Point-like constituent quarks and scattering equivalences

Evan Sengbusch* and W. N. Polyzou†
Department of Physics and Astronomy,
The University of Iowa
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Experiments can only determine a theory up to an overall scattering equivalence. The freedom to use different scattering equivalent theories is a useful tool for simplifying the structure of dynamical models of physical systems. This paper illustrates how scattering equivalences can be used to simplify current operators in relativistic constituent quark models.

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I. INTRODUCTION

Scattering equivalences are unitary transformations that leave the scattering matrix unchanged [5]. They parameterize the freedom to change the Hamiltonian and its eigenfunctions without changing experimental observables. In models with confining interactions, where scattering is not relevant, scattering equivalences are distinguished because they leave the constituent particle masses unchanged. In this paper a scattering equivalence is applied to a constituent quark model to construct an equivalent model with a point-like quark current operator.

The constituent quark model of Carlson, Kogut and Pandharipande [1] is used to illustrate the construction. This model is designed to fit the meson mass spectrum. The Hamiltonian in [1] is interpreted as the mass operator of a relativistic constituent quark model with a light-front kinematic subgroup. The construction of a consistent dynamical representation of the Poincaré group can be done following ref. [2][3][4].

It is possible [5] to find a conserved covariant quark current operator that is consistent with this model and fits the data [6][7] for the charge form factor of the pion. That this current cannot be a point-like impulse current is seen by direct calculation. The dashed curves in figures 1 and 2 show the pion charge form factor that would be predicted in this model if the quark current was a point-like current in the light-front impulse approximation[8][9]. Both the low- and high-momentum transfer predictions are inconsistent with the data.

A scattering equivalence is used to construct an equivalent model, with the same meson mass spectrum, where the quark current operator is transformed to an operator that can be accurately approximated by a point-like light-front impulse current operator.

The mass operator in the Carlson, Kogut, and Pandharipande model has the form

TABLE I: Interaction Parameters

Interaction	$m\,[GeV]$	$\lambda_1\,[GeV]$	λ_2	$\lambda_3\,[GeV]^2$	$\lambda_4\,[GeV]^{-1}$
V_c	.360	$-.777$	$-.5$	.197	.66
$\bar{V}_c$	.360	$-.911$	$-.14$	.049	.35

$$M = M_0 + V_c \qquad M_0 = \sqrt{m_q^2 + k^2} + \sqrt{m_{\bar{q}}^2 + k^2} \quad (1)$$

$$V_c(\lambda_1, \cdots, \lambda_4) = \lambda_1 + \frac{\lambda_2}{r} + \lambda_3 r + \delta \vec{s}_q \cdot \vec{s}_{\bar{q}} e^{-\frac{r^2}{4\lambda_4^2}} \quad (2)$$

$$\delta = -\frac{\lambda_2}{3 m_q^2 \lambda_4^3 \sqrt{\pi}}. \quad (3)$$

The parameters $\lambda_1, \cdots, \lambda_4$ of the original Carlson Kogut and Pandharipande model are listed on the first line of table 1.

The eigenvectors and eigenvalues of M are denoted by

$$M|m_n\rangle = m_n|m_n\rangle \quad (4)$$

where the $n = 0$ state is the π meson state and $m_0 = m_\pi$.

A scattering equivalence is a unitary operator of the form $A = I + \Delta$, where Δ satisfies [12]

$$\lim_{\tau \to \pm\infty} \|\Delta e^{-iM_0\tau}|\psi\rangle\| = 0. \quad (5)$$

In this paper Δ is taken to be proportional to a one-dimensional projection operator.

To construct Δ let $|\bar{m}_\pi\rangle$ be the lowest mass eigenstate of an operator $\bar{M}$ obtained from M by adjusting parameters $\{\lambda_1, \cdots, \lambda_4\} \to \{\bar{\lambda}_1, \cdots, \bar{\lambda}_4\}$ in the confining interaction:

$$\bar{M} = M_0 + \bar{V}_c \qquad \bar{M}|\bar{m}_\pi\rangle = \bar{m}_\pi|\bar{m}_\pi\rangle \quad (6)$$

$$\bar{V}_c = V_c(\bar{\lambda}_1, \cdots, \bar{\lambda}_4). \quad (7)$$

until the lowest mass eigenstate of $\bar{M}$ fits the pion charge form factor data using a point-like quark current in the

*Also at Department of Physics and Astronomy, The University of Iowa.
†Electronic address: polyzou@uiowa.edu

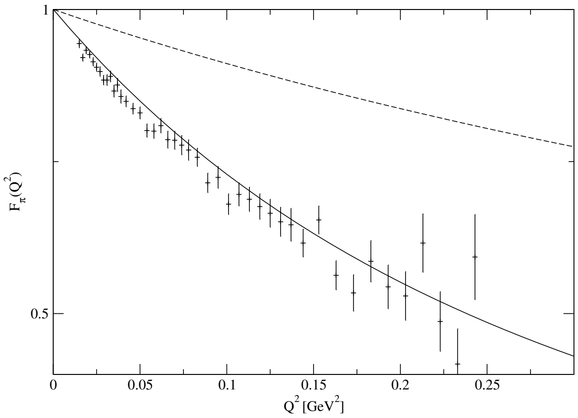

FIG. 1: Pion form factor at low Q^2.

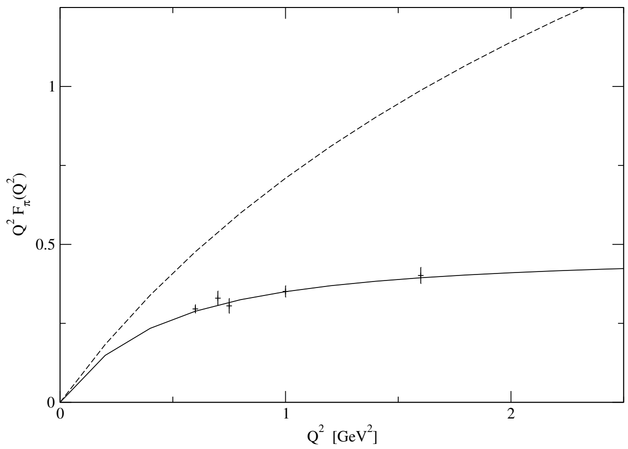

FIG. 2: Pion form factor at high Q^2.

light-front impulse approximation. Denote this vector by $|\bar{m}_\pi\rangle$. The spectrum of $\bar{M}$ does not agree with the observed meson masses. It should not be interpreted as a mass operator; its role is to generate candidates for the transformed ground state wave function that depend on a small number of parameters.

In this model the parameters on the second line of table 1 lead to a state $|\bar{m}_\pi\rangle$ that predicts the charge form factors given by the solid lines in figures 1 and 2 in the light-front impulse approximation. These curves are in better agreement with the experimental results.

To construct the desired transformation define two orthogonal bases on the two-dimensional subspace spanned by the vectors $|m_\pi\rangle$ and $|\bar{m}_\pi\rangle$. The orthonormal basis functions are

$$|m_\pi\rangle, \qquad |m_\perp\rangle := \frac{|\bar{m}_\pi\rangle - |m_\pi\rangle\cos(\theta)}{\sin(\theta)} \tag{8}$$

and

$$|\bar{m}_\pi\rangle, \qquad |\bar{m}_\perp\rangle := \frac{|m_\pi\rangle - |\bar{m}_\pi\rangle\cos(\theta)}{\sin(\theta)} \tag{9}$$

respectively, where

$$\cos(\theta) = \cos(\theta)^* := \langle m_\pi|\bar{m}_\pi\rangle \qquad \sin(\theta) > 0. \tag{10}$$

The overlap $\langle m_\pi|\bar{m}_\pi\rangle = \cos(\theta)$ can be chosen to be real as a consequence of the time-reversal invariance of V_c and $\bar{V}_c$.

The operator A is constructed to satisfy

$$A|m_\pi\rangle = |\bar{m}_\pi\rangle \tag{11}$$

$$A|m_\perp\rangle = |\bar{m}_\perp\rangle \tag{12}$$

and

$$A|\psi\rangle = |\psi\rangle \qquad \text{for} \qquad \langle m_\pi|\psi\rangle = \langle m_\perp|\psi\rangle = 0. \tag{13}$$

A scattering equivalence A satisfying (11),(12) and (13) is given by:

$$A = I + \Delta =$$

$$I - |m_\pi\rangle\langle m_\pi| - |m_\perp\rangle\langle m_\perp| + |\bar{m}_\pi\rangle\langle m_\pi| + |\bar{m}_\perp\rangle\langle m_\perp|. \tag{14}$$

After some algebra Δ can be expressed as multiple of a one-dimensional projection operator:

$$\Delta = -\rho(|m_\pi\rangle - |\bar{m}_\pi\rangle)(\langle m_\pi| - \langle\bar{m}_\pi|) \tag{15}$$

where

$$\rho = \frac{\cos(\theta)+1}{\sin^2(\theta)}. \tag{16}$$

In this example the overlap parameter is

$$\cos(\theta) := \langle m_\pi|\bar{m}_\pi\rangle = .731. \tag{17}$$

The scattering equivalence A is used to define a transformed mass operator

$$M' := A^\dagger(M_0 + V_c)A = M_0 + V_c' \tag{18}$$

which leads to an equivalent quark model with confining interaction

$$V_c' = V_c + \Delta M + M\Delta^\dagger + \Delta M \Delta^\dagger =$$

$$V_c + \rho(|m_\pi\rangle - |\bar{m}_\pi\rangle)\langle\bar{m}_\pi|(V_c - \bar{V}_c)$$

$$+\rho(V_c - \bar{V}_c)|\bar{m}_\pi\rangle(\langle m_\pi| - \langle\bar{m}_\pi|)$$

$$+\rho^2(|m_\pi\rangle - |\bar{m}_\pi\rangle)\langle\bar{m}_\pi|(V_c - \bar{V}_c)|\bar{m}_\pi\rangle(\langle m_\pi| - \langle\bar{m}_\pi|). \tag{19}$$

The method of constructing $|\bar{m}_\pi\rangle$ leads to a confining interaction with separable terms that has the same structure as the original confining interaction.

By construction the mass operator M':

a.) Has the same spectrum as M.

b.) Has the same pion wave function as $\bar{M}$

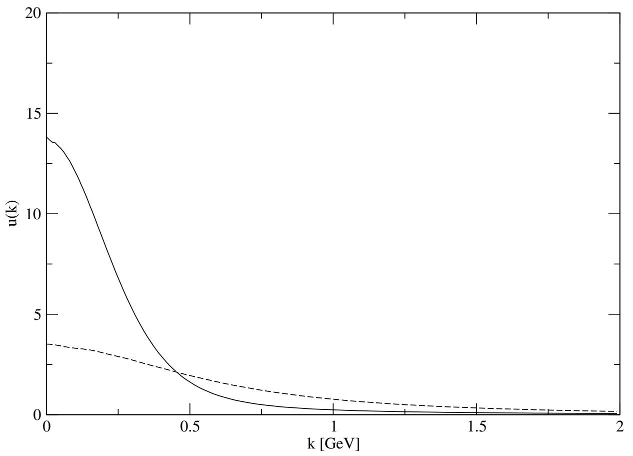

FIG. 3: Comparison of k-space wave functions.

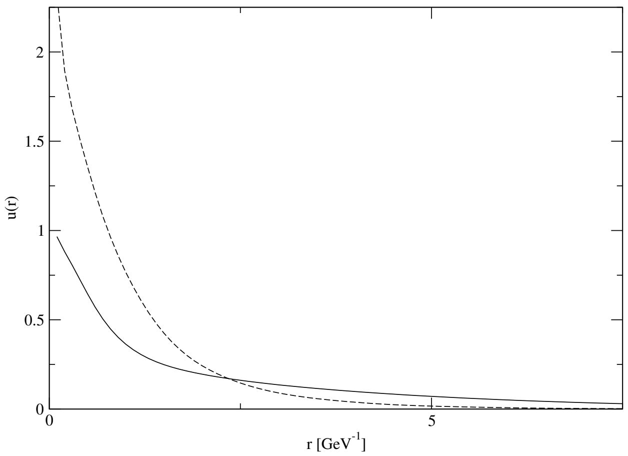

FIG. 4: Comparison of r-space wave functions.

c.) Differs from M by the short range modification (19) to the confining interaction.

Because the operator Δ is kinematically invariant, the operator M' is also a mass operator for a unitary representation of the Poincaré group with the light-front kinematic subgroup.

Even though the scattering equivalence A is constructed to transform the ground state wave function, it also transforms the $n > 0$ states to preserve orthogonality. The transformed states have the form

$$|m'_n\rangle = A|m_n\rangle = |m_n\rangle + \rho\langle\bar{m}_\pi|m_n\rangle\left(|m_\pi\rangle - |\bar{m}_\pi\rangle\right). \quad (20)$$

The operator M' has the same mass spectrum as the original Carlson, Kogut, Pandharipande mass operator and has a pion eigenstate that can be used with a point-quark impulse quark current to obtain the measured pion charge form factor. The pion charge form factor in this model is given by the solid curves in figures 1 and 2.

Figures 3 and 4 compare the coordinate (figure 3) and momentum space (figure 4) wave functions of the pion for the mass operators M (dotted curve) and M' (solid curve). The calculation of the wave functions are done using the method described in [10]. The Carlson, Kogut, and Pandharipande wave functions have a smaller size in configuration space and more high-momentum components than the wave-functions of the transformed mass operator.

The transformation A has the overall effect of softening the wave functions of the original Carlson, Kogut, and Pandharipande model. This is consistent with the calculations of Cardarelli et. al. [11] who use a similar constituent quark model and introduce single-quark form factors to obtain measured pion charge form factors.

The effect of the transformation A is to add an additional short range structure to the original confining interaction. Compared to the original Carlson, Kogut, and Pandharipande interaction, the additional short-ranged part contains non-localities. In a relativistic quantum theory there is no preferred reason to favor a local over a non-local interaction except for mathematical simplicity. Local and non-local interactions have similar strengths and defects. In relativistic models with short-ranged interactions, both the local and non-local interactions are consistent with two-body cluster properties, but both fail to be consistent with microscopic locality. Microscopic derivations of local interactions necessarily make implicit assumptions that lead to local interaction, however these assumptions are not based on physics principles. There is no reason to consider the transformed interaction to be any more or less fundamental than the original interaction.

II. CONCLUSIONS

In this paper the freedom to change the mass operator and wave functions without changing the underlying physics is used to construct a constituent quark model that fits the meson mass spectrum and reproduces the pion form factor using only point-like constituent quarks.

In this example the desired scattering equivalence is an easily constructed rank-one perturbation of the identity. The only required input is the new state vector $|\bar{m}_\pi\rangle$ and the overlap $\langle m_\pi|\bar{m}_\pi\rangle$. The new representation does not require constituent quark/antiquark form factors.

While two-body currents are implicitly generated in the light-front impulse approximation [8], they are not explicitly needed to compute the form factor.

The choice of Δ affects the predictions for elastic and transition form factors involving other mesons. Scattering equivalences with higher rank Δs can be used to further simplify the current in these models if it is warranted by the physics.

In general it may not be possible to find a scattering equivalence that completely removes the many-body contributions to a current operator. In this example, while figure 1 shows considerable improvement in the agreement with experiment when it is compared to the original model, the agreement at lower values of momentum transfer is not within the small experimental error bars. Wave functions that provide good fits to both the high and low-momentum transfer data using point-like impulse currents are easily constructed in constituent quark models models with slightly lower constituent quark masses [9]. While these wave functions could be used in the construction of Δ, the results would be different because the constituent quark mass also appears in the Clebsch-Gordan coefficients [3] of the Poincaré group that are used to compute matrix elements of the point-like current operators.

This example started with a relativistic constituent quark model with a light-front kinematic symmetry, and produced another relativistic model with the same light-front kinematic symmetry, the same mass spectrum, where the pion charge form factor can be computed in the point-quark impulse approximation. This example illustrates that the constraints imposed by the choice of kinematic subgroup and mass spectrum do not determine the form factors.

The freedom to choose a particular scattering equivalent representation for a dynamical computation is analogous to choosing a convenient coordinate system. This freedom may prove to be important because scattering equivalences in few-body models naturally lead to modifications of the corresponding many-body theory. This freedom could be used to reduce the strength of many-body interactions and current operators.

Scattering equivalences are also known to exist [12] between models with different kinematic subgroups. These relationships can also be exploited to construct equivalent current operators for models with different forms of dynamics. Existing calculations show that the representation of the currents in different forms of dynamics can be very different [8][9][11][13][14][15][16]. This same freedom also exists in local quantum field theory, where theories in the same Borchers class [17] are scattering equivalent. While the ambiguities in representations of the dynamics are sometimes considered a liability, this paper shows that they lead to a flexibility that can lead to a simplification of the dynamics.

Acknowledgments

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