1. Consider the function \( f(x) = (1 - x^2) \) on the interval \([-1, 1]\). Calculate a 4-th degree Weierstrass polynomial approximation to this function. Compare the exact and approximate functions.

2. Use the Rodrigues formula to calculate the first four Hermite polynomials.

3. Use the Gram Schmid method to calculate the first three orthogonal polynomials on \([-1, 1]\) with weight \( w(x) = 1 \). Compare these to first three Legendre polynomials generated.

4. Assume that \( |f(x) - p_n(x)| < \epsilon \) for all \( x \in [a, b] \), where \( p_n(x) \) is a polynomial. Let \( M \) be a 2 \times 2 Hermitian matrix with real eigenvalues \( \lambda \) satisfying \( a < \lambda < b \). Show that

\[
\| (f(M) - p_n(M)) \| < \epsilon
\]

where \( \| O \| \) is the matrix or operator norm of \( O \).