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Constraints of cluster separability and covariance on current operators

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Abstract Realistic models of hadronic systems should be defined by a dynamical unitary representation of the Poincaré group that is also consistent with cluster properties and a spectral condition. All three of these requirements constrain the structure of the interactions. These conditions can be satisfied in light-front quantum mechanics, maintaining the advantage of having a kinematic subgroup of boosts and translations tangent to a light front. The most straightforward construction of dynamical unitary representations of the Poincaré group due to Bakamjian and Thomas fails to satisfy the cluster condition for more than two particles. Cluster properties can be restored, at significant computational expense, using a recursive method due to Sokolov. In this work we report on an investigation of the size of the corrections needed to restore cluster properties in Bakamjian-Thomas models with a light-front kinematic symmetry. Our results suggest that for models based on nucleon and meson degrees of freedom these corrections are too small to be experimentally observed.

Keywords Relativistic quantum mechanics · cluster properties · light-front quantum mechanics

1 Introduction

Light-front quantum mechanics (1) has the desirable feature that it is easy to construct realistic quantum mechanical models that are exactly Poincaré invariant(2), with a kinematic subgroup that leaves a light front invariant. The most straightforward construction, due to Bakamjian and Thomas (3)(4), achieves this by requiring both the generators of the kinematic subgroup and the total light-front spin to be non-dynamical. A dynamical mass operator is defined by adding interactions that commute with the kinematic generators and the kinematic light-front spin to the non-interacting invariant mass. The dynamical Poincaré generators are well-defined functions of this mass operator and these kinematical operators.

While the original Bakamjian-Thomas construction was intended for two-body systems, the construction gives a dynamical representation of the Poincaré group for any number of particles. However,

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while this construction satisfies exact Poincaré invariance, the Poincaré generators do not satisfy cluster properties for systems of more than two particles.

Sokolov (5) introduced an inductive construction that starts with Bakamjian-Thomas two-body models and builds a many-body unitary representation of the Poincaré group consistent with cluster properties. Sokolov's construction can be formulated to preserve the light-front (4) kinematic subgroup. The key elements in the Sokolov construction are unitary transformations that map tensor products of the Poincaré group into S-matrix equivalent light-front Bakamjian-Thomas representations. These transformations preserve the S matrix but they do not preserve cluster properties.

In Sokolov's construction many-body Poincaré generators that satisfy cluster properties are expressed as functions of several of these unitary transformations and Bakamjian-Thomas interactions. The Bakamjian-Thomas generators are recovered in the limit that all of these unitary transformations become the identity. For systems of four or more particles this limit does not preserve the S-matrix.

Because of its complexity, there have been no dynamical few-body calculations that utilize the full Sokolov construction. On the other hand there have been many calculations based on the Bakamjian-Thomas construction, some involving more than two particles. The size of the unitary transformations discussed in the previous paragraph relative to the identity provides one estimate on the size of the terms needed to restore cluster properties in Bakamjian-Thomas models.

2 Model and results

In this work we use a simple model to investigate how close these unitary transformations are to the identity. We consider a system where an electron scatters off of a proton in the presence of a bound state of a proton and neutron (a deuteron). We assume that the deuteron does not interact with the struck proton or electron. We treat the three-nucleon system in two ways. We first represent the three-body dynamics by a unitary representation of the Poincaré group that is a tensor product of a one-body representation with an interacting two-body representation. This representation satisfies cluster properties by construction. We also consider a S-matrix equivalent Bakamjian-Thomas representation of the 2+1 system. This representation has a non-interacting light-front spin. The interactions are constructed to give identical S matrices. This is done by taking the same internal two-body interaction and multiplying by different delta functions that ensure that the interaction in the three-particle Hilbert space either commutes with the spectator unitary representation of the Poincaré group (tensor product representation) or the non-interacting spin of the three-body system (Bakamjian-Thomas representation). Both models give the same S-matrix because the internal two-body interactions are identical. To study systems with nuclear physics scales we choose the model two-body mass Casimir operator to be $M_0^2 + 4mV$ (6)(7) where V is a Malfliet-Tjon (8) nucleon-nucleon interaction, M_0 is the non-interacting two-body invariant mass, and m is the nucleon mass. This interaction is a sum of an attractive and short-range repulsive Yukawa interaction that supports a deuteron bound state. The existence of Sokolov's unitary operator A relating these two representations follows from the identity of the two S-matrices as a consequence of a theorem of Ekstein's (9).

In order to focus only on the essential features associated with the violation of cluster properties we ignore all particle spins and we replace the four-vector current that interacts with the electron current in the one-photon-exchange-approximation with a scalar current. The interaction with the electron makes this a four-body problem. The relevant feature is that the scalar current is evaluated between three-nucleon states with different total energy and momenta.

The Bakamjian-Thomas and tensor product initial and final states are related by

$$A|d, p_3, p_{12}\rangle_{bt} = |d, p_3, p_{12}\rangle_{tp}, \tag{1}$$

where A is the Sokolov operator, bt stands for the Bakamjain-Thomas states and tp stands for the tensor product states. For both models we calculate the quantity

$$F_x(p'_3, p_3, p_{12}) := \int {}_x\langle d, p'_3, p'_{12}|J(0)|d, p_3, p_{12}\rangle_x dp'_{12} \tag{2}$$

where $x \in \{bt, tp\}$. We use the same variables in both models, even though the variables used in eq. (2) are not the most natural choice in the Bakamjian-Thomas model.

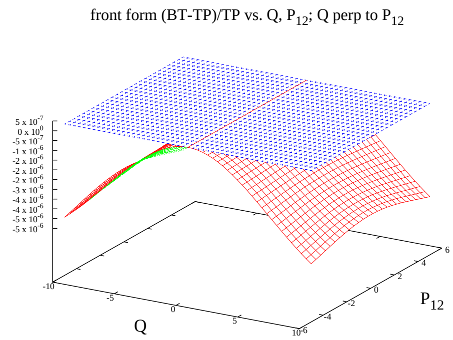

Fig. 1 Front form - $p_{12} \perp Q$

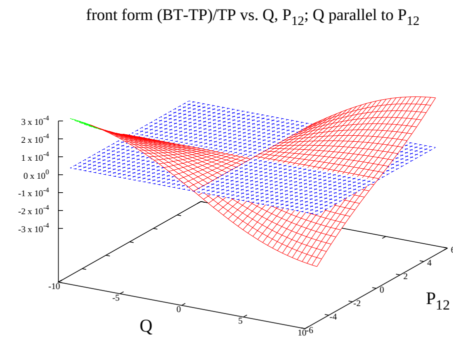

Fig. 2 Front form - $p_{12} \| Q$

In the tensor product model $F_{tp}(\cdots)$ is independent of p_{12} and only depends on the momentum transfer $Q = p_3' - p_3$. This is the expected behavior. In the Bakamjian-Thomas model we find a non-trivial dependence on p_{12}. Because of equation (1) we can express the correct tensor product result in terms of the Bakamjian-Thomas states and the unitary operators A:

$$F_{tp}(p_3', p_3, p_{12}) := \int {}_{bt}\langle d, p_3', p_{12}' | A^\dagger J(0) A | d, p_3, p_{12} \rangle_{bt} dp_{12}'. \tag{3}$$

In the limit $A \to I$ this becomes the Bakamjian-Thomas result. In figure's 1 and 2 we plot the difference $(F_{tb} - F_{bt})/F_{tp}$ as a function of $Q = P' - P$ and p_{12}. We consider frame where the + component of the momentum transfer is zero, which is always possible for spacelike momentum transfers. We choose the light front $z + t = 0$, assume that the momentum transfer Q is in the x direction and investigate the dependence on p_{12} in the x (parallel) or y (perp) directions.

The results are show in figures 1 and 2. In both cases the figures exhibit a non-trivial (and unphysical) dependence on p_{12} in the Bakamjain-Thomas case. However, for this problem, which uses parameters that have scales expected in nuclear physics models with meson-exchange interactions, the size of the corrections needed to restore cluster properties is too small to be measured in laboratory experiments. This investigation suggests that it is reasonable to construct light-front quantum mechanical models of few-nucleon systems using only the Bakamjian-Thomas representation of the Poincaré group, without including the corrections due to the Sokolov operators.

Had we instead constructed our deuteron out of sub-nuclear degrees of freedom involving stronger binding and larger internal momenta the size of these corrections could large enough to be observable.

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