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Few-Body Systems

Relativity in the 3-nucleon system

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Abstract.
A Poincaré invariant formulation of the 3-body system is used. The 2-body force embedded in the 3- particle Hilbert space is generated out of the high-precision NN forces by solving a nonlinear equation. The solution of the relativistic 3N Faddeev equation for ^3H reveals less binding energy than for the nonrelativistic one. The effect of the Wigner spin rotation on the binding energy is very small.

1 Introduction

The nonrelativistic NN force v^{nr} is added to the nonrelativistic kinetic energy $\frac{k^2}{m}$, while the relativistic NN force v is added to the relativistic kinetic energy $2\omega(k) = 2\sqrt{k^2+m^2}$. They give the same NN S-matrix, $S(k)$, if they are related by:

$$v = \sqrt{4mv^{nr} + 4(k^2+m^2)} - 2\sqrt{k^2+m^2} = \sqrt{4mv^{nr} + 4\omega^2} - 2\omega \tag{1}$$

In former work [1, 2] we employed an analytic scale transformation of the momenta

$$2m + (k^{nr})^2/m \equiv 2\sqrt{q^2+m^2} \tag{2}$$

such that the relativistic and nonrelativistic NN phase shifts are exactly equal at the same energy, but not at the same c.m. momenta as they should [3]. We introduce in section 2 a new technique [4] to solve Eq.(1) and show relativistic and nonrelativististic results for the ^3H binding energy based on high precision NN forces in section 3.

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Table 1. Convergence of the $v_p^{(n)}$ iteration in Eq. (5) at total momentum $p = 0$. We choose the coupled partial waves ($^3S_1$-$^3D_1$) of the Argonne V18 potential. The momenta k and k' are 1.0 fm^{-1} and the potential unit is [fm^2].

n	$v_0^{(n)}(^3S_1\text{-}^3S_1)$	$v_0^{(n)}(^3S_1\text{-}^3D_1)$	$v_0^{(n)}(^3D_1\text{-}^3D_1)$
0	0.084232	0.044709	0.016853
1	0.067716	0.044628	0.016785
2	0.059933	0.044597	0.016744
3	0.056135	0.044587	0.016719
10	0.052194	0.044595	0.016684
20	0.052126	0.044597	0.016684
30	0.052126	0.044597	0.016684

2 The nonlinear equation and its solution

The NN interactions enter the 3N mass operator in the form [6]

$$\begin{aligned} v_p &= \sqrt{(2\omega(k)+v)^2+p^2} - \sqrt{(2\omega(k))^2+p^2} \\ &= \sqrt{4mv^{nr}+4\omega^2+p^2} - \sqrt{(2\omega(k))^2+p^2} \\ &= \sqrt{4mv^{nr}+4\omega_p^2} - 2\omega_p \end{aligned} \tag{3}$$

where p is the invariant relative momenta related to the 3N invariant mass by $M_0 = 2\omega_p = 2\sqrt{k^2+m^2+p^2/4}$. The p-dependence arises because in a 3-body system the NN subsystems are not at rest.

The new way [4] is to directly solve the quadratic operator equation (3). After partial wave decomposition in momentum space it reads

$$\begin{aligned} &< k|v_p|k\,'>= \frac{2m < k|v^{nr}|k\,'>}{\omega_p(k)+\omega_p(k\,')} \\ &-\frac{1}{2(\omega_p(k)+\omega_p(k\,'))}\int_0^\infty dk\,''k\,''^2 < k|v_p|k\,''>< k\,''|v_p|k\,'> \end{aligned} \tag{4}$$

We verified numerically that for all the realistic high precision potentials the following very simple iterative scheme works

$$\begin{aligned} &< k|v_p|k\,'>^{(0)}= \frac{2m < k|v^{nr}|k\,'>}{\omega_p(k)+\omega_p(k\,')}, \\ &< k|v_p|k\,'>^{(n+1)}= \frac{1}{2(\omega_p(k)+\omega_p(k\,'))}\{4m < k|v^{nr}|k\,'> \\ &-\int_0^\infty dk\,''k\,''^2 < k|v_p|k\,''>^{(n)}< k\,''|v_p|k\,'>^{(n)}\} \end{aligned} \tag{5}$$

with $n = 0, 1, 2, \cdots$. We demonstrate in Table 1. the convergence of of this Eq. (5) in the case of $p = 0$.

3 Relativistic versus nonrelativistic binding energies of ^{3}H

In our calculation of the ^{3}H binding energy the relativistic Faddeev equation (Eq. (3.7) in [2]) is solved in a partial wave basis. The feasibility of our approach is

Table 2. The relativistic (E_b)and nonrelativistic (E_b^{nr})triton binding energies in MeV together with $\Delta \equiv E_b - E_b^{nr}$. The superscripts of (1) and (2) correspond to Eqs. (1) (direct connection) and (2) (scale transformation), respectively.

Potential	E_b^{nr}	$E_b^{(1)}$	$\Delta^{(1)}$	$E_b^{(2)}$	$\Delta^{(2)}$
RSC	-7.02	-6.97	0.05	-6.59	0.43
CD-Bonn	-8.33	-8.22	0.11	-7.98	0.35
Nijmegen II	-7.65	-7.58	0.07	-7.22	0.43
Nijmegen I	-8.00	-7.90	0.10	-7.71	0.29
Nijmegen 93	-7.76	-7.68	0.08	-7.46	0.30
AV18	-7.66	-7.59	0.07	-7.23	0.43
Exp. (-8.48)					

demonstrated in Table 2 for a five-channel calculation (NN forces only in the states $^1S_0$ and $^3S_1$-$^3D_1$). There are two kinds of relativistic results, $E_b^{(1)}$ and $E_b^{(2)}$, in comparison to the nonrelativistic result E_b^{nr}. The present results, $E_b^{(1)}$, based on (5) show a significantly smaller reduction of the binding energy than $E_b^{(2)}$, what we obtained by the momentum scale transformation in [5] and [2]. The Wigner spin rotation is handled using the Balian-Brezin method [7]. Its effect in reducing the binding energy is about 1 Kev and thus marginal.

For the relativistic calculation of 3N scattering and the basic relativistic formulation see [8], H.Witala, and W.N.Polyzou in this conference.

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