1. The commutator and anti-commutator of two linear operators are defined by

\[ [A, B] := AB - BA \quad \{A, B\} := AB - BA \]

Prove the following identities

\[ [A[B, C]] + [B[C, A]] + [C[A, B]] = 0 \]


\[ [A, BC] = \{A, B\}C - B\{A, C\} \]

2. Let \( K \) be a linear Hermitian operator. Define

\[ W := (I + iK)(I - iK)^{-1} \]

Show that \( W \) is a unitary operator.

Express \( K \) in terms of \( W \). (\( K \) is called the Cayley transform of \( W \))

3. Let \( P \) be an orthogonal projection operator. Let \( Q := I - P \).

Show that \( Q \) is an orthogonal projection operator.

Evaluate \( QP \).

4. A linear operator \( N \) is Nilpotent if for some finite \( n \), \( N^n = 0 \). Show that \( e^N \) is a finite degree polynomial in \( N \) if \( N \) is nilpotent. Show that \( e^{\alpha N}e^{\beta N} = e^{(\alpha + \beta)N} \) still holds when \( N \) is nilpotent.

5. Let \( A \) be a bounded linear operator on a normed linear space. Define the partial sums

\[ F_n(A) = I + \sum_{m=1}^{n} \frac{1}{m!}A^m \]

Show that this is a Cauchy sequence of operators.

6. Show that if \([A, B] = 0\) that

\[ exp(A + B) = exp(A)exp(B) = exp(B)exp(A) \]

What happens to these relations if \([A, B] = \alpha I\) where \( \alpha \) is complex and \( I \) is the identity operator?

7. Let \( P \) be a positive operator. Prove the generalized Cauchy Schwartz inequality:

\[ |\langle a|P|b\rangle|^2 \leq \langle a|P|a\rangle \langle b|P|b\rangle \]