1.) Show that
\[ \int \theta(x)\delta(x)f(x)dx \]
does not make sense on the space of Schwartz functions.

2. Let \( A \) be a bounded operator with the property that \( A^2 \) is compact. Under what conditions does
\[ (I - A)^{-1} \]
exist. Hint use the Fredholm alternative.

3. Show that if \( A \) is compact that \( A^\dagger \) is also compact.

4. Let \( \{ |\chi_n\rangle \}_{n=1}^\infty \) be any orthonormal basis and let \( K \) be compact. Consider the finite rank approximations
\[ K_N = \sum_{m,n=1}^{N} |\chi_n\rangle\langle\chi_n|K|\chi_m\rangle\langle\chi_m| \]
Show that
\[ \| |K - K_N| \| \]
can be made as small as desired by choosing a large enough \( N \).

5. Show that the integral operator \( K \) defined by
\[ \langle x|K|f \rangle = \int_{-\infty}^{\infty} \langle x|K|y \rangle dy \langle y|f \rangle \]
with
\[ \langle x|K|y \rangle = e^{-x^2-y^2} \]
is compact.

6. Show that the linear operator \( \frac{d}{dx} \) is not bounded on the space of square integrable functions on the real line.