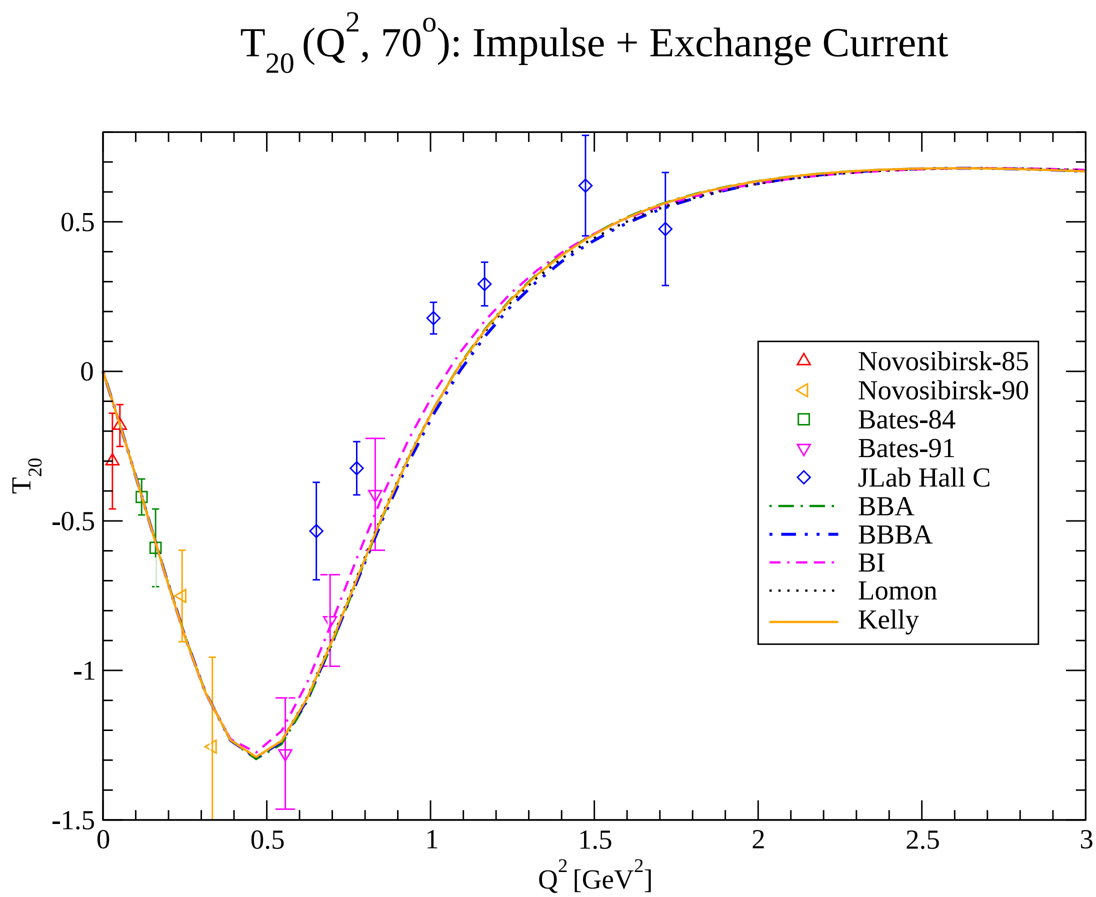

FIG. 13: $T_{20}(Q^2, 70^o)$: Generalized impulse approximation plus exchange current I

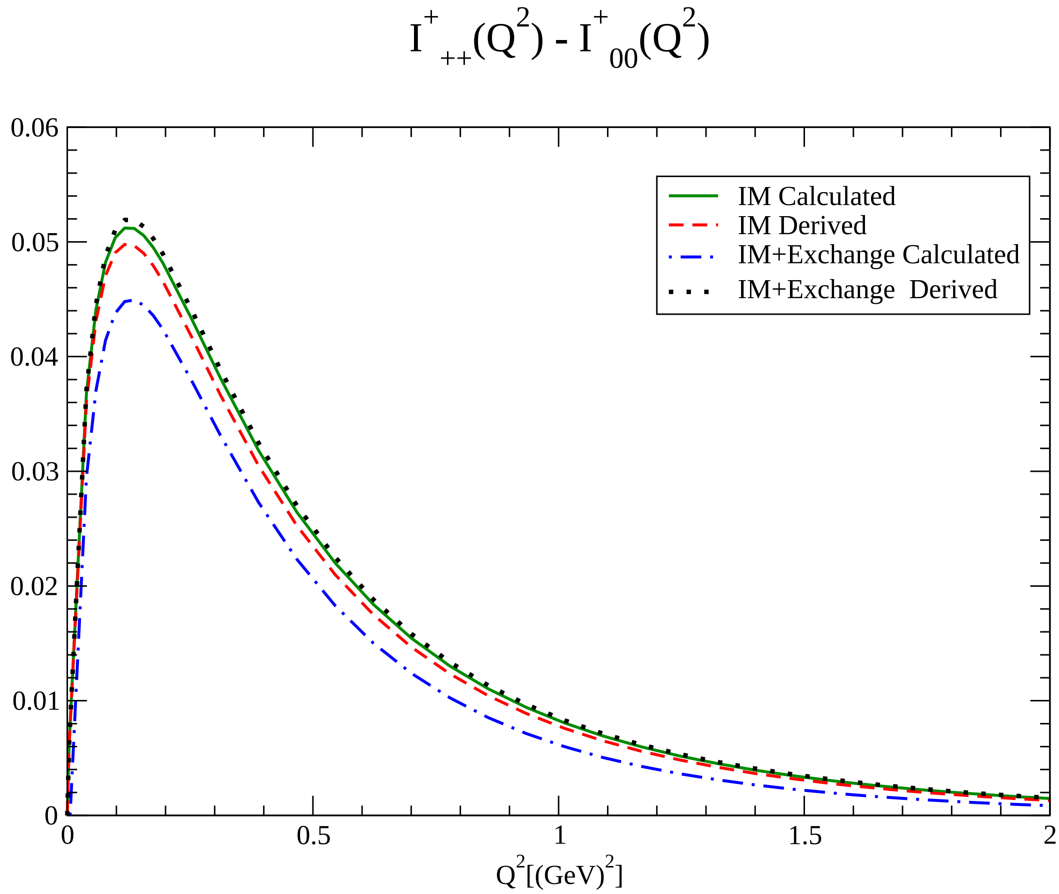

FIG. 14: Calculated vs. derived $I^{+}_{++}(0) - I^{+}_{00}(0)$ with and without exchange currrent I

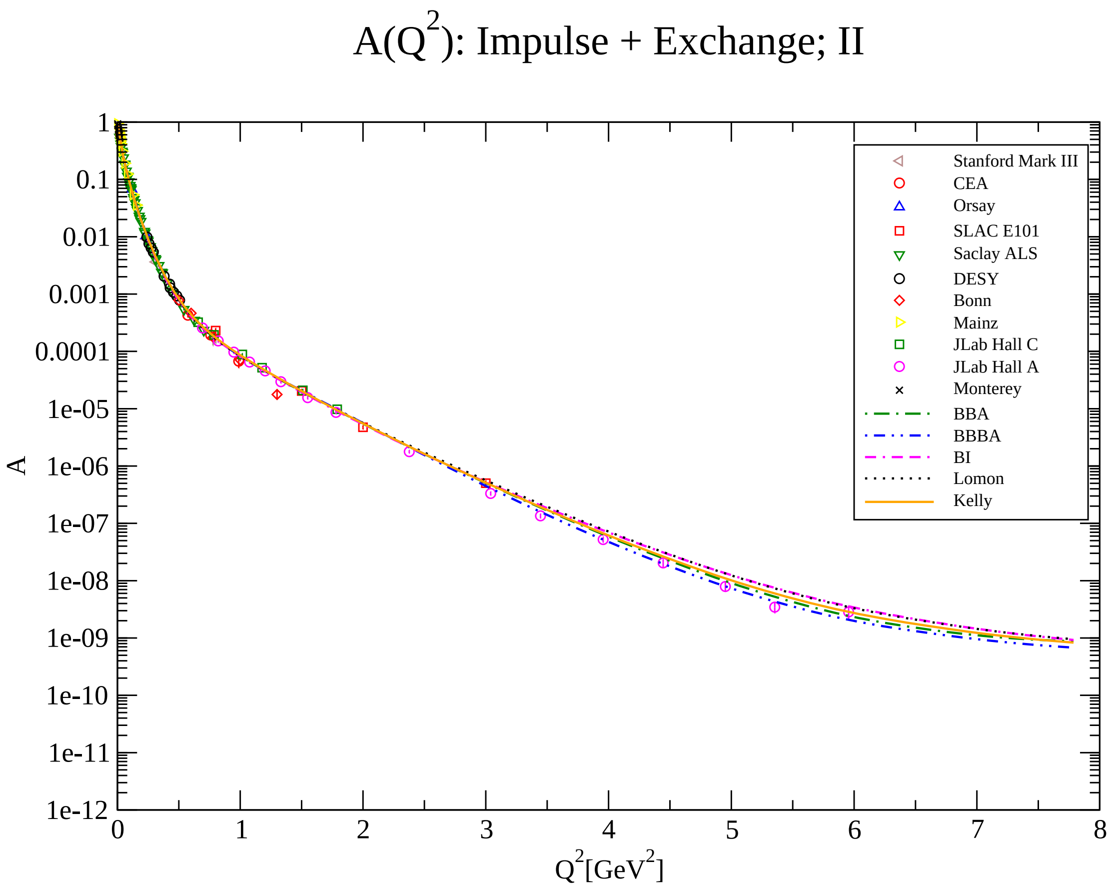

FIG. 15: $A(Q^2)$: Generalized impulse approximation plus exchange current II

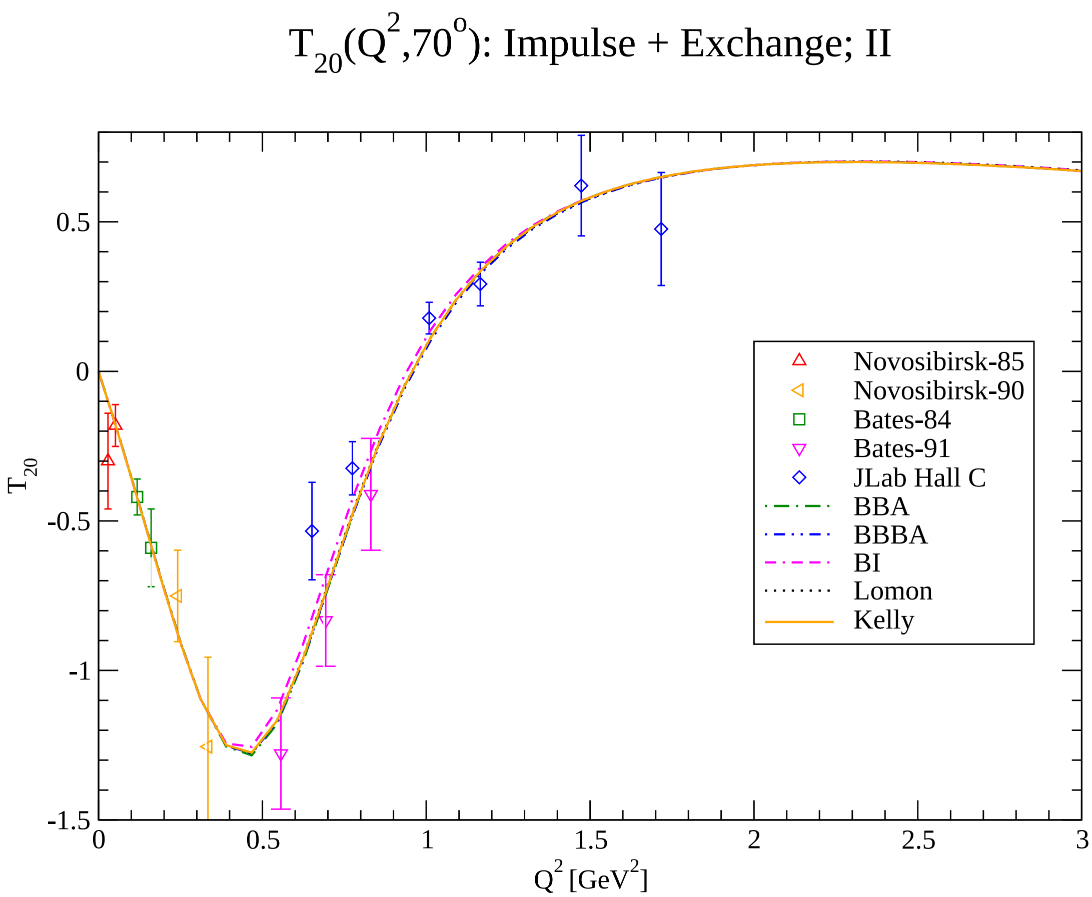

FIG. 16: $T_{20}(Q^2, 70^o)$: Generalized impulse approximation plus exchange current II

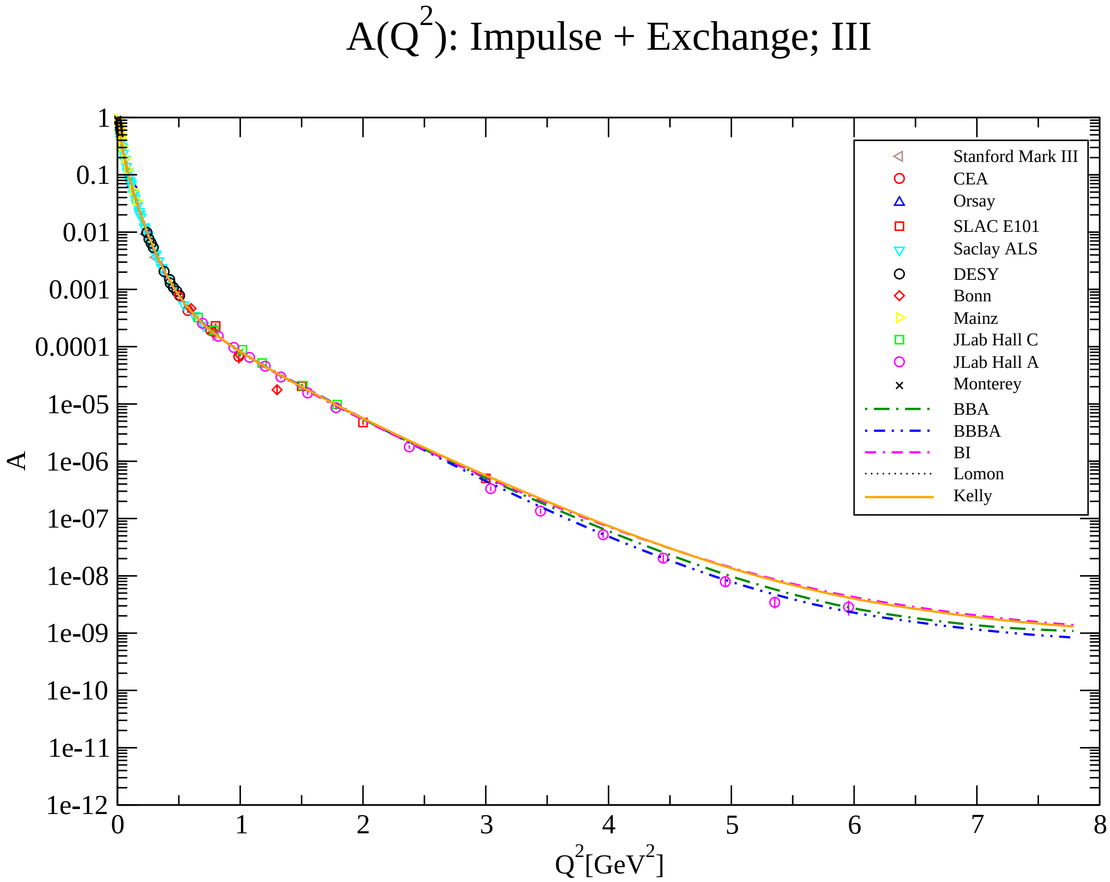

FIG. 17: $A(Q^2)$: Generalized impulse approximation plus exchange current III

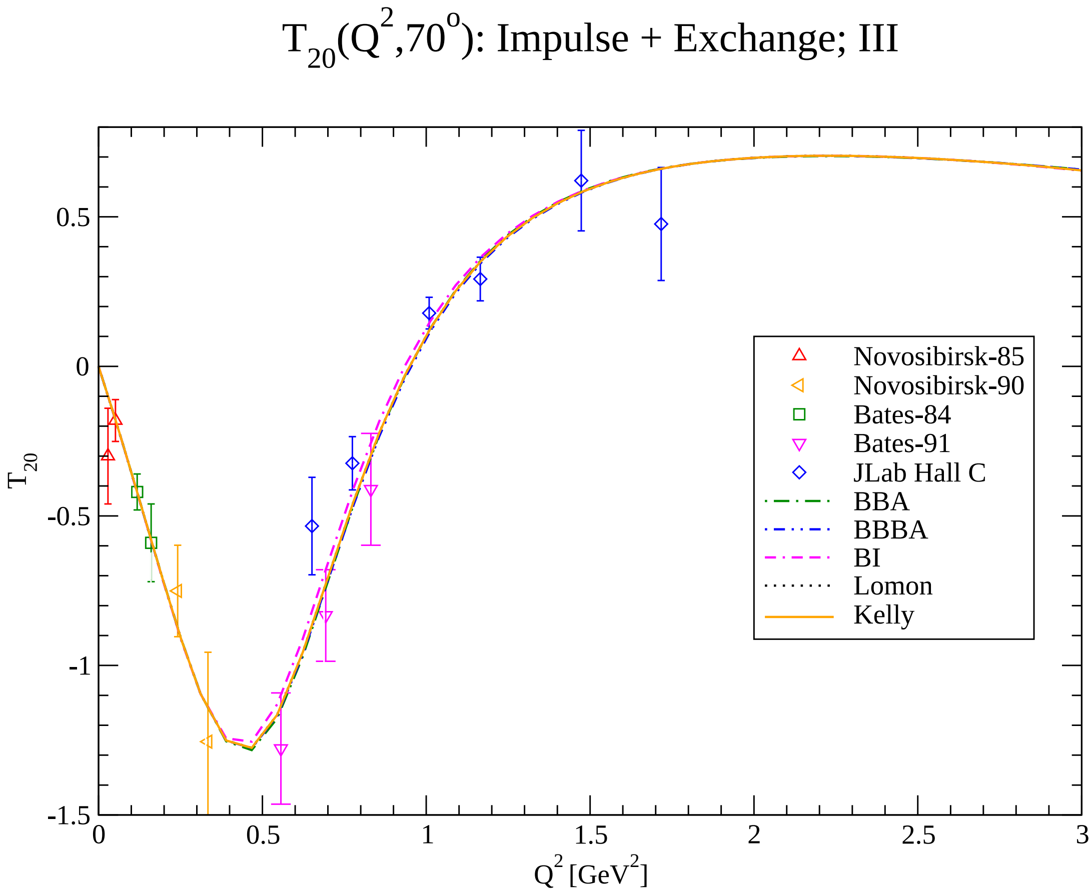

FIG. 18: $T_{20}(Q^2, 70^o)$: Generalized impulse approximation plus exchange current III

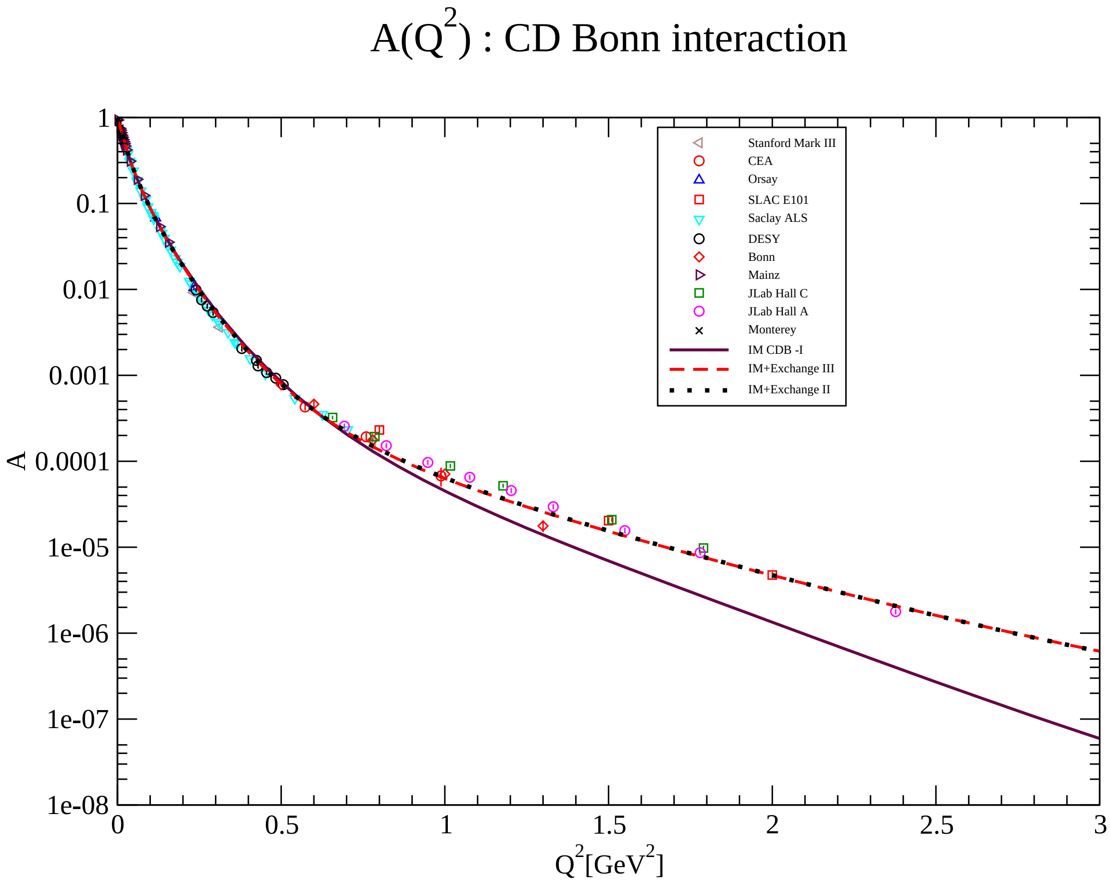

FIG. 19: $A(Q^2)$: CD Bonn with and without exchange current

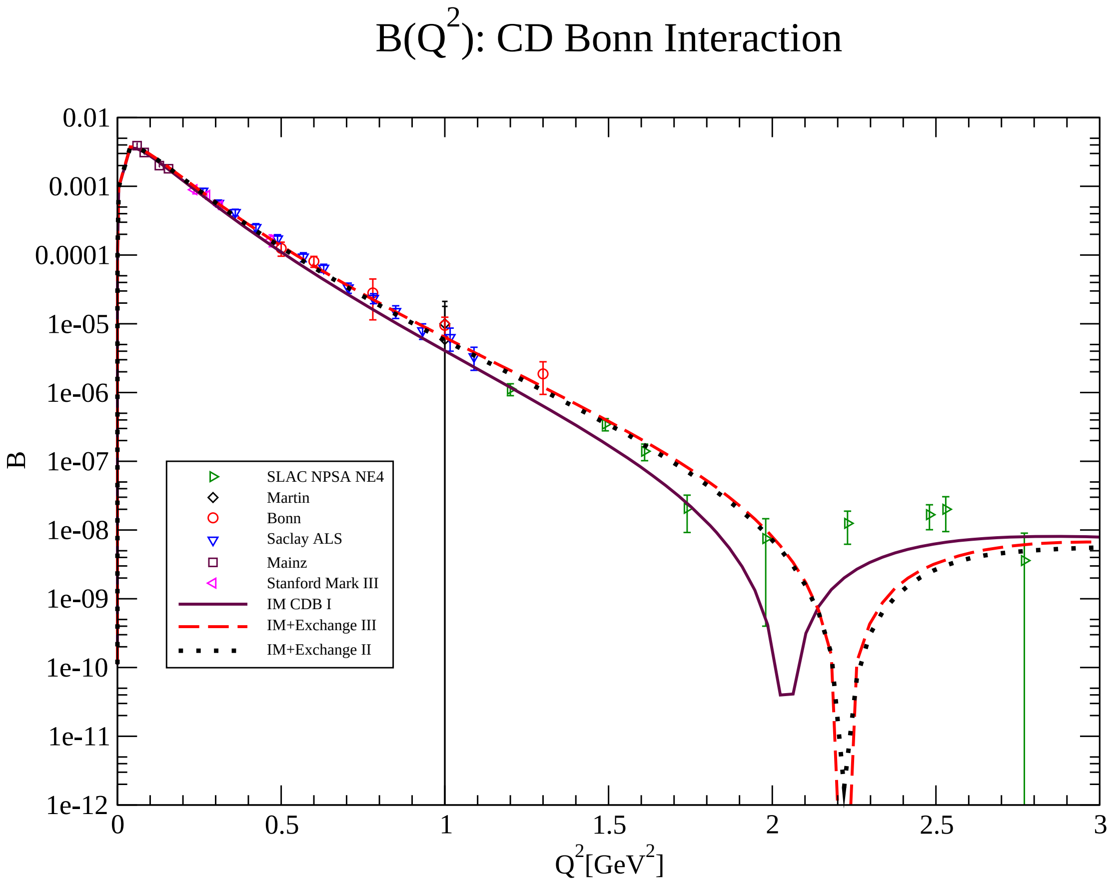

FIG. 20: $B(Q^2)$: CD Bonn with and without exchange current

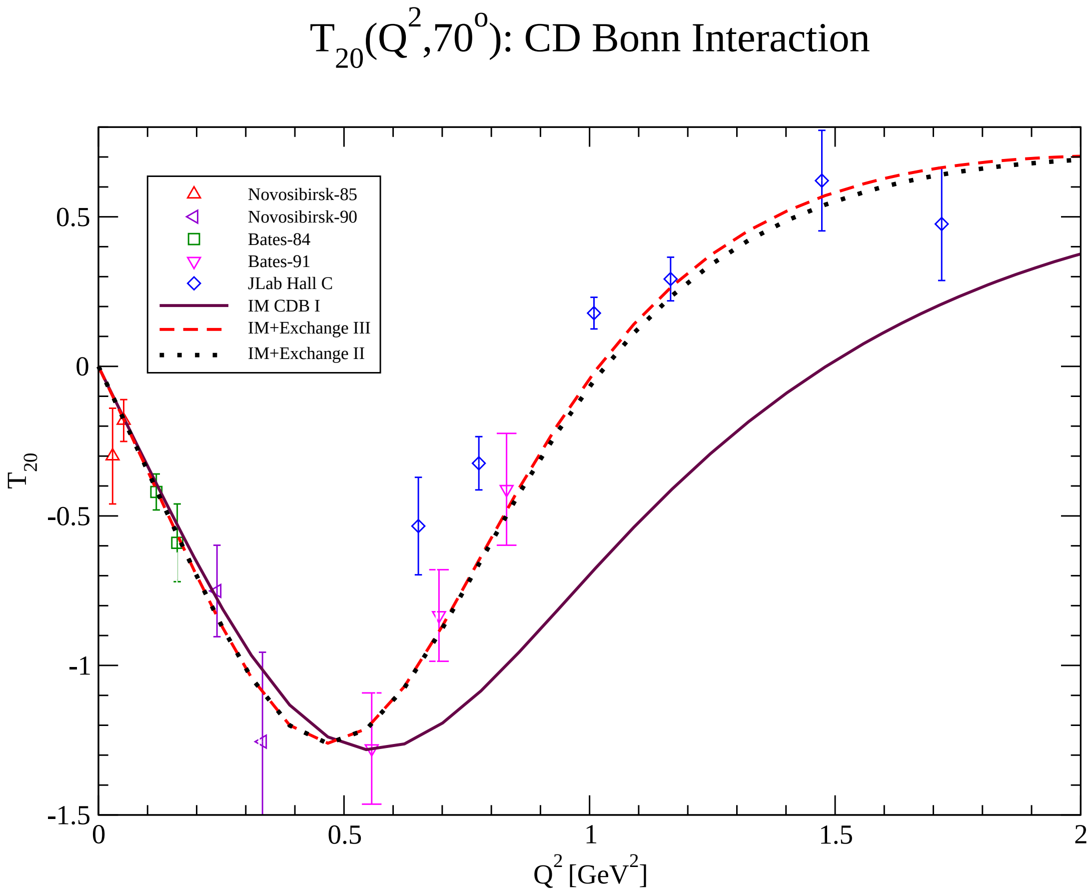

FIG. 21: $T_{20}(Q^2, 70^o)$: CD Bonn with and without exchange current

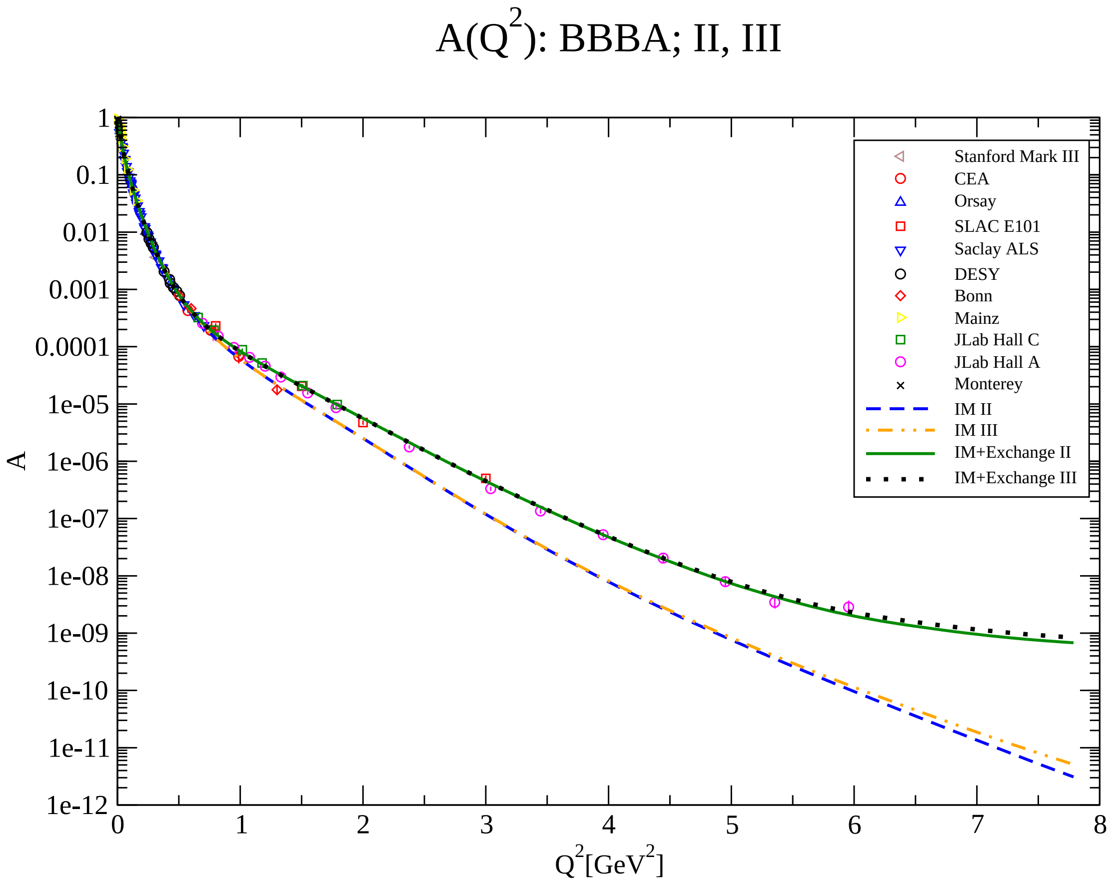

FIG. 22: $A(Q^2)$, V18,II, III, BBBA Form factors with and without exchange current

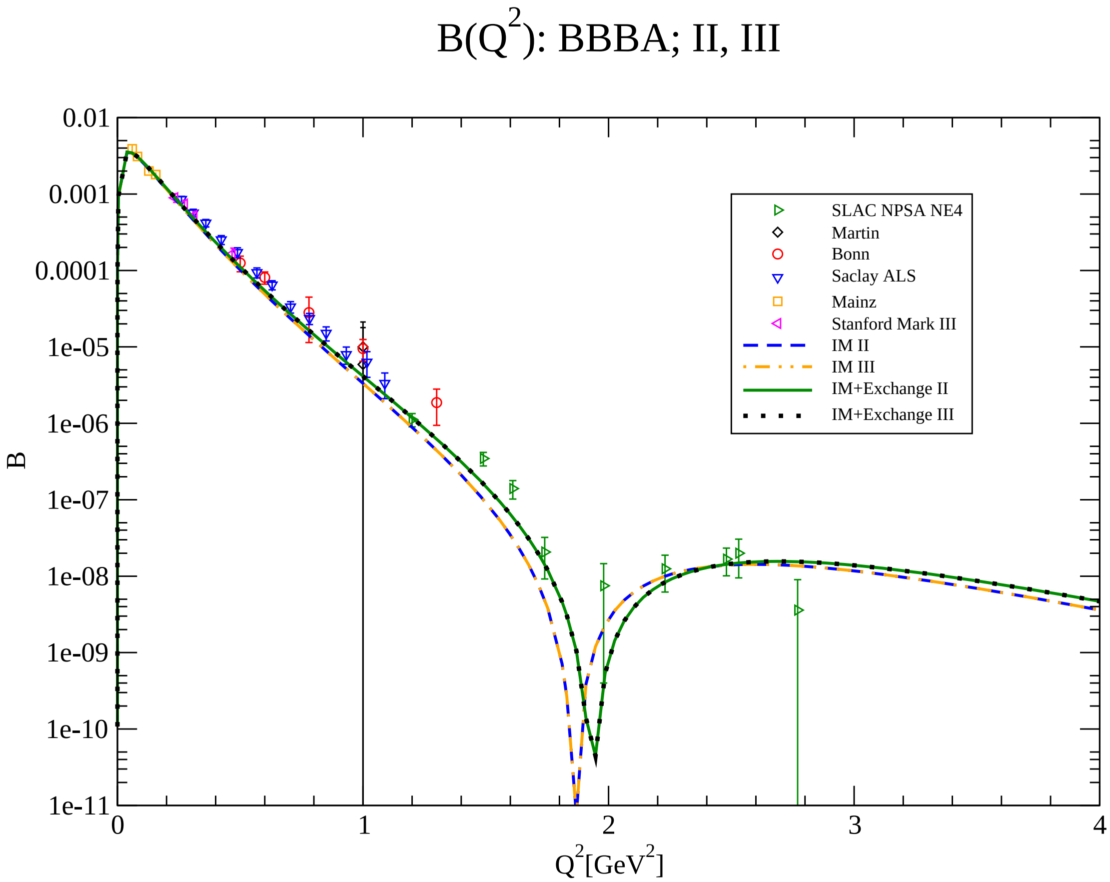

FIG. 23: $B(Q^2)$, V18,II, III, BBBA Form factors with and without exchange current

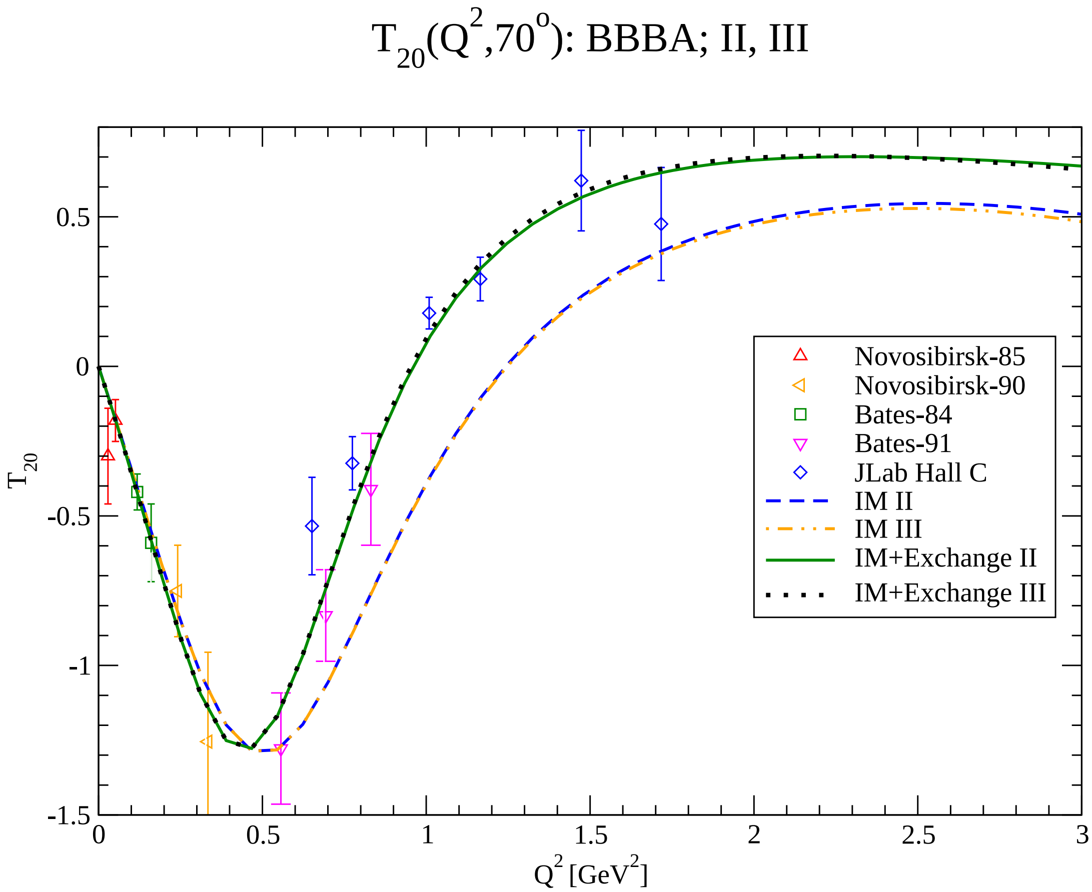

FIG. 24: $T_{20}(Q^2, 70^o)$, V18,II, III, BBBA Form factors with and without exchange current