1.) Fourier Series: In class I developed the Fourier series for periodic functions on the interval $[-\pi, \pi]$. I constructed the orthonormal basis functions

$$\langle \theta | n \rangle = \frac{1}{\sqrt{2\pi}} e^{in\theta}$$

a. Find the corresponding orthonormal basis functions for periodic functions on the interval $[0, L]$?

b. If $f(\theta)$ is a real valued periodic function, how can you recognize that it is a real functions by looking at the expansion coefficients:

$$f(\theta) = \sum_n f_n \langle \theta | n \rangle$$

2.) Calculate $\int_0^\infty \frac{\sin(ax)}{x} dx$ for $a > 0$.

3.) Calculate the Fourier Transform of $e^{-ax^2}$

4.) Let $f(x)$ and $g(x)$ have Fourier Transforms.

$$\hat{f}(k) = \frac{1}{\sqrt{2\pi}} \int dy e^{i ky} f(y) \quad \hat{g}(k) = \frac{1}{\sqrt{2\pi}} \int dy e^{i ky} g(y)$$

Find an expression for the Fourier transform of the product $f(x)g(x)$ in terms of their individual Fourier transforms, $\hat{f}(k)$ and $\hat{g}(k)$,

5.) Show

$$\lim_{\lambda \to \infty} \int_{-\infty}^{\infty} e^{i\lambda x} f(x) dx = 0$$

if $f$ is absolutely integrable and differentiable for every $x$.

6.) Let $f(\theta)$ be 1 for $0 < \theta < \pi$ and -1 for $-\pi < \theta < 0$. Find the coefficients $f_n$ in the Fourier series

$$f(\theta) = \sum_n f_n \langle \theta | n \rangle$$