Consider the matrix

\[ M := \begin{pmatrix} 1 & -i & 0 \\ i & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

1. Find the characteristic polynomial of \( M \)
2. Find the eigenvalues of \( M \)
3. Find the polynomials \( \phi_i(\lambda) \)
4. Calculate \( \phi_i(M) \).
5. Let \( |v\rangle \) be any vector such that \( \phi_i(M)|v\rangle \neq 0 \). Show that

\[ M\phi_i(M)|v\rangle = \lambda_i \phi_i(M)|v\rangle \]

where \( \lambda_i \) is the \( i \)-th eigenvalue.
6. Calculate \( \sin(M) \)
7. Show that \( \sum_i \phi_i(M) = I \)
8. Find a similarity transform that diagonalizes \( M \)
9. Find \( M^{-1} \)