Comment on the equivalence of Bakamjian-Thomas mass operators in different forms of dynamics

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We discuss the scattering equivalence of the generalized Bakamjian-Thomas construction of dynamical representations of the Poincaré group in all of Dirac's forms of dynamics. The equivalence was established by Sokolov in the context of proving that the equivalence holds for models that satisfy cluster separability. The generalized Bakamjian Thomas construction is used in most applications, even though it only satisfies cluster properties for systems of less than four particles. Different forms of dynamics are related by unitary transformations that remove interactions from some infinitesimal generators and introduce them to other generators. These unitary transformation must be interaction dependent, because they can be applied to a non-interacting generator and produce an interacting generator. This suggests that these transformations can generate complex many-body forces when used in many-body problems. It turns out that this is not the case. In all cases of interest the result of applying the unitary scattering equivalence results in representations that have simple relations, even though the unitary transformations are dynamical. This applies to many-body models as well as models with particle production. In all cases no new many-body operators are generated by the unitary scattering equivalences relating the different forms of dynamics. This makes it clear that the various calculations used in applications that emphasize one form of the dynamics over another are equivalent. Furthermore, explicit representations of the equivalent dynamical models in any form of dynamics are easily constructed. Where differences do appear is when electromagnetic probes are treated in the one-photon exchange approximation. This approximation is different in each of Dirac's forms of dynamics.

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I. INTRODUCTION

One of the most straightforward constructions of exactly Poincaré invariant quantum mechanical models of systems of a finite number of degrees of freedom is based on a method introduced by Bakamjian and Thomas [1]. The construction can be summarized as follows. Particles are represented by irreducible representations of the Poincaré group. The model Hilbert space, which is determined by the particle content of the system, is the direct sum of tensor products of irreducible representation spaces for the Poincaré group. The kinematic (non-interacting) unitary representation of the Poincaré group, $U_0(\Lambda, a)$, on this space is the direct sum of tensor products of unitary irreducible representations of the Poincaré group. The kinematic representation of the Poincaré group is decomposed into a direct integral of irreducible representations of the Poincaré group using Poincaré group Clebsch-Gordan coefficients[2][3][4]. Wave functions in this direct integral representation are square integrable functions of the eigenvalues of (1) the Casimir operators, (m, j), of the Poincaré group (2) commuting observables, $\mathbf{v}$, that label different vectors in an irreducible subspace and (3) invariant degeneracy operators, $\mathbf{d}$, that distinguish multiple copies of the same irreducible representation. Wave functions in this representation are square integrable functions, $\psi(m, j, \mathbf{v}, \mathbf{d})$, of the eigenvalues of these operators.

The goal of the Bakamjian-Thomas construction is to add interactions to the Poincaré generators in a manner that preserves the Poincaré Lie algebra. This is non-trivial because the Hamiltonian appears on the right side of the commutator of the translation and boost generators,

$$[P^j, K^k] = i\delta_{jk} H, \tag{1.1}$$

which cannot be satisfied for an interacting H unless some combination of $\mathbf{P}$ and $\mathbf{K}$ also include interactions. The full set of commutation relations imposes additional non-linear constraints on the interactions.

Bakamjian and Thomas solve this problem by adding interactions to the mass Casimir operator, m. The allowed interactions are represented by kernels that have the form,

$$\langle (m, j), \mathbf{v}, \mathbf{d}|V|(m', j'), \mathbf{v}', \mathbf{d}'\rangle = \delta(\mathbf{v} : \mathbf{v}')\delta_{jj'}\langle m, \mathbf{d}\|V^j\|m', \mathbf{d}'\rangle \tag{1.2}$$

in the kinematic irreducible representation, where $\delta(\mathbf{v} : \mathbf{v})$ denotes a product of Dirac delta functions in the continuous variables and Kronecker delta functions in the discrete variables. If $m_d = m_d^\dagger := m + V > 0$ then m_d becomes the mass Casimir operator for a dynamical representation of the Poincaré group. The structure of the interaction and the requirement $m_d > 0$ implies that simultaneous eigenstates of m_d, j^2 and $\mathbf{v}$, denoted by $|(\lambda, j), \mathbf{v}\rangle$, are complete, and transform irreducibly with respect to a dynamical representation of the Poincaré group. Simultaneous eigenfunctions of $\{m_d, j, \mathbf{v}\}$ in the kinematic irreducible basis have the form

$$\langle (m, j), \mathbf{v}, \mathbf{d}|(\lambda', j'), \mathbf{v}'\rangle = \delta(\mathbf{v} : \mathbf{v}')\delta_{jj'}\psi_{\lambda',j'}(m, \mathbf{d}) \tag{1.3}$$

where the internal wave-function, $\psi_{\lambda',j'}(m, \mathbf{d})$, is the solution of the eigenvalue equation:

$$(\lambda - m)\psi_{\lambda',j'}(m, \mathbf{d}) = \sum\!\!\!\!\!\!\int{}' \langle m, \mathbf{d}\|V^j\|m', \mathbf{d}'\rangle dm' d\mathbf{d}' \psi_{\lambda',j'}(m', \mathbf{d}'). \tag{1.4}$$

Note that the variables $\mathbf{v}$, which define the choice of basis on each irreducible subspace, do not appear in the equation for the internal wave function, $\psi_{\lambda',j'}(m', \mathbf{d}')$. In addition, the variables $\mathbf{v}$ play no role in formulating the asymptotic conditions for scattering solutions of equation (1.4).

This means that the internal wave function $\psi_{\lambda',j'}(m, \mathbf{d})$ is independent of the choice of basis for the kinematic irreducible representation. The dynamical unitary representation of the Poincaré group on this complete set of eigenstates is

$$\langle (m, j), \mathbf{v}, \mathbf{d}|U(\Lambda, a)|(\lambda, j), \mathbf{v}'\rangle = \sum\!\!\!\!\!\!\int{}'' \langle (m, j), \mathbf{v}, \mathbf{d}|(\lambda, j), \mathbf{v}''\rangle d\mathbf{v}'' \mathcal{D}^{\lambda;j}_{\mathbf{v}'',\mathbf{v}'}[\Lambda, a] \tag{1.5}$$

where

$$\mathcal{D}^{\lambda;j}_{\mathbf{v}'',\mathbf{v}'}[\Lambda, a] := \langle (\lambda, j), \mathbf{v}''|U(\Lambda, a)|(\lambda, j), \mathbf{v}'\rangle \tag{1.6}$$

is the Poincaré group Wigner function, which is the known mass λ spin j irreducible representation of the Poincaré group in the basis $\{|(\lambda, j), \mathbf{v}'\rangle\}$, which we call the "$\mathbf{v}$-basis". The Wigner function is dynamical because it depends on the mass eigenvalue λ, which requires solving eq. (1.4).

This is a short summary of the Bakamjian-Thomas construction. This construction gives an explicit representation of finite Poincaré transformations. Dynamical generators can be constructed by differentiating with respect to the group parameters. Bakamjian and Thomas actually construct the generators, but they are difficult to exponentiate, while the finite transformations discussed above can be used directly in applications.

The Bakamjian-Thomas construction is not limited to two-particle or fixed number of particle systems. In more complex systems the interaction is a sum of interactions that may be more naturally expressed in bases with the same $\mathbf{v}$ but different degeneracy parameters. For example, in the three-body problem it is natural to construct three-body kinematic irreducible representation using successive pairwise coupling. Different orders of pairwise coupling lead to irreducible representations with the same overall $\mathbf{v}$ but different choices of degeneracy parameters, $\mathbf{d}$. For example, interactions involving the $i - j$ pair of particles are most naturally described in a representation where the $i - j$ pair are coupled first.

Because the degeneracy parameters are kinematically invariant, the coefficients of the transformation that relates bases with degeneracy parameters $\mathbf{d}_b$ to bases with degeneracy parameters $\mathbf{d}_a$ necessarily have the form

$$\langle (m, j), \mathbf{v}, \mathbf{d}_a|(m', j'), \mathbf{v}', \mathbf{d}'_b\rangle = \delta(\mathbf{v} : \mathbf{v}')\delta_{jj'}\delta(m : m')R^{jm}(\mathbf{d}_a, \mathbf{d}'_b). \tag{1.7}$$

The coefficients $R^{jm}(\mathbf{d}_a, \mathbf{d}'_b)$ of the unitary operator that transforms invariant degeneracy parameters are Racah coefficients for the Poincaré group. The important observation is that these coefficients commute with and are independent of the variables, $\mathbf{v}$.

In the general case the interaction kernel, (1.2), has the form

$$\langle (m, j), \mathbf{v}, \mathbf{d}|V|(m', j'), \mathbf{v}', \mathbf{d}'\rangle =$$

$$= \delta(\mathbf{v} : \mathbf{v}')\delta_{jj'} \sum\!\!\!\!\!\!\int R^{jm}(\mathbf{d}, \mathbf{d}_b) d\mathbf{d}_b \langle m, \mathbf{d}_b\|V^j_b\|m', \mathbf{d}'_b\rangle d\mathbf{d}'_b R^{jm}(\mathbf{d}'_b, \mathbf{d}'). \tag{1.8}$$

The relevant observation is that in general the interaction still has the form (1.2) with

$$\langle m, \mathbf{d}\|V^j\|m', \mathbf{d}'\rangle = \sum\!\!\!\!\!\!\int R^{jm}(\mathbf{d}, \mathbf{d}_b) d\mathbf{d}_b \langle m, \mathbf{d}_b\|V^j_b\|m', \mathbf{d}'_b\rangle d\mathbf{d}'_b R^{jm}(\mathbf{d}'_b, \mathbf{d}'). \tag{1.9}$$

To make the connection with Dirac's forms of dynamics note that for some choice of bases, $|(m, j)\mathbf{v}, \mathbf{d}\rangle$, the Poincaré group Wigner function $\mathcal{D}^{\lambda;j}_{\mathbf{v}'',\mathbf{v}'}[\Lambda, a]$ is independent of the mass λ when (Λ, a) is restricted to a subgroup of the Poincaré group. The kinematic subgroup only depends on the choice of basis, $\mathbf{v}$. This is because the Poincaré group Wigner function does not depend on the degeneracy parameters, $\mathbf{d}$. This subgroup is called the kinematic subgroup associated with the basis $\mathbf{v}$. Dirac identified the three largest kinematic subgroups, which are the three-dimensional Euclidean group (instant-form dynamics), the Lorentz group (point-form dynamics), and the subgroup that leaves a plane tangent to the light-cone invariant (front-form dynamics). In our presentation, each kinematic subgroup is uniquely associated with a preferred basis for irreducible subspaces. This characterization exists even in the absence of interactions.

The natural bases for the irreducible subspaces associated with Dirac's [5] forms of dynamics are simultaneous eigenstates of

Table 1.

form	vector variables
instant form: $\mathbf{v} \to$	$(\mathbf{p}, \mathbf{j}_c \cdot \hat{\mathbf{z}})$
point form: $\mathbf{v} \to$	$(\mathbf{u} := \mathbf{p}/m, \mathbf{j}_c \cdot \hat{\mathbf{z}})$
front form: $\mathbf{v} \to$	$(p^+ := p^0 + p^3, p^1, p^2, \mathbf{j}_f \cdot \hat{\mathbf{z}})$

where the $\mathbf{p}$ are momentum operators, p^0 is the Hamiltonian and $\mathbf{j}_x$ are different spin operators, which are related by momentum-dependent rotations[6].

The connection with Dirac's notion of kinematic subgroup is that when (Λ, a) is an element of the kinematic subgroup then $U(\Lambda, a)$ can either act to the right on the parameters or to the left on the arguments of the wave function:

$$\langle (m,j)\mathbf{v}, \mathbf{d}| U[\Lambda, a] |(\lambda, j)\mathbf{v}'\rangle$$

$$= \sum\!\!\!\!\!\!\!\int{}'' \mathcal{D}^{m,j}_{\mathbf{v},\mathbf{v}''}[\Lambda, a] d\mathbf{v}'' \langle (m,j)\mathbf{v}'', \mathbf{d}|(\lambda, j)\mathbf{v}'\rangle = \sum\!\!\!\!\!\!\!\int{}'' \langle (m,j)\mathbf{v}, \mathbf{d}|(\lambda, j)\mathbf{v}''\rangle d\mathbf{v}'' \mathcal{D}^{\lambda,j}_{\mathbf{v}'',\mathbf{v}'}[\Lambda, a]. \tag{1.10}$$

In this case $\mathcal{D}^{\lambda,j}_{\mathbf{v}'',\mathbf{v}'}[\Lambda, a] = \mathcal{D}^{m,j}_{\mathbf{v}'',\mathbf{v}'}[\Lambda, a]$ because the Wigner functions are independent of m or λ for (Λ, a) kinematic.

Thus, while the computation of a general Poincaré transformation,

$$\langle (m,j)\mathbf{v}, \mathbf{d}| U[\Lambda, a] |\Psi\rangle = \sum\!\!\!\!\!\!\!\int{}' \psi_{\lambda,j}(m, \mathbf{d}) \mathcal{D}^{\lambda,j}_{\mathbf{v},\mathbf{v}'}[\Lambda, a] \psi^*_{\lambda,j}(m', \mathbf{d}') dm' d\mathbf{v}' d\mathbf{d}' \langle (m', j)\mathbf{v}', \mathbf{d}'|\Psi\rangle \tag{1.11}$$

requires solutions of the eigenvalue problem (1.4), for (Λ, a) in the kinematic subgroup, we get an equivalent, but simpler result that does not require solutions of (1.4):

$$\langle (m,j)\mathbf{v}, \mathbf{d}| U[\Lambda, a] |\Psi\rangle = \sum\!\!\!\!\!\!\!\int d\mathbf{v}' \mathcal{D}^{m,j}_{\mathbf{v},\mathbf{v}'}[\Lambda, a] \langle (m', j)\mathbf{v}', \mathbf{d}'|\Psi\rangle. \tag{1.12}$$

II. THE EQUIVALENCE

We call two theories scattering equivalent if (1) the unitary representations of the Poincaré group are related by a unitary transformation and (2) both theories have the same S-matrix elements. Note the that the first condition does not imply the second. Two-body models with different repulsive potentials are unitarily equivalent, but they do not necessarily have identical phase shifts.

We consider a model Hilbert space defined by finite direct sums of tensor products of irreducible representations. We consider two different single particle bases: $|(m,j)\mathbf{v}_a\rangle$ and $|(m,j)\mathbf{v}_b\rangle$. These could be any pair of bases from the table above or more generally the $\mathbf{v}$ could be any set of observables that label vectors in an irreducible subspace. In all cases these kinematic bases are related by a matrix for the form

$$\langle (m,j)\mathbf{v}_a | (m', j')\mathbf{v}'_b\rangle = \delta_{mm'} \delta_{jj'} A^{mj}(\mathbf{v}_a; \mathbf{v}'_b) \tag{2.1}$$

The kinematic irreducible bases are constructed out of direct sums of tensor products of single-particle irreducible representations. For our purposes it is enough to consider successive pairwise coupling. The coefficients of the unitary transformation relating tensor products in the a (resp b) basis to irreducible representation in the a (resp b) basis are Clebsch-Gordan coefficients of the Poincaré group:

$$\langle (m_1, j_1), \mathbf{v}_{a1}, (m_2, j_2)\mathbf{v}_{a2} | (m, j), \mathbf{v}_a, \mathbf{d} \rangle.$$

These coefficients have the intertwining property

$$\int \langle (m_1, j_1), \mathbf{v}_{a1}, (m_2, j_2)\mathbf{v}_{a2} | (m', j'), \mathbf{v}''_a, \mathbf{d}'' \rangle d\mathbf{v}'' \mathcal{D}^{m',j'}_{\mathbf{v}'',\mathbf{v}'}[\Lambda, a] =$$

$$\int'' \prod \mathcal{D}^{m_1,j_1}_{\mathbf{v}_{a1},\mathbf{v}''_{a1}}[\Lambda, a] \mathcal{D}^{m_2,j_2}_{\mathbf{v}_{a2},\mathbf{v}''_{a2}}[\Lambda, a] d\mathbf{v}''_{a1} d\mathbf{v}''_{a2} \langle (m_1, j_1), \mathbf{v}''_{a1}, (m_2, j_1)\mathbf{v}''_{a2} | (m, j), \mathbf{v}', \mathbf{d} \rangle. \tag{2.2}$$

Depending on details of the construction of the Clebsch-Gordan coefficients there are different possible choices of degeneracy quantum numbers, $\mathbf{d}$.

Keeping $\mathbf{d}$ fixed we can construct a Clebsch-Gordan coefficient in the b basis using

$$\langle (m_1, j_1), \mathbf{v}_{b1}, (m_2, j_2)\mathbf{v}_{b2} | (m, j), \mathbf{v}_b, \mathbf{d} \rangle$$

$$= \sum\!\!\!\!\!\!\int' d\mathbf{v}'_{a1} d\mathbf{v}'_{a2} d\mathbf{v}'_a A^{m_1 j_1}(\mathbf{v}_{b1}; \mathbf{v}'_{a1}) A^{m_2 j_2}(\mathbf{v}_{b2}; \mathbf{v}'_{a2}) \langle (m_1, j_1), \mathbf{v}'_{a1}, (m_2, j_2)\mathbf{v}'_{a2} | (m, j), \mathbf{v}'_a, \mathbf{d} \rangle A^{mj}(\mathbf{v}'_a; \mathbf{v}_b). \tag{2.3}$$

What is relevant is that this is a Clebsch Gordan coefficient in the b basis with the same degeneracy parameters as the original one in the a basis.

We can continue successively pairwise coupling until the entire Hilbert space is represented by a direct integral of irreducible representations in the a or b basis with *identical degeneracy parameters* $\mathbf{d}$. We write these bases as

$$|(m, j)\mathbf{v}_a, \mathbf{d}\rangle \qquad |(m, j)\mathbf{v}_b, \mathbf{d}\rangle. \tag{2.4}$$

What Sokolov established [7] was that the Bakamjian Thomas construction using the interactions

$$\langle (m, j), \mathbf{v}_a, \mathbf{d} | V_a | (m', j'), \mathbf{v}'_b, \mathbf{d}' \rangle = \delta(\mathbf{v}_a : \mathbf{v}'_a) \delta_{jj'} \langle m, \mathbf{d} \| V^j \| m', \mathbf{d}' \rangle \tag{2.5}$$

and

$$\langle (m, j), \mathbf{v}_b, \mathbf{d} | V_b | (m', j'), \mathbf{v}'_b, \mathbf{d}' \rangle = \delta(\mathbf{v}_b : \mathbf{v}'_b) \delta_{jj'} \langle m, \mathbf{d} \| V^j \| m', \mathbf{d}' \rangle \tag{2.6}$$

are scattering equivalent. At first glance it looks like V_a and V_b are related by a simple variable change. This is not the case because V_a and V_b commute with different kinematic subgroups. This property cannot be changed by a change of variables.

It is apparent from equations (2.5-2.6) that both representations have identical internal wave functions. The relevant unitary transformation is

$$\sum\!\!\!\!\!\!\int_{\lambda j} A^{\lambda j}(\mathbf{v}_a; \mathbf{v}_b) |\psi_{\lambda,j}\rangle \langle \psi_{\lambda,j}| \tag{2.7}$$

where this involves a sum over the eigenvalues λ of the internal mass operator.

To establish that both models give the same S matrices we first observe that the structure of the interactions implies that the scattering matrices in both representations are

$$\langle (m, j), \mathbf{v}_a, \mathbf{d} | S_a | (m', j'), \mathbf{v}'_a, \mathbf{d}' \rangle = \delta(\mathbf{v}_a, \mathbf{v}'_a) \delta_{jj'} \delta(m, m') \langle \mathbf{d} \| S^{mj} \| \mathbf{d}' \rangle \tag{2.8}$$

and

$$\langle (m, j), \mathbf{v}_b, \mathbf{d} | S_b | (m', j'), \mathbf{v}'_b, \mathbf{d}' \rangle = \delta(\mathbf{v}_b, \mathbf{v}'_b) \delta_{jj'} \delta(m, m') \langle \mathbf{d} \| S^{mj} \| \mathbf{d}' \rangle. \tag{2.9}$$

where the reduced S-matrices $\langle \mathbf{d} \| S^{mj} \| \mathbf{d}' \rangle$ are identical.

If we change variables in the first of these equations we get

$$\langle (m, j), \mathbf{v}_b, \mathbf{d} | S_a | (m', j'), \mathbf{v}'_b, \mathbf{d}' \rangle =$$

$$\sum\!\!\!\!\!\!\!\int A^{mj}(\mathbf{v}_b;\mathbf{v}_a)d\mathbf{v}_a\langle(m,j),\mathbf{v}_a,\mathbf{d}|S_a|\langle(m',j'),\mathbf{v}'_a,\mathbf{d}'\rangle d\mathbf{v}'_a A^{m'j'}(\mathbf{v}'_a;\mathbf{v}'_b) =$$

$$\sum\!\!\!\!\!\!\!\int A^{mj}(\mathbf{v}_b;\mathbf{v}_a)d\mathbf{v}_a\delta(\mathbf{v}_a,\mathbf{v}'_a)\delta_{jj'}\delta(m,m')d\mathbf{v}'_a A^{m'j'}(\mathbf{v}'_a;\mathbf{v}'_b)\langle\mathbf{d}\|S^{mj}\|\mathbf{d}'\rangle =$$

$$\delta(\mathbf{v}_b,\mathbf{v}'_b)\delta_{jj'}\delta(m,m')\langle\mathbf{d}\|S^{mj}\|\mathbf{d}'\rangle =$$

$$\langle(m,j),\mathbf{v}_b,\mathbf{d}|S_b|(m',j'),\mathbf{v}'_a,\mathbf{d}'\rangle \tag{2.10}$$

which proves the equivalence.

III. SUMMARY

To summarize, Bakamjian-Thomas constructions of dynamical representations of the Poincaré group have the general form

$$U_b(\Lambda,a)|(\lambda,j),\mathbf{v}_b\rangle = \sum\!\!\!\!\!\!\!\int d\mathbf{v}'|(\lambda,j),\mathbf{v}'_b\rangle\mathcal{D}^{\lambda',j}_{\mathbf{v}'_b,\mathbf{v}_b}[\Lambda,a] \tag{3.1}$$

where

$$\langle(m,j),\mathbf{v}_b|(\lambda,j'),\mathbf{v}'_b\rangle = \delta(\mathbf{v}_b,\mathbf{v}'_b)\psi_{\lambda,j}(m,\mathbf{d}) \tag{3.2}$$

and $\psi_{\lambda,j}(m,\mathbf{d})$ is the solution of the mass eigenvalue equation:

$$(\lambda-m)\psi_{\lambda',j'}(m,\mathbf{d}) = \sum\!\!\!\!\!\!\!\int{}' \langle m,\mathbf{d}\|V^j\|m',\mathbf{d}'\rangle dm' d\mathbf{d}'\psi_{\lambda',j'}(m',\mathbf{d}') \tag{3.3}$$

which is identical in all forms of dynamics. Equivalent models with different kinematic symmetries differ only in the choice of the variables $\mathbf{v}_b$ in equations (3.1-3.2). While different choices of $\mathbf{v}_b$ lead to different interactions (1.2) with different kinematic symmetries, the resulting dynamical models are all equivalent.

Irreducible vectors in the different forms of dynamics are related by

$$|(\lambda,j),\mathbf{v}_b\rangle = \sum\!\!\!\!\!\!\!\int{}' |(\lambda,j),\mathbf{v}'_c\rangle d\mathbf{v}'_c A^{\lambda j}(\mathbf{v}'_c;\mathbf{v}'_b) \tag{3.4}$$

and the Wigner functions in different representations are related by

$$\mathcal{D}^{\lambda,j}_{\mathbf{v}'_c,\mathbf{v}_c}[\Lambda,a] = \sum\!\!\!\!\!\!\!\int d\mathbf{v}_b d\mathbf{v}'_b A^{\lambda j}(\mathbf{v}'_c;\mathbf{v}'_b)\mathcal{D}^{\lambda,j}_{\mathbf{v}'_b,\mathbf{v}_b}[\Lambda,a]A^{\lambda j}(\mathbf{v}_{b'};\mathbf{v}_c). \tag{3.5}$$

The transformation relating the different kinematic subgroups are dynamical because the mass eigenvalues λ that appear in both $A^{\lambda j}(\mathbf{v}_b;\mathbf{v}_c)$ and $\mathcal{D}^{\lambda,j}_{\mathbf{v}'_b,\mathbf{v}_b}[\Lambda,a]$ are determined by solving the dynamical equation. The important observation is that the physical observables (binding energies, S-matrix elements) are obtained by solving (3.3) which is independent of the choice of kinematic subgroup.

The conclusion of this work is that Poincaré invariant quantum models should be considered as being defined without reference to any specific kinematic subgroup, and any Poincaré invariant model can be transformed to a representation that exhibits any kinematic symmetry. This conclusion is not limited to two-body models or models that conserve particle number - nor is it limited to the maximal kinematic subgroups discussed by Dirac. The non-trivial dynamical equation that must be solved is the same in all cases. The different choices of representation have no effect on bound state or scattering observables.

The one class of applications where using different forms of dynamics has dynamical consequences is when they are used in the one photon-exchange approximation. This is because the initial and final hadronic states are in different frames, and have different invariant masses. The equivalence proof breaks down when $m \neq m'$. While the equivalence can be recovered by transforming the impulse current in one representation to another representation, the transformed current will generally have many-body contributions.

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